



# Spin dependent spectral functions of ${}^3\text{He}$

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# Why? 12 GeV Experiments @JLab, with $^3\text{He}$

## ● DIS regime, e.g.

Hall A, <http://halloweb.jlab.org/12GeV/>

**MARATHON Coll. E12-10-103 (Rating A):** Measurement of the  $F_{2n}/F_{2p}$ , d/u Ratios and A=3 EMC Effect in Deep Inelastic Electron Scattering Off the Tritium and Helium Mirror Nuclei

Hall C, <https://www.jlab.org/Hall-C/>

**J. Arrington, et al PR12-10-008 (Rating A<sup>-</sup>):** Detailed studies of the nuclear dependence of  $F_2$  in light nuclei

## ● SIDIS regime, e.g.

Hall A, <http://halloweb.jlab.org/12GeV/>

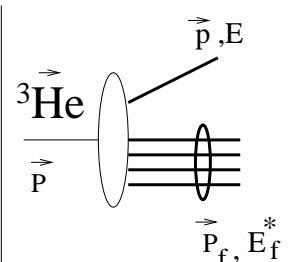
**H. Gao et al, PR12-09-014 (Rating A):** Target Single Spin Asymmetry in Semi-Inclusive Deep-Inelastic ( $e, e'\pi^\pm$ ) Reaction on a Transversely Polarized  $^3\text{He}$  Target

**J.P. Chen et al, PR12-11-007 (Rating A):** Asymmetries in Semi-Inclusive Deep-Inelastic ( $e, e'\pi^\pm$ ) Reactions on a Longitudinally Polarized  $^3\text{He}$  Target

## ● Others? DVCS, spectator tagging...

In  $^3\text{He}$  conventional nuclear effects under control... Exotic ones disentangled

# The spectral function (Impulse Approximation)





$$\mathbf{P}_{\mathcal{M}\sigma\sigma}^N(\vec{p}, E) = \sum_f \left| \left\langle \begin{array}{c} \vec{p}, E \\ \text{---} \\ \vec{P} \\ \text{---} \\ \vec{P}_f, E_f^* \end{array} \right\rangle_{\text{He}} \right|^2 =$$


intrinsic overlaps

$$\sum_f \delta(E - E_{min} - E_f^*) \langle \Psi_A; J_A \mathcal{M} \pi_A | \vec{p}, \sigma; \phi_f(E_f^*) \rangle \langle \phi_f(E_f^*); \sigma \vec{p} | \pi_A J_A \mathcal{M}'; \Psi_A \rangle_{S_A}$$

- probability distribution to find a nucleon with given 3-momentum and missing energy in the nucleus. It arises in q.e., DIS, SIDIS, DVCS...
- In general, if spin is involved, a 2x2 matrix,  $\mathbf{P}_{\mathcal{M}\sigma\sigma'}^N(\vec{p}, E)$ , not a density;
- the two-body recoiling system can be either the deuteron or a scattering state: when a deeply bound nucleon, with high removal energy  $E = E_{min} + E_f^*$ , leaves the nucleus, the recoiling system is left with high excitation energy  $E_f^*$ ;
- **Realistic** Spectral Function: 3-body bound state and 2-body final state evaluated within the same **Realistic** interaction (in our case, **Av18**, from the **Pisa** group (Kievsky, Viviani)). Extension to heavier nuclei very difficult





# Status (Impulse Approximation and beyond)

	Impulse Approximation		including FSI	
	unpolarized	spin dep.	unpolarized	spin dep.
Non Relativistic	✓	✓	✓	✓
Light-Front	Def: ✓	Def: ✓		
	Calc: 	Calc: 		

Selected contributions from Rome-Perugia:

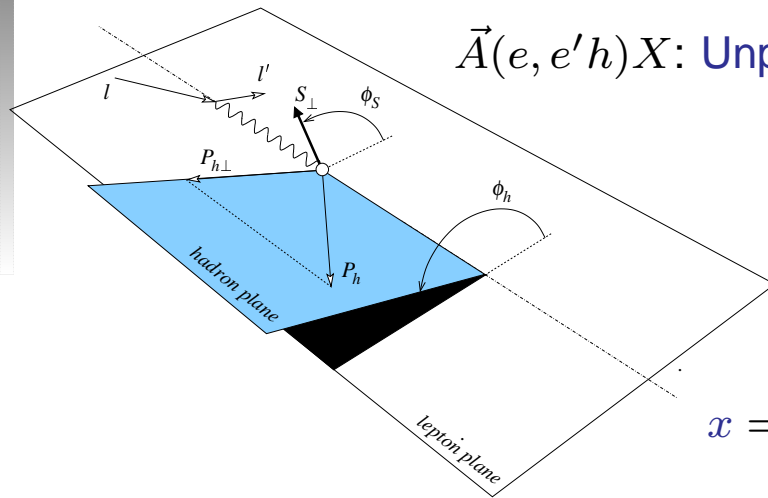
- Ciofi, Pace, Salmè PRC 21 (1980) 505 ...
- Ciofi, Pace, Salmè PRC 46 (1991) 1591: spin dependence
- Pace, Salmè, S.S., Kievsky PRC 64 (2001) 055203, first Av18 calculation
- Ciofi, Kaptari, PRC 66 (2002) 044004, unpolarized with FSI (q.e.)
- S.S. PRC 70 (2004) 015205, non diagonal SF for DVCS
- Kaptari, Del Dotto, Pace, Salmè, S.S., PRC 89 (2014), spin dependent with FSI
- LF, preliminary, see, e.g., S.S., Del Dotto, Kaptari, Pace, Rinaldi, Salmè, Few Body Syst. 56 (2015) 6, 425 and references therein

# Outline

	Impulse Approximation		including FSI	
	unpolarized	spin dep.	unpolarized	spin dep.
Non Relativistic	✓	✓	✓	✓
Light-Front	Def: ✓	Def: ✓		
	Calc: 	Calc: 		

- Extracting the **neutron** information from **SiDIS** off  ${}^3\vec{H}e$ .  
 Basic approach: Impulse Approximation in the Bjorken limit  
 ( S.S., PRD 75 (2007) 054005 )
  
- **Main topic:**
  - \* **Evaluation of Final state interactions (FSI): distorted spectral function and spectator SIDIS**  
 (L. Kaptari, A. Del Dotto, E. Pace, G. Salmè, S.S., PRC 89 (2014) 035206 )
  - \* **Evaluation of FSI: distorted spectral function and full treatment of SIDIS**  
 (A. Del Dotto, L. Kaptari, E. Pace, G. Salmè, S.S., “ready” for submission)
  
- A short Light-Front update & Conclusions

# Single Spin Asymmetries (SSAs) - 1



$\vec{A}(e, e'h)X$ : Unpolarized beam and T-polarized target  $\rightarrow \sigma_{UT}$

$$d^6\sigma \equiv \frac{d^6\sigma}{dx dy dz d\phi_S d^2 P_{h\perp}}$$

$$x = \frac{Q^2}{2P \cdot q} \quad y = \frac{P \cdot q}{P \cdot l} \quad z = \frac{P \cdot h}{P \cdot q} \quad \boxed{\hat{q} = -\hat{e}_z}$$

The number of emitted hadrons at a given  $\phi_h$  depends on the orientation of  $\vec{S}_\perp$ !  
 In SSAs 2 different mechanisms can be experimentally distinguished

$$A_{UT}^{\text{Sivers(Collins)}} = \frac{\int d\phi_S d^2 P_{h\perp} \sin(\phi_h - (+)\phi_S) d^6\sigma_{UT}}{\int d\phi_S d^2 P_{h\perp} d^6\sigma_{UU}}$$

with  $d^6\sigma_{UT} = \frac{1}{2}(d^6\sigma_{U\uparrow} - d^6\sigma_{U\downarrow})$   $d^6\sigma_{UU} = \frac{1}{2}(d^6\sigma_{U\uparrow} + d^6\sigma_{U\downarrow})$

