Data generation equations

\[ X = \theta_{\text{Normal}}(0.1) \]
\[ M = aX + e_{\text{Normal}}(0.1) \]
\[ Y = bM + c'X + e_{\text{Poisson}} \]

Results

For standard Poisson outcomes, the \( a'b \) and \( c'c \) estimates of the mediated effect were generally similar. However, compared to the true population value of the mediated effect, the \( c'c \) method tended to underestimate the mediated effect, particularly as the \( b \) path increased in magnitude; this effect was attenuated as sample size increased.

\[ \text{Sample size } = 100 \]
\[ \text{The } a'b \text{ estimate of the mediated effect increases linearly with values of the } b \text{ path and closely follows the true mediated effect.} \]
\[ \text{The } c'c \text{ estimate tends to underestimate the mediated effect by a large margin, particularly at larger values of the } b \text{ path.} \]

\[ \text{Sample size } = 1000 \]
\[ \text{The } a'b \text{ estimate continues to be very close to the true mediated effect.} \]
\[ \text{The } c'c \text{ estimate continues to underestimate the mediated effect, but the discrepancy is somewhat attenuated.} \]

Discussion

These findings for mediation with a count outcome largely parallel those of MacKinnon et al. (2007) for mediation with a binary outcome. We found that the \( a'b \) and \( c'c \) methods of estimating the mediated effect can lead to different estimates of the mediated effect, with the \( a'b \) path (larger values of \( b \)) having a larger discrepancy.

Although we found a discrepancy between the \( a'b \) and \( c'c \) estimates of the mediated effect for count outcomes, it is notable that the magnitude of this discrepancy relates to that found for binary outcomes. The effect size for the logistic regression models is based on the linear, additive effect of the predictors on the latent variable underlying the observed binary outcome. The logistic model is based on the log odds of the binary outcome, and the Poisson model is based on the logarithm of the expected value of the count outcome. The discrepancy might be expected to be greater for Poisson regression models, particularly for count outcomes with smaller values of \( b \).

However, compared to the true population values of the mediated effect, the \( c'c \) method tended to under-estimate the mediated effect, particularly as the \( b \) path increased in magnitude; this effect was attenuated as sample size increased. For example, for the largest \( b \) path (larger values of \( b \)), the \( c'c \) estimate of the mediated effect is attenuated.

Note that all conditions are on or below the diagonal line. This suggests that mediation models are used to evaluate the mechanism by which an intervention has an effect. In the present study, mediation models were used to examine the causal chain through which a one variable has its effect on another variable. The mediation models used were based on the ratios of residual variances to equate the effect sizes across logistic regression and Poisson regression models. However, compared to the true population values of the mediated effect, the \( c'c \) method tended to under-estimate the mediated effect, particularly as the \( b \) path increased in magnitude; this effect was attenuated as sample size increased. For example, for the largest \( b \) path (larger values of \( b \)), the \( c'c \) estimate of the mediated effect is attenuated.

This study examines the combination of two methods that are commonly used in many areas of the social sciences: Poisson regression for count outcomes and statistical mediation analysis. Court outcomes, addressed within the generalized linear model (GLM) framework with Poisson regression analysis, are of increasing interest in psychology and related behavioral sciences (Coxe, West, & Aiken, 2009). Examples of count variables that are relevant to prevention research include number of depressive symptoms (Schaffer et al., 2000), number of alcoholic drinks per day (Saitz, Fisher, P., Lucas, C. P., & Walters, M. R., Winter, M., Winter, M., & Samet, J. 2005), Relationship of depressive symptoms and mental health functioning to repeat detoxification. (Shanahan et al., 2005). Additionally, we found that the degree of discrepancy between the \( a'b \) and \( c'c \) methods depends on both the size of the \( b \) path (larger values of \( b \)) and the sample size (larger sample size = smaller discrepancy).