

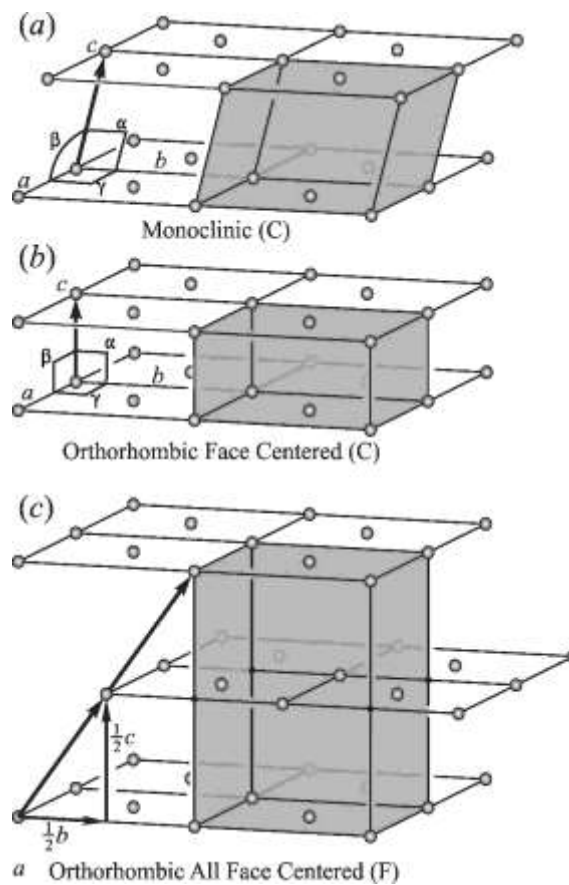


Figure 2.7 Bravais lattices produced by translation of a centered rectangular plane lattice where a and b are different lengths and $\gamma = 90^\circ$.

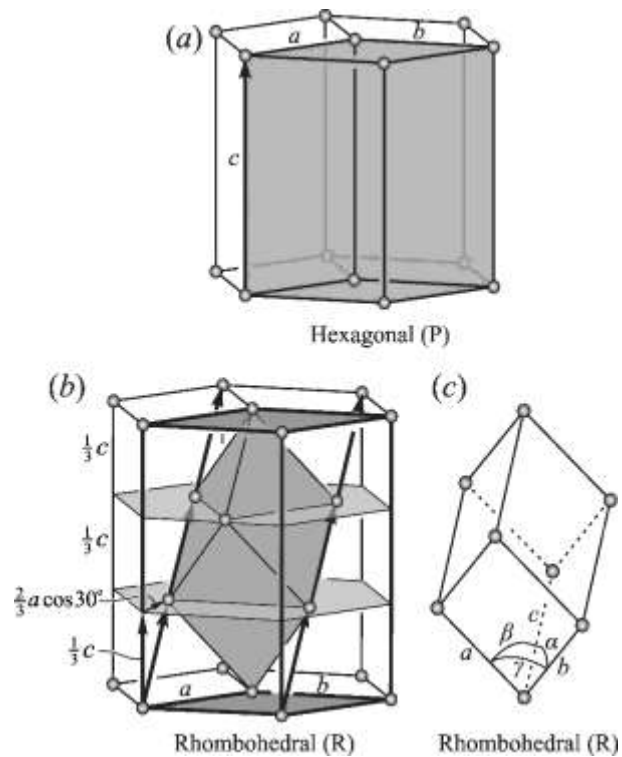


Figure 2.8 Bravais lattices produced by translation of a hexagonal plane lattice.