## SSAs - 2

SSAs in terms of parton distributions and fragmentation functions:

$$A_{UT}^{Sivers} = N^{Sivers} / D \quad A_{UT}^{Collins} = N^{Collins} / D$$

$$N^{Sivers} \propto \sum_q e_q^2 \int d^2\kappa_T d^2\mathbf{k}_T \delta^2(\mathbf{k}_T + \mathbf{q}_T - \kappa_T) \frac{\hat{\mathbf{P}}_{h\perp} \cdot \mathbf{k}_T}{M} f_{1T}^{\perp q}(x, \mathbf{k}_T^2) D_1^{q,h}(z, (z\kappa_T)^2)$$

$$N^{Collins} \propto \sum_q e_q^2 \int d^2\kappa_T d^2\mathbf{k}_T \delta^2(\mathbf{k}_T + \mathbf{q}_T - \kappa_T) \frac{\hat{\mathbf{P}}_{h\perp} \cdot \kappa_T}{M_h} h_1^q(x, \mathbf{k}_T^2) H_1^{\perp q,h}(z, (z\kappa_T)^2)$$

$$D \propto \sum_q e_q^2 f_1^q(x) D_1^{q,h}(z)$$

LARGE  $A_{UT}^{Sivers}$  measured in  $\vec{p}(e, e'\pi)x$  HERMES PRL 94, 012002 (2005)

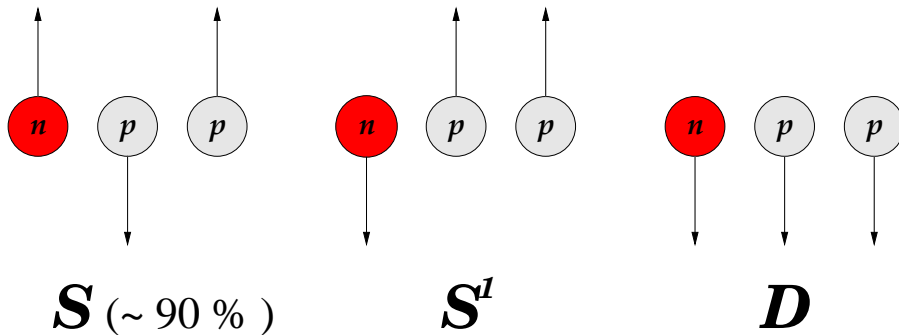
SMALL  $A_{UT}^{Sivers}$  measured in  $\vec{D}(e, e'\pi)x$ ; COMPASS PRL 94, 202002 (2005)

**A strong flavor dependence**

**Importance of the neutron for flavor decomposition!**

# The neutron information from $^3\text{He}$

$^3\text{He}$  is the ideal target to study the polarized neutron:



In  $S$ -wave  
 ${}^3\vec{H}e = \vec{n}$  !

... But the bound nucleons in  $^3\text{He}$  are moving!

Dynamical nuclear effects in inclusive DIS ( ${}^3\vec{H}e(e, e')X$ ) were evaluated with a realistic spin-dependent spectral function for  ${}^3\vec{H}e$ ,  $P_{\sigma, \sigma'}(\vec{p}, E)$ . It was found that the formula

$$A_n \simeq \frac{1}{p_n f_n} (A_3^{exp} - 2p_p f_p A_p^{exp}), \quad (\text{Ciofi degli Atti et al., PRC48(1993)R968})$$

( $f_p, f_n$  dilution factors)

can be safely used  $\longrightarrow$  widely used by experimental collaborations.

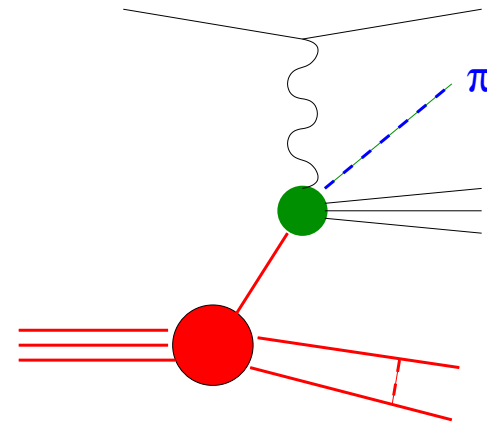
The nuclear effects are hidden in the “effective polarizations”

$$p_p = -0.023 \quad (Av18) \quad p_n = 0.878 \quad (Av18)$$



## $\vec{n}$ from ${}^3\vec{H}e$ : SIDIS case, IA

Can one use the same formula to extract the SSAs ?  
 in SiDIS also the fragmentation functions can be modified  
 by the nuclear environment !



The process  ${}^3\vec{H}e(e, e' \pi) X$  has been evaluated :  
 in the Bjorken limit

in IA → no FSI between the measured fast, ultrarelativistic  $\pi$   
 the remnant and the two nucleon recoiling system

$E_\pi \simeq 2.4 \text{ GeV}$  in JLAB exp at  $6 \text{ GeV}$  - Qian et al., PRL 107 (2011) 072003

SSAs involve convolutions of the spin-dependent nuclear spectral function,  $\vec{P}(\vec{p}, E)$ , with  
 parton distributions AND fragmentation functions [S.Scopetta, PRD 75 (2007) 054005] :

$$A \simeq \int d\vec{p} dE \dots \vec{P}(\vec{p}, E) f_{1T}^{\perp q} \left( \frac{Q^2}{2p \cdot q}, \mathbf{k}_T^2 \right) D_1^{q,h} \left( \frac{p \cdot h}{p \cdot q}, \left( \frac{p \cdot h}{p \cdot q} \kappa_T \right)^2 \right)$$

The nuclear effects on fragmentation functions are new with respect to the DIS case  
 and have to be studied carefully

# The IA @ JLab kinematics: a few words more

The convolution formulae for a generic structure function can be cast in the form

$$\mathcal{F}^A(x_{Bj}, Q^2, \dots) = \sum_N \int_{x_{Bj}}^A f_N^A(\alpha, Q^2, \dots) \mathcal{F}^N(x_{Bj}/\alpha, Q^2, \dots) d\alpha$$

with the **light-cone momentum distribution**:

$$f_N^A(\alpha, Q^2, \dots) = \int dE \int_{p_{min}(\alpha, Q^2, \dots)}^{p_{max}(\alpha, Q^2, \dots)} P_N^A(\mathbf{p}, \mathbf{E}) \delta\left(\alpha - \frac{\mathbf{p} \cdot \mathbf{q}}{m\nu}\right) \theta\left(\mathbf{W}_x^2 - (M_N + M_\pi)^2\right) d^3\mathbf{p}$$



**Bjorken limit:**

$p_{min,max}$  not dependent on  $Q^2, x$ :

$f_N^A(\alpha)$  depends on  $\alpha$  only,

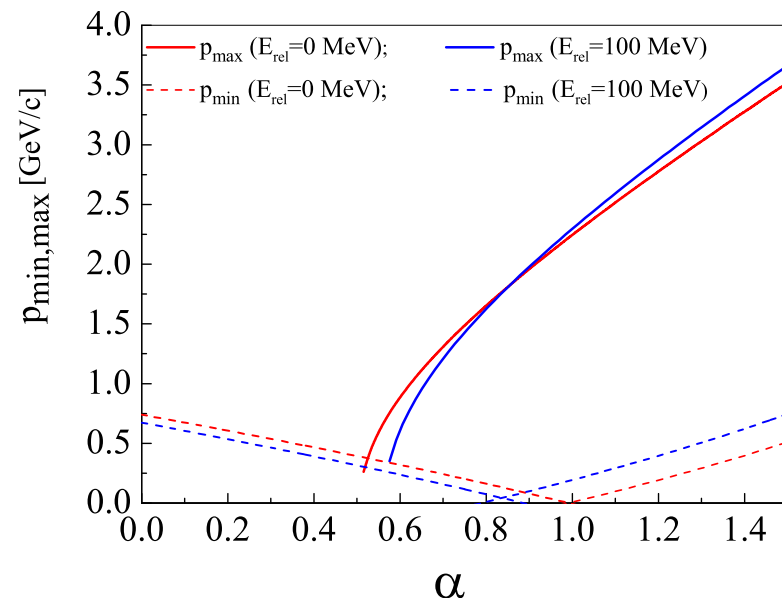
$$0 \leq \alpha \leq A$$



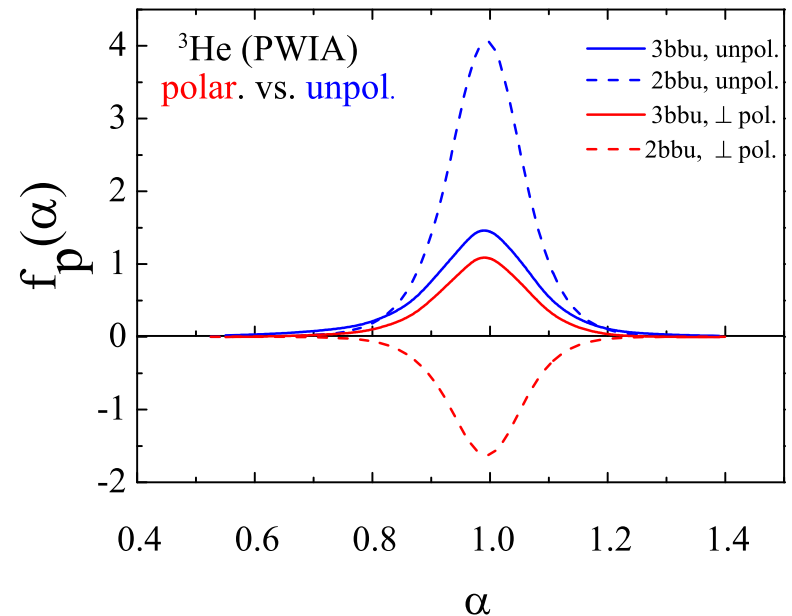
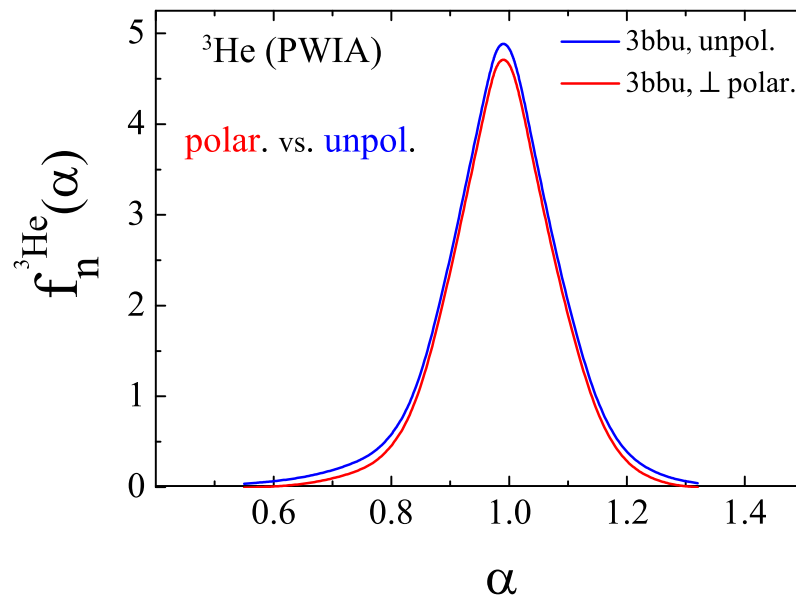
**@ JLab kinematics,**

$(E = 8.8 \text{ GeV}, E' \simeq 2 \div 3 \text{ GeV},$

$\theta_e \simeq 30^\circ) q \neq \nu$  and  $\alpha_{min} \neq 0$

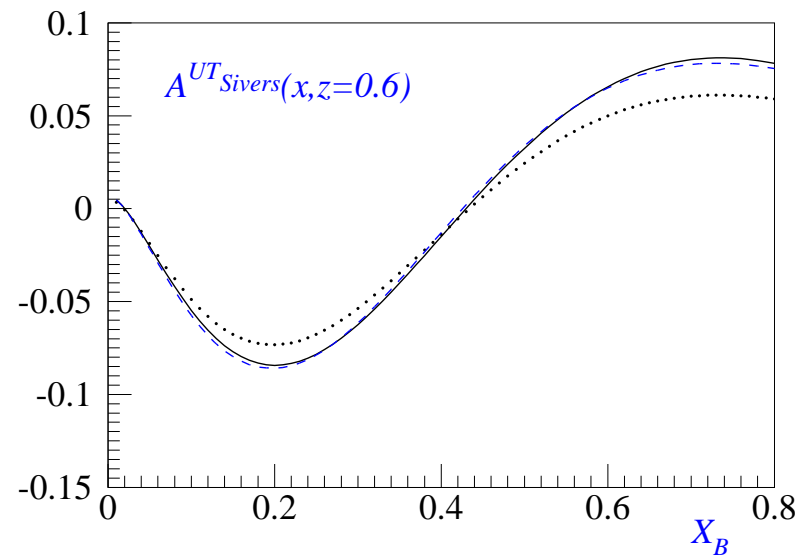
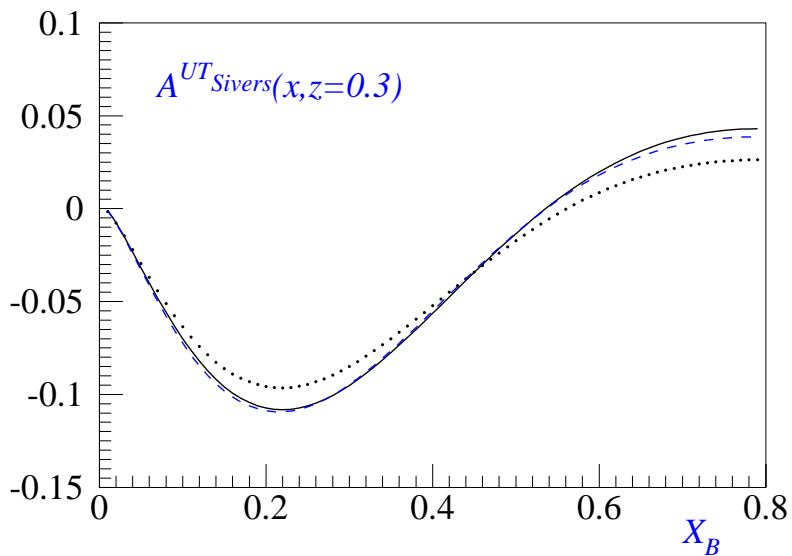


# Light-cone momentum distributions in IA



- weak depolarization of the neutron
- strong depolarization of the protons  
(cancellation between contributions in the 2-body and 3-body channels)