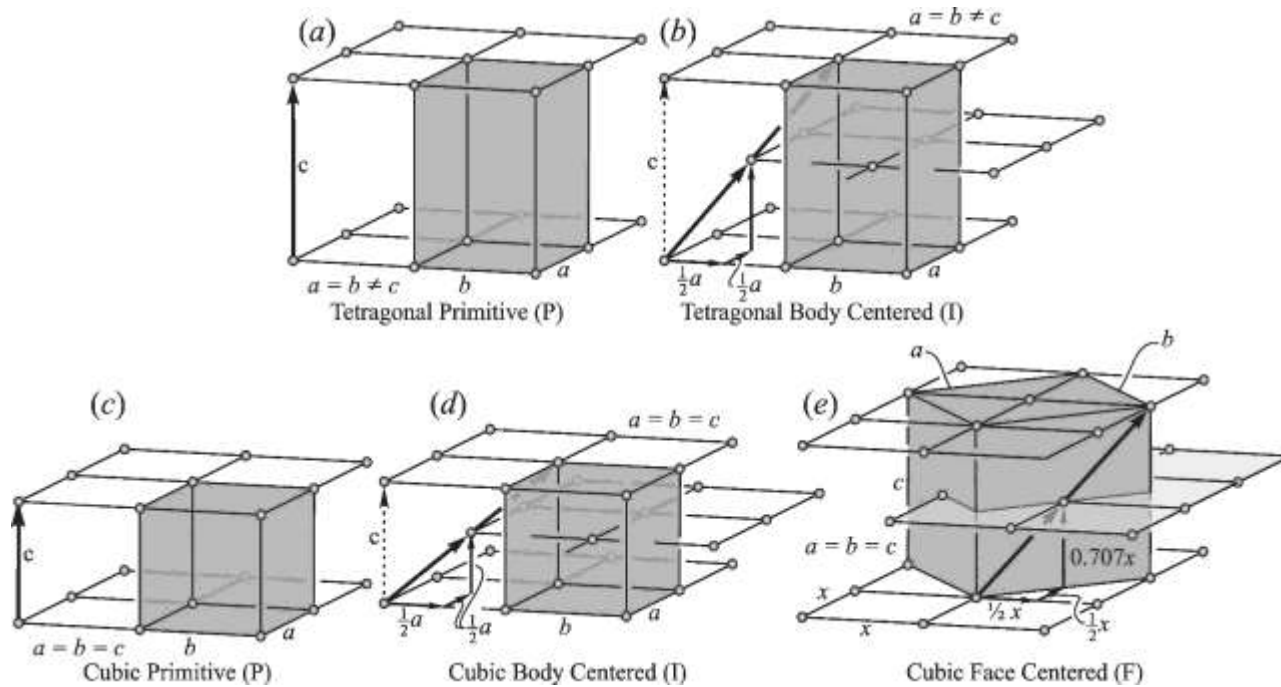


Figure 2.9 Bravais lattices produced by translation of a square plane lattice whose dimensions are $a = b$.

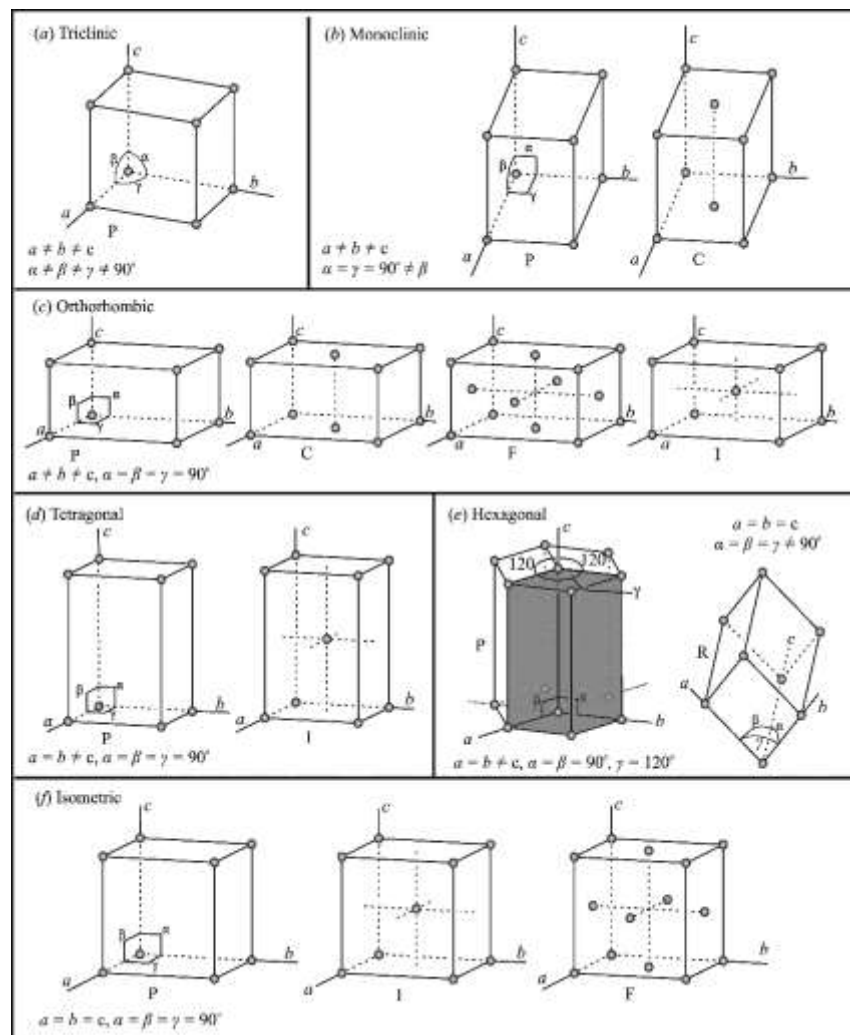


Figure 2.10 The 14 Bravais lattices define six different three-dimensional volumes (a–f) that correspond to the unit cells of the six crystal systems.