# Results: $\vec{n}$ from ${}^3\vec{H}e$ : $A_{UT}^{Sivers}$ , @ JLab, in IA



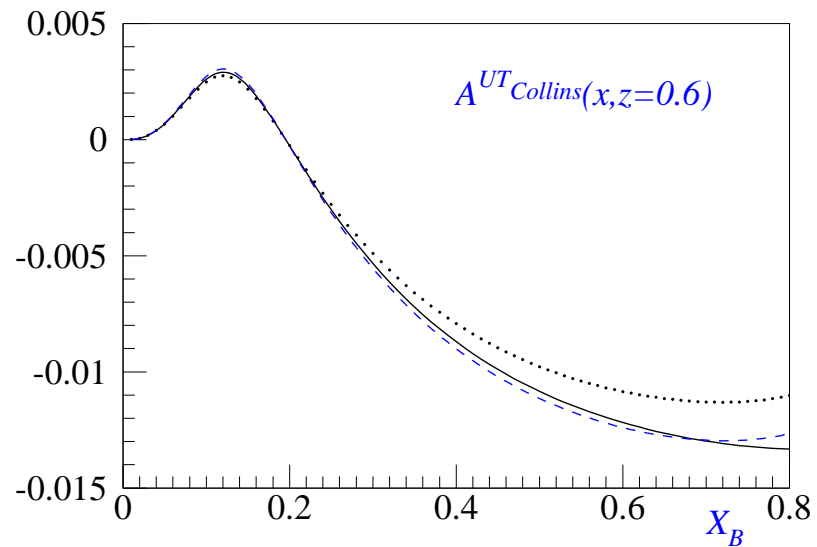
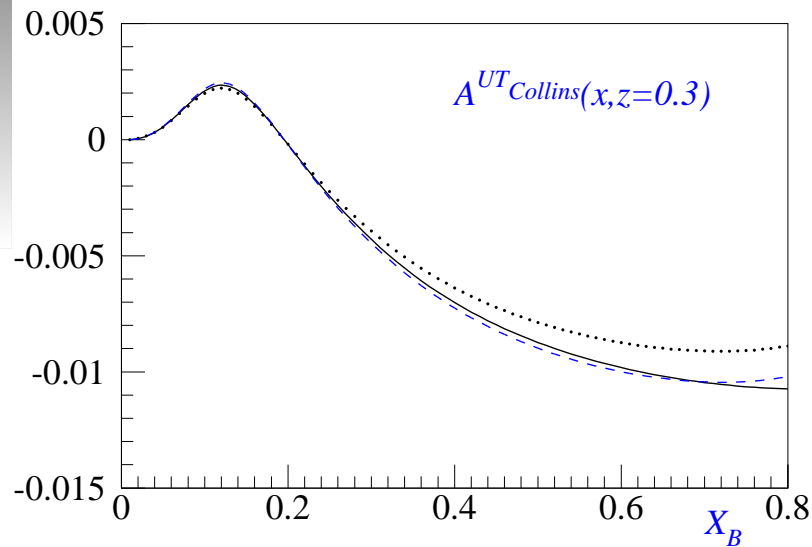
**FULL:** Neutron asymmetry (model: from parameterizations or models of TMDs and FFs)

**DOTS:** Neutron asymmetry extracted from  ${}^3He$  (calculation) neglecting the contribution of the proton polarization  $\bar{A}_n \simeq \frac{1}{f_n} A_3^{calc}$

**DASHED :** Neutron asymmetry extracted from  ${}^3He$  (calculation) taking into account nuclear structure effects through the formula:

$$A_n \simeq \frac{1}{p_n f_n} \left( A_3^{calc} - 2p_p f_p A_p^{model} \right)$$

# Results: $\vec{n}$ from ${}^3\vec{H}e$ : $A_{UT}^{Collins}$ , @ JLab



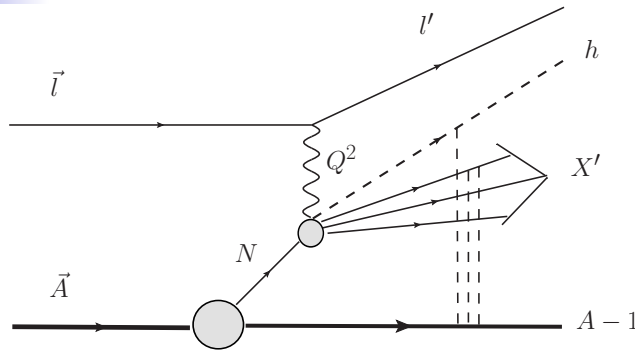
**In the Bjorken limit** the extraction procedure successful in **DIS** works also in **SiDIS**, for both the Collins and the Sivers **SSAs** !

**What about FSI effects ?**

(thinking to E12-09-018, A.G. Cates et al., approved with rate A @JLab 12)

# FSI: Generalized Eikonal Approximation (GEA)

L. Kaptari, A. Del Dotto, E. Pace, G. Salmè, S.S., PRC 89 (2014) 035206



Relative energy between  $A - 1$  and the remnants: a few GeV

→ **eikonal** approximation.

$$d\sigma \simeq l^{\mu\nu} W_{\mu\nu}^A(S_A)$$

$$W_{\mu\nu}^A(S_A) \approx \sum_{S_{A-1}, S_X} J_{\mu}^A J_{\nu}^A$$

$$J_{\mu}^A \simeq \langle S_A \mathbf{P} | \hat{\mathbf{J}}_{\mu}^A(0) | S_X, S_{A-1}, \mathbf{P}_{A-1} \mathbf{E}_{A-1}^f \rangle$$

$$\langle S_A \mathbf{P} | \mathbf{r}_1 \mathbf{r}_2 \mathbf{r}_3 \rangle = \Phi_{3\text{He}}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = \mathcal{A} e^{i\mathbf{P}\mathbf{R}} \Psi_3^{S_A}(\rho, \mathbf{r})$$

$$\langle \mathbf{r}_1 \mathbf{r}_2 \mathbf{r}_3 | S_X, S_{A-1} \mathbf{P}_{A-1} \mathbf{E}_{A-1}^f \rangle = \Phi_f^*(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) \approx \hat{S}_{GI}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) \Psi^{*f}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)$$

$\hat{S}_{GI} = \text{Glauber operator}$

$$\approx \hat{S}_{GI}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) \sum_{\mathbf{j} > \mathbf{k}} \chi_{S_X}^+ \phi^*(\xi_{\mathbf{x}}) e^{-i\mathbf{p}\mathbf{x}\mathbf{r}_i} \Psi_{\mathbf{jk}}^{*f}(\mathbf{r}_j, \mathbf{r}_k),$$

$$J_{\mu}^A \approx \int d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{r}_3 \Psi_{23}^{*f}(\mathbf{r}_2, \mathbf{r}_3) e^{-i\mathbf{p}\mathbf{x}\mathbf{r}_i} \chi_{S_X}^+ \phi^*(\xi_{\mathbf{x}}) \cdot \hat{S}_{GI}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) \hat{j}_{\mu}(\mathbf{r}_1, X) \vec{\Psi}_3^{S_A}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)$$

IF ( *FACTORIZED* FSI ! )  $\left[ \hat{S}_{GI}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3), \hat{j}_{\mu}(\mathbf{r}_1) \right] = 0$  THEN:

$$W_{\mu\nu}^A = \sum_{N, \lambda, \lambda'} \int dE d\mathbf{p} w_{\mu\nu}^{N, \lambda \lambda'}(\mathbf{p}) P_{\lambda \lambda'}^{FSI, A, N}(E, \mathbf{p}, \dots) \quad \text{CONVOLUTION!}$$

# FSI: distorted spin-dependent spectral function of ${}^3\text{He}$

L. Kaptari, A. Del Dotto, E. Pace, G. Salmè, S.S., PRC 89 (2014) 035206

Relevant part of the (GEA-distorted) spin dependent spectral function:

$$\mathcal{P}_{\parallel}^{IA(FSI)} = \mathcal{O}_{\frac{1}{2}\frac{1}{2}}^{IA(FSI)} - \mathcal{O}_{-\frac{1}{2}-\frac{1}{2}}^{IA(FSI)}; \quad \text{with:}$$

$$\mathcal{O}_{\lambda\lambda'}^{IA(FSI)}(p_N, E) = \sum_{\epsilon_{A-1}^*} \rho(\epsilon_{A-1}^*) \langle S_A, \mathbf{P}_A | (\hat{S}_{Gl}) \{ \Phi_{\epsilon_{A-1}^*}, \lambda', \mathbf{p}_N \} \rangle \times \\ \langle (\hat{S}_{Gl}) \{ \Phi_{\epsilon_{A-1}^*}, \lambda, \mathbf{p}_N \} | S_A, \mathbf{P}_A \rangle \delta(E - B_A - \epsilon_{A-1}^*).$$

Glauber operator:  $\hat{S}_{Gl}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = \prod_{i=2,3} [1 - \theta(z_i - z_1) \Gamma(\mathbf{b}_1 - \mathbf{b}_i, z_1 - z_i)]$

(generalized) profile function:  $\Gamma(\mathbf{b}_{1i}, z_{1i}) = \frac{(1-i\alpha) \sigma_{eff}(z_{1i})}{4\pi b_0^2} \exp\left[-\frac{\mathbf{b}_{1i}^2}{2b_0^2}\right],$

GEA ( $\Gamma$  depends also on the traveled longitudinal distance  $z_{1i}$ !) very successful in q.e. semi-inclusive and exclusive processes off  ${}^3\text{He}$

see, e.g., Alvioli, Ciofi & Kaptari PRC 81 (2010) 02100

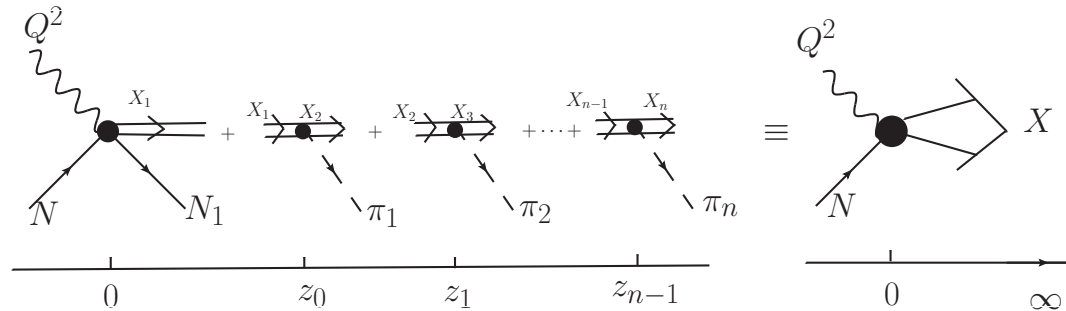
A hadronization model is necessary to define  $\sigma_{eff}(z_{1i})\dots$

## FSI: the hadronization model

Hadronization model (Kopeliovich et al., NPA 2004)

+  $\sigma_{eff}$  model for SIDIS (Ciofi & Kopeliovich, EPJA 2003)

GEA + hadronization model successfully applied to unpolarized SIDIS  $^2H(e, e'p)X$   
(Ciofi & Kaptari PRC 2011).



$$\sigma_{eff}(z) = \sigma_{tot}^{NN} + \sigma_{tot}^{\pi N} [n_M(z) + n_g(z)]$$

- The hadronization model is phenomenological: parameters are chosen to describe the scenario of JLab experiments (e.g.,  $\sigma_{NN}^{tot} = 40$  mb,  $\sigma_{\pi N}^{tot} = 25$  mb,  $\alpha = -0.5$  for both  $NN$  and  $\pi N$ ...).



# FSI: distorted spin-dependent spectral function of ${}^3\text{He}$

L. Kaptari, A. Del Dotto, E. Pace, G. Salmè, S.S., PRC 89 (2014) 035206

- While  $P^{IA}$  is “static”, i.e. depends on ground state properties,  $P^{FSI}$  is dynamical ( $\propto \sigma_{eff}$ ) and process dependent;
- For each experimental point (given  $x, Q^2 \dots$ ), a different spectral function has to be evaluated!
- Quantization axis (w.r.t. which polarizations are fixed) and eikonal direction (fixing the “longitudinal” propagation) are different)... States have to be rotated...
- $P^{FSI}$ : a really cumbersome quantity, a very demanding evaluation ( $\approx 1$  Mega CPU\*hours @ “Zefiro” PC-farm, PISA, INFN “gruppo 4”).

The convolution formulae for a generic structure function can be cast in the form

$$\mathcal{F}^A(x_{Bj}, Q^2, \dots) = \sum_N \int_{x_{Bj}}^A f_N^A(\alpha, Q^2, \dots) \mathcal{F}^N(x_{Bj}/\alpha, Q^2, \dots) d\alpha$$

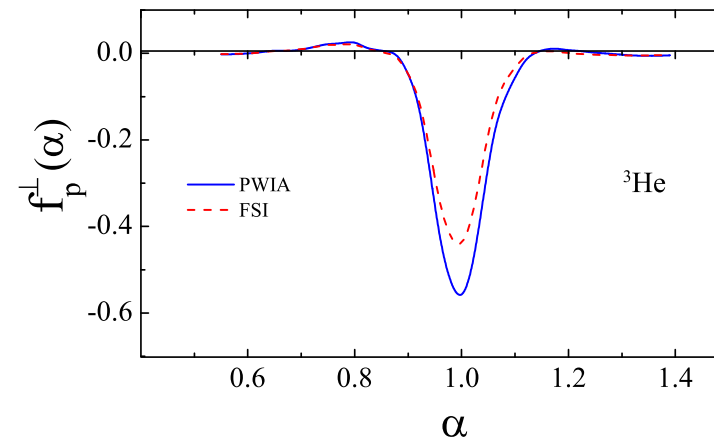
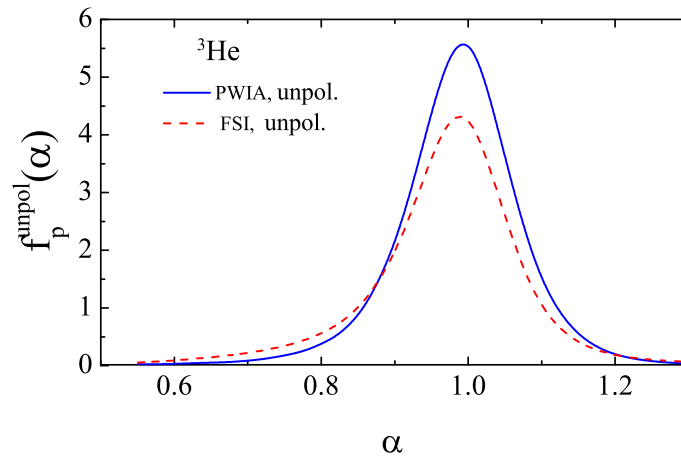
with the **distorted light-cone momentum distribution**:

$$f_N^A(\alpha, Q^2, \dots) = \int dE \int_{p_m(\alpha, Q^2, \dots)}^{p_M(\alpha, Q^2, \dots)} P_N^{A,FSI}(\mathbf{p}, E, \sigma..) \delta\left(\alpha - \frac{pq}{m\nu}\right) \theta\left(W_x^2 - (M_N + M_\pi)^2\right) d^3\mathbf{p}$$