Point Symmetry

- Point symmetry deals with how a pattern or motif is repeated around a point
- In case of minerals the motif that is repeated can be crystal faces or a particular arrangement of atoms that make up a crystal
- Possible point symmetry operations are:
 - **Reflection, rotation and inversion**

Reflection:

produced by a mirror plane (m) so that one side of the crystal is a mirror image of the other side

Monoclinic system: only one mirror plane

Triclinic system: no mirror plane

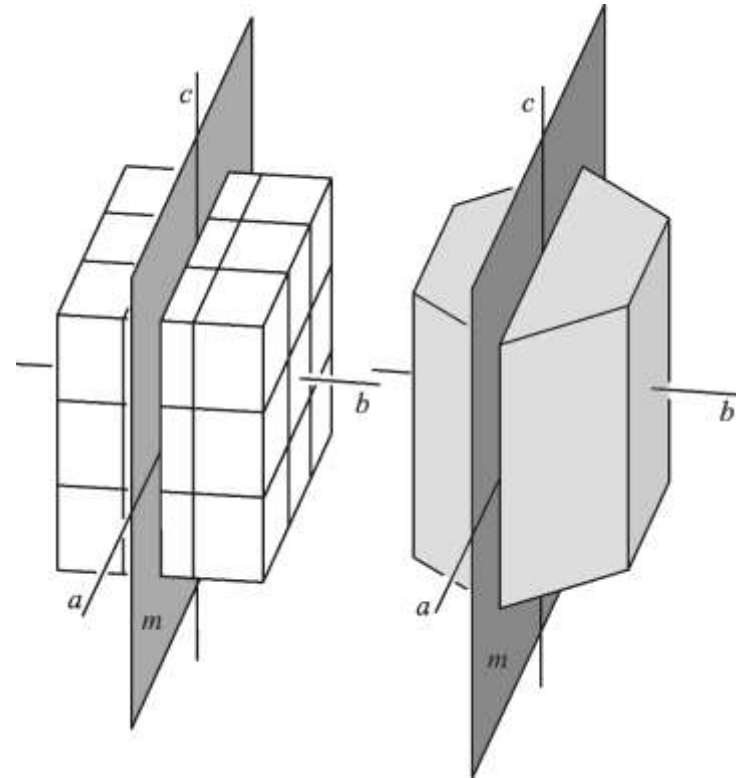


Figure 2.11 Symmetry by a mirror (m) plane or reflection. The mirror plane operates on the lattice, structure, and crystal faces of the mineral.

Rotation:

Repeating a motif by a set of uniform rotation

Possible rotational symmetry operations are:

6 fold (A_6) involves 60° rotation

4 fold (A_4) 90° involves rotation

3 fold (A_3) 120° involves rotation

two fold (A_2) 180° involves rotation

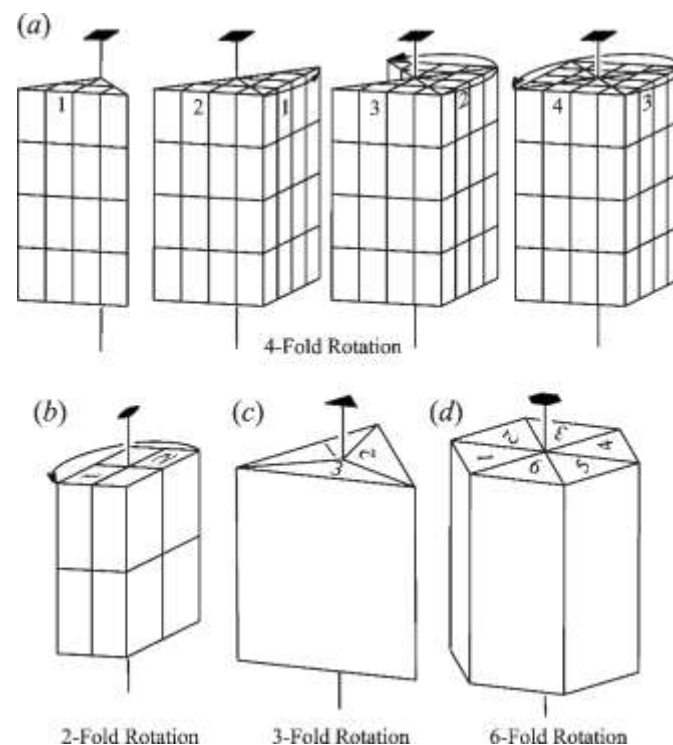