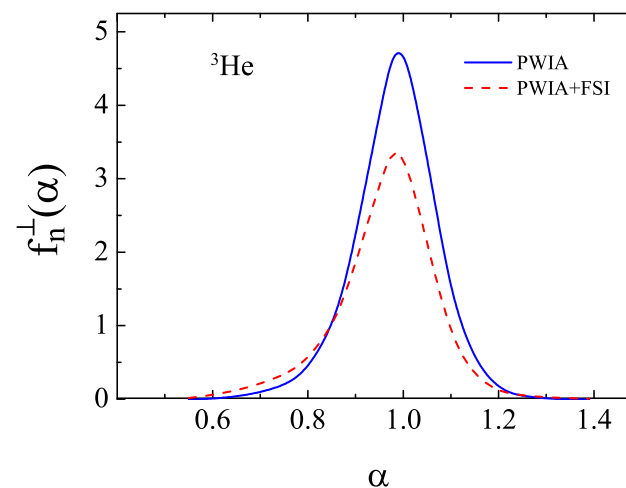
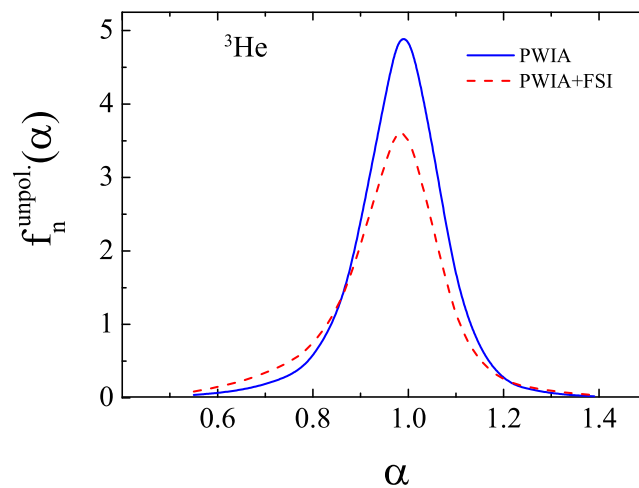
# light-cone momentum distributions with FSI:

A. Del Dotto, L.P. Kaptari, E. Pace, G. Salmè, S.S., "ready" for submission

## PROTON @ $E_i = 8.8$ GeV



## NEUTRON @ $E_i = 8.8$ GeV



Actually, one should also consider the effect on dilution factors...

## DILUTION FACTORS

$$A_3^{exp} \simeq \frac{\Delta \vec{\sigma}_3^{exp.}}{\sigma_{unpol.}^{exp.}} \Rightarrow \frac{\langle \vec{s}_n \rangle \Delta \vec{\sigma}(\mathbf{n}) + 2 \langle \vec{s}_p \rangle \Delta \vec{\sigma}(\mathbf{p})}{\langle \mathbf{N}_n \rangle \sigma_{unpol.}(\mathbf{n}) + 2 \langle \mathbf{N}_p \rangle \sigma_{unpol.}(\mathbf{p})} = \langle \vec{s}_n \rangle f_n A_n + 2 \langle \vec{s}_p \rangle f_p A_p$$

PWIA:  $\langle \vec{s}_{n(p)} \rangle = \int dE \int d^3p P_{||}(E, \mathbf{p}) = \mathbf{p}_{n(p)}$ ;  
 $\langle N \rangle = \int dE \int d^3p P_{unpol.}(E, \mathbf{p}) = 1.$

$$f_{n,(p)}(\mathbf{x}, \mathbf{z}) = \frac{\sum_q e_q^2 f_1^{q,n(p)}(\mathbf{x}) D_1^{q,h}(\mathbf{z})}{\sum_N \sum_q e_q^2 f_1^{q,N}(\mathbf{x}) D_1^{q,h}(\mathbf{z})}$$

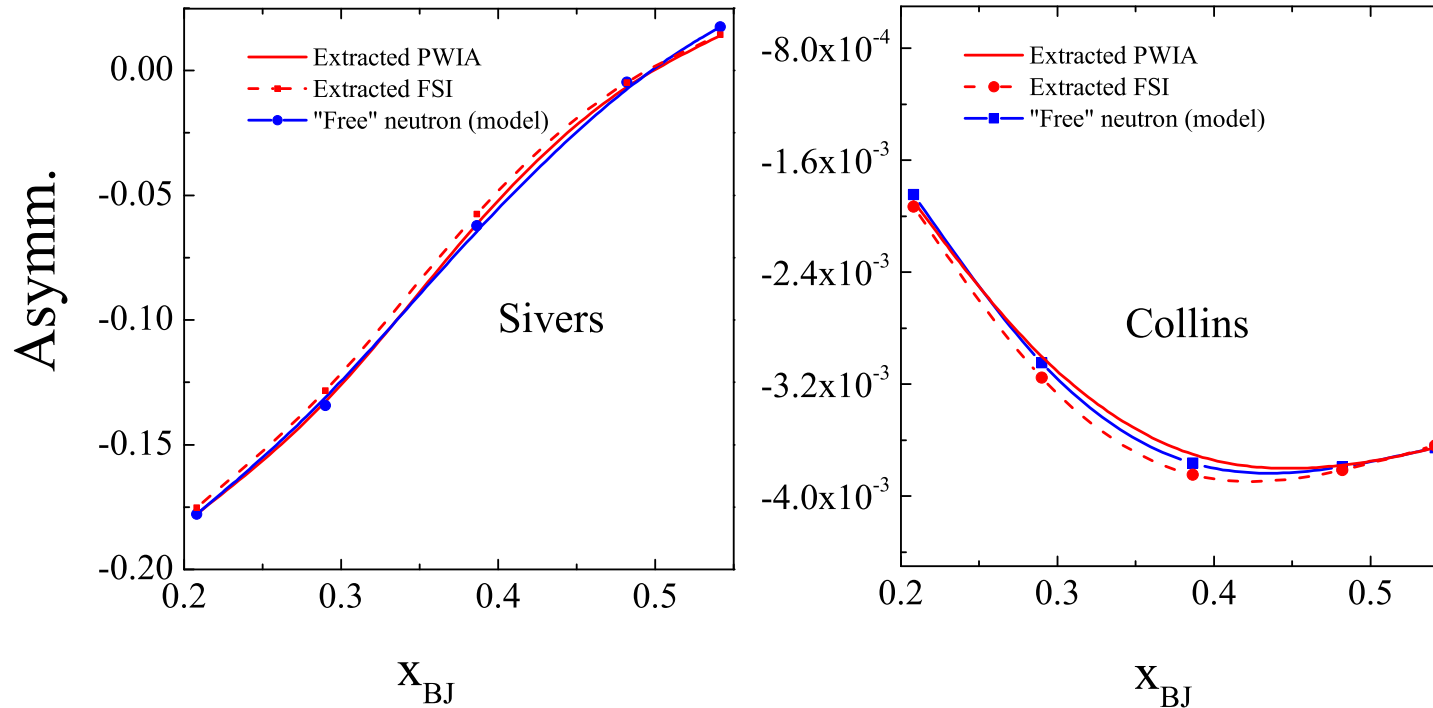
FSI:  $\langle \vec{s}_{n(p)} \rangle = \int dE \int d^3p P_{||}^{FSI}(E, \mathbf{p}) = \mathbf{p}_{n(p)}^{FSI}$ ;  
 $\langle N \rangle = \int dE \int d^3p P_{unpol.}^{FSI}(E, \mathbf{p}) < 1.$

$$f_{n,(p)}^{FSI}(\mathbf{x}, \mathbf{z}) = \frac{\sum_q e_q^2 f_1^{q,n(p)}(\mathbf{x}) D_1^{q,h}(\mathbf{z})}{\sum_N \langle \mathbf{N} \rangle \sum_q e_q^2 f_1^{q,N}(\mathbf{x}) D_1^{q,h}(\mathbf{z})}$$

$$A_n \approx \frac{1}{p_n^{FSI} f_n^{FSI}} \left( A_3^{exp} - 2 p_p^{FSI} f_p^{FSI} A_p^{exp} \right) \approx \frac{1}{p_n f_n} \left( A_3^{exp} - 2 p_p f_p A_p^{exp} \right)$$

2

## Good news from GEA studies of FSI!



Effects of GEA-FSI (shown at  $E_i = 8.8$  GeV) in the dilution factors and in the effective polarizations compensate each other to a large extent: the **usual extraction** is safe!

$$A_n \approx \frac{1}{p_n^{FSI} f_n^{FSI}} \left( A_3^{exp} - 2p_p^{FSI} f_p^{FSI} A_p^{exp} \right) \approx \frac{1}{p_n f_n} \left( A_3^{exp} - 2p_p f_p A_p^{exp} \right)$$

L. Kaptari, A. Del Dotto, E. Pace, G. Salmè, S.S., "ready" for submission

# What about relativity ?

## Good *preliminary* news

We are now going to evaluate the SSAs using the **LF hadronic tensor**, to check whether the proposed extraction procedure still holds within the **LF approach**. We have preliminary encouraging indications:

- **LF longitudinal** and **transverse** polarizations change little from the NR ones:

	<i>proton NR</i>	<i>proton LF</i>	<i>neutron NR</i>	<i>neutron LF</i>
$\int dE d\vec{p} \frac{1}{2} \text{Tr}(\mathcal{P})$	0.999	0.999	0.999	0.999
$\int dE d\vec{p} \frac{1}{2} \text{Tr}(\mathcal{P} \sigma_z) \vec{S}_A = \hat{z}$	-0.023	-0.022	0.878	0.873
$\int dE d\vec{p} \frac{1}{2} \text{Tr}(\mathcal{P} \sigma_y) \vec{S}_A = \hat{y}$	-0.023	-0.023	0.878	0.875

The difference between the effective **longitudinal** and **transverse** polarizations is a **measure of the relativistic content of the system** (in a proton, it would correspond to the difference between **axial** and **tensor** charges).

The extraction procedure should work well within **the LF approach** as it does in the non relativistic case... **BUT WE ARE STILL WORKING...**

# LF Nucleon Spectral Function for ${}^3\text{He}$

$$\mathcal{P}_{\sigma'\sigma}^T(\kappa^+, \boldsymbol{\kappa}_\perp, \epsilon_S, S_{He}) = \rho(\epsilon_S) \sum_{J_S J_z S \alpha T_S \tau_S} \sum_{LF} \langle \tau_S, T_S, \alpha, \epsilon_S J_S J_z S; \tau \sigma', \tilde{\boldsymbol{\kappa}} | \Psi_0 S_{He} \rangle$$

$$\times \langle S_{He}, \Psi_0 | \tilde{\boldsymbol{\kappa}}, \sigma \tau; J_S J_z S \epsilon_S, \alpha, T_S, \tau_S \rangle_{LF}$$

●  $\kappa^+ = \xi \mathcal{M}_0(1, 23)$  and

$$\mathcal{M}_0^2(1, 23) = \frac{m^2 + |\boldsymbol{\kappa}_\perp|^2}{\xi} + \frac{M_S^2 + |\boldsymbol{\kappa}_\perp|^2}{(1 - \xi)}$$

$\mathcal{M}_0(1, 23)$ ="free mass", value of the total  $P^+$  in the LF intrinsic frame of the (1,23) cluster, in terms of which the spectral function is defined

●  $M_S = 2\sqrt{m^2 + m\epsilon_S}$

●  $\rho(\epsilon_S) \equiv$  density of the two-body states (1 for the bound state, and  $m\sqrt{m\epsilon_S}/2$  for the excited ones)

● what about the overlap  ${}_{LF} \langle \tau_S, T_S, \alpha, \epsilon_S J_z S J_S; \tau \sigma', \tilde{\boldsymbol{\kappa}} | \Psi_0 S_{He} \rangle$  ?

## LF overlaps for ${}^3\text{He}$ from the IF ones

$$\begin{aligned}
 & \text{LF} \langle \tau_S, T_S, \alpha, \epsilon_S J_S J_{zS}; \tau\sigma, \tilde{\boldsymbol{\kappa}} | \Psi_0 S_{He} \rangle = \\
 & = \sum_{\tau_2, \tau_3} \sum_{\sigma'_1} D^{\frac{1}{2}} [\mathcal{R}_M^\dagger(\tilde{\boldsymbol{\kappa}})]_{\sigma\sigma'_1} \int d\mathbf{k}_{23} \sum_{\sigma_2, \sigma_3} \sqrt{\frac{\kappa^+ E_{23}}{k^+ E_S}} \sqrt{(2\pi)^3 k^+ \frac{\partial k_z}{\partial k^+}} \times
 \end{aligned}$$

$$\text{IF} \langle \tau_S, T_S, \alpha, \epsilon_S J_S J_{zS} | \mathbf{k}_{23}; \sigma_2, \sigma_3; \tau_2, \tau_3 \rangle \langle \tau_3, \tau_2, \tau_1; \sigma_3, \sigma_2, \sigma'_1; \mathbf{k}, \mathbf{k}_{23} | \Psi_0 S_{He} \rangle \text{IF}$$

- $\mathbf{k}_\perp = \boldsymbol{\kappa}_\perp$ , since the  ${}^3\text{He}$  transverse momentum is  $\mathbf{P}_\perp = 0$ , by choice
- $k^+ = \xi M_0(123) = \kappa^+ M_0(123) / \mathcal{M}_0(1, 23)$

$$\text{with } M_0^2(123) = \frac{m^2 + |\boldsymbol{\kappa}_\perp|^2}{\xi} + \frac{M_{23}^2 + |\boldsymbol{\kappa}_\perp|^2}{(1 - \xi)}$$

and  $M_{23}^2 = 4(m^2 + |\mathbf{k}_{23}|^2)$  the mass of the spectator pair without interaction !  
 Recall that in  $\mathcal{M}_0(1, 23)$  the spectator pair is interacting,  $M_{23} \rightarrow M_S$

- $k_z = \frac{1}{2} \left[ k^+ - \frac{m^2 + |\boldsymbol{\kappa}_\perp|^2}{k^+} \right]$ ,  $E_{23} = \sqrt{M_{23}^2 + |\mathbf{k}|^2}$  and  $E_S = \sqrt{M_S^2 + |\boldsymbol{\kappa}|^2}$
- In the preliminary results,  $\mathcal{M}_0(1, 23) = M_0(123)$

In the actual calculations, we have identified the IF overlaps with the NR ones

$$\langle \tau_3, \tau_2, \tau_1; \sigma_3, \sigma_2, \sigma'_1; \mathbf{k}, \mathbf{k}_{23} | \Psi_0 S_{He} \rangle_{IF} \Rightarrow \langle \tau_3, \tau_2, \tau_1; \sigma_3, \sigma_2, \sigma'_1; \mathbf{k}, \mathbf{k}_{23} | \Psi_0 S_{He} \rangle_{NR}$$

$$IF \langle \tau_S, T_S, \alpha, \epsilon_S J_S J_{zS} | \mathbf{k}_{23}; \sigma_2, \sigma_3; \tau_2, \tau_3 \rangle \Rightarrow_{NR} \langle \tau_S, T_S, \alpha, \epsilon_S J_S J_{zS} | \mathbf{k}_{23}; \sigma_2, \sigma_3; \tau_2, \tau_3 \rangle$$

- Formally set up E. Pace, A. Del Dotto, M. Rinaldi, G. Salmè, S.S., **Few-Body Systems (2016)**  
Light-Cone Conference 2015, Frascati, Proceedings, in press;



- (difficult) calculation in progress



# Conclusions

**We are studying**  ${}^3\vec{H}e(e, e'\pi)X$  **beyond** the realistic, **NR**, **IA** approach in the Bjorken limit, to check if **nuclear effects** in the extraction of the **neutron** information are **under control**. We have preliminary encouraging results concerning:

## ● **FSI effects**

Evaluated through the **GEA** – a distorted spin dependent spectral function is studied

## ● **An analysis at finite $Q^2$ with a LF spectral function** (in IA); (briefly discussed today)

E. Pace, A. Del Dotto, M. Rinaldi, G. Salmè, S.S., *Few-Body Systems* 54 (2013) 1079-1082;

S.S., Del Dotto, Kaptari, Pace, Rinaldi, Salmè, *Few Body Syst.* 56 (2015) 6, 425 and references there in

**Next steps:** 1) **complete** this program! 2) **relativistic FSI?**

3) **Towards 4-body systems:** DVCS off  ${}^4\text{He}$  (in collaboration with M. Viviani, Pisa), non diagonal spectral function...

## BACKUP - $\vec{n}$ from ${}^3\vec{H}e$ : SiDIS case

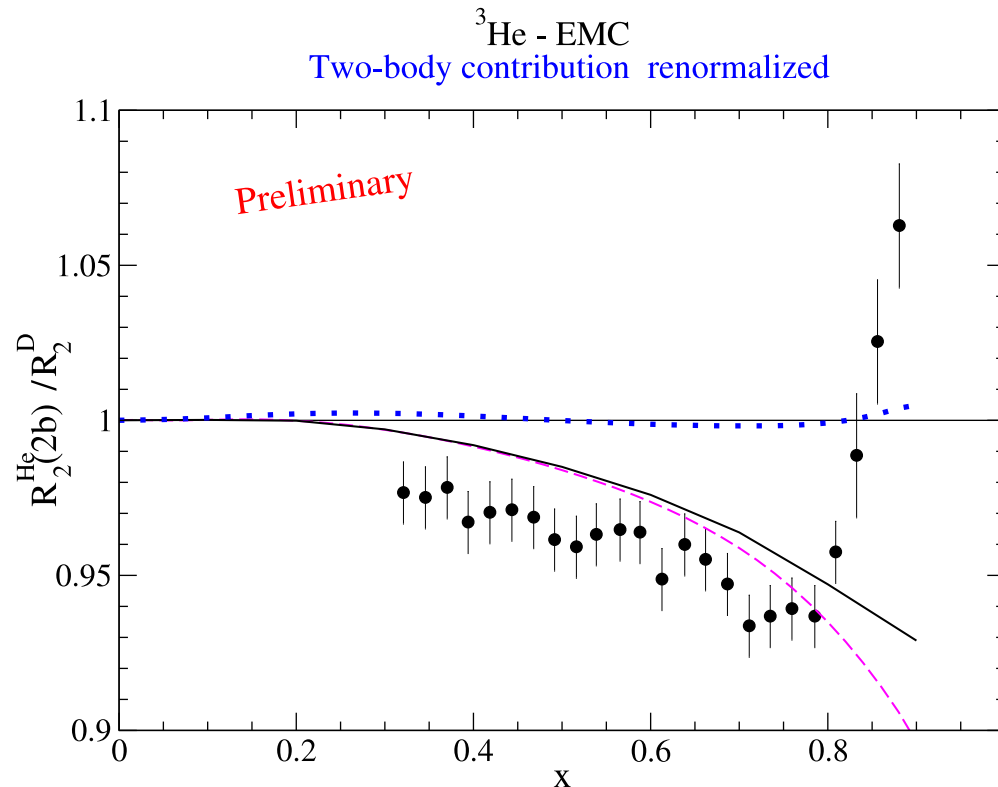
Ingredients of the calculations :

- A realistic **spin-dependent spectral function** of  ${}^3\text{He}$  (C. Ciofi degli Atti et al., PRC 46 (1992) R 1591; A. Kievsky et al., PRC 56 (1997) 64) obtained using the **AV18** interaction and the **wave functions** evaluated by the **Pisa** group [ A. Kievsky et al., NPA 577 (1994) 511 ] (**small effects from 3-body interactions**)
- Parametrizations of data for **pdfs** and **fragmentation functions** whenever available:  
 $f_1^q(x, \mathbf{k}_T^2)$ , Glueck et al., EPJ C (1998) 461 ,  
 $f_{1T}^{\perp q}(x, \mathbf{k}_T^2)$ , Anselmino et al., PRD 72 (2005) 094007,  
 $D_1^{q,h}(z, (z\kappa_T)^2)$ , Kretzer, PRD 62 (2000) 054001
- Models for the unknown **pdfs** and **fragmentation functions**:  
 $h_1^q(x, \mathbf{k}_T^2)$ , Glueck et al., PRD 63 (2001) 094005,  
 $H_1^{\perp q,h}(z, (z\kappa_T)^2)$  Amrath et al., PRD 71 (2005) 114018

Results will be model dependent. Anyway, the aim for the moment is **to study nuclear effects**.

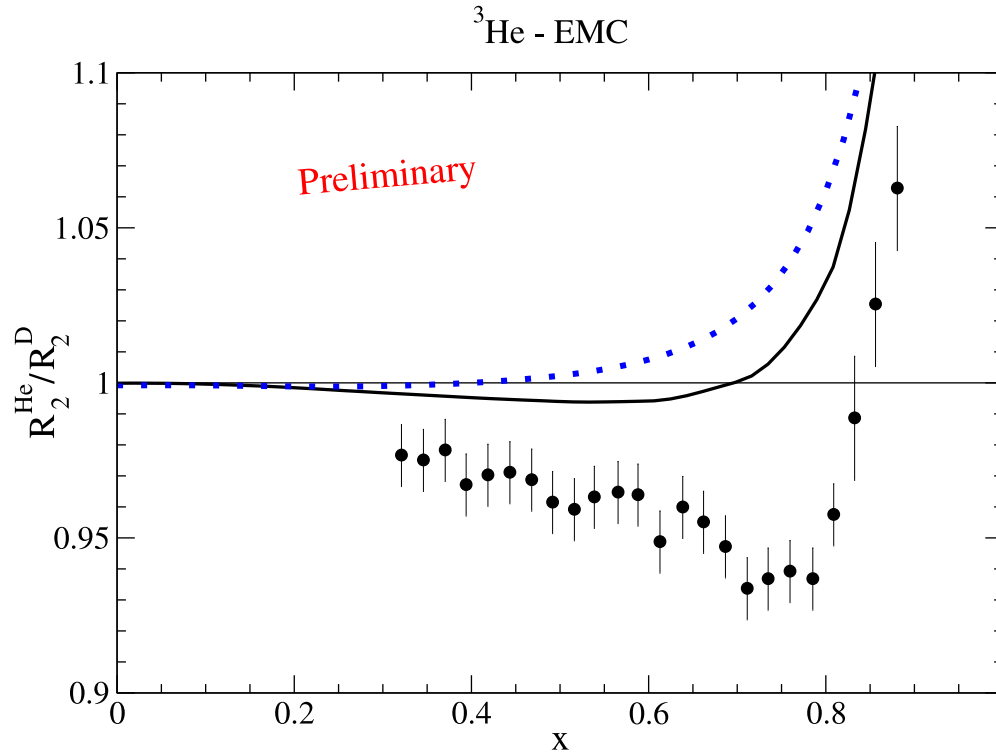
# Preliminary Results for ${}^3\text{He}$ EMC effect

We have first calculated the contribution from the **2B channel**, with the spectator pair in a **deuteron state**



- Solid line: calculation with the **LF Spectral Function**.
- Dashed line: as the solid line, but with  $\sqrt{\bar{k}_{23}^2} = 136.37 \text{ MeV}$  for D (AV18).
- Dotted line: **LF Momentum Distribution** with only two-body contribution

# Calculation of $R_2^{He}(x)/R^D(x)$ : 2-body and 3-body contributions



- Solid line: **LF Spectral Function**, with the exact calculation for the 2-body channel, and an average energy in the 3-body contribution:  $\langle \bar{k}_{23} \rangle = 113.53 \text{ MeV}$  (proton),  $\langle \bar{k}_{23} \rangle = 91.27 \text{ MeV}$  (neutron).
- Dotted line: **LF momentum distribution**

Within the LF framework normalization and momentum sum rule are fulfilled automatically.

Big difference from the IF approach !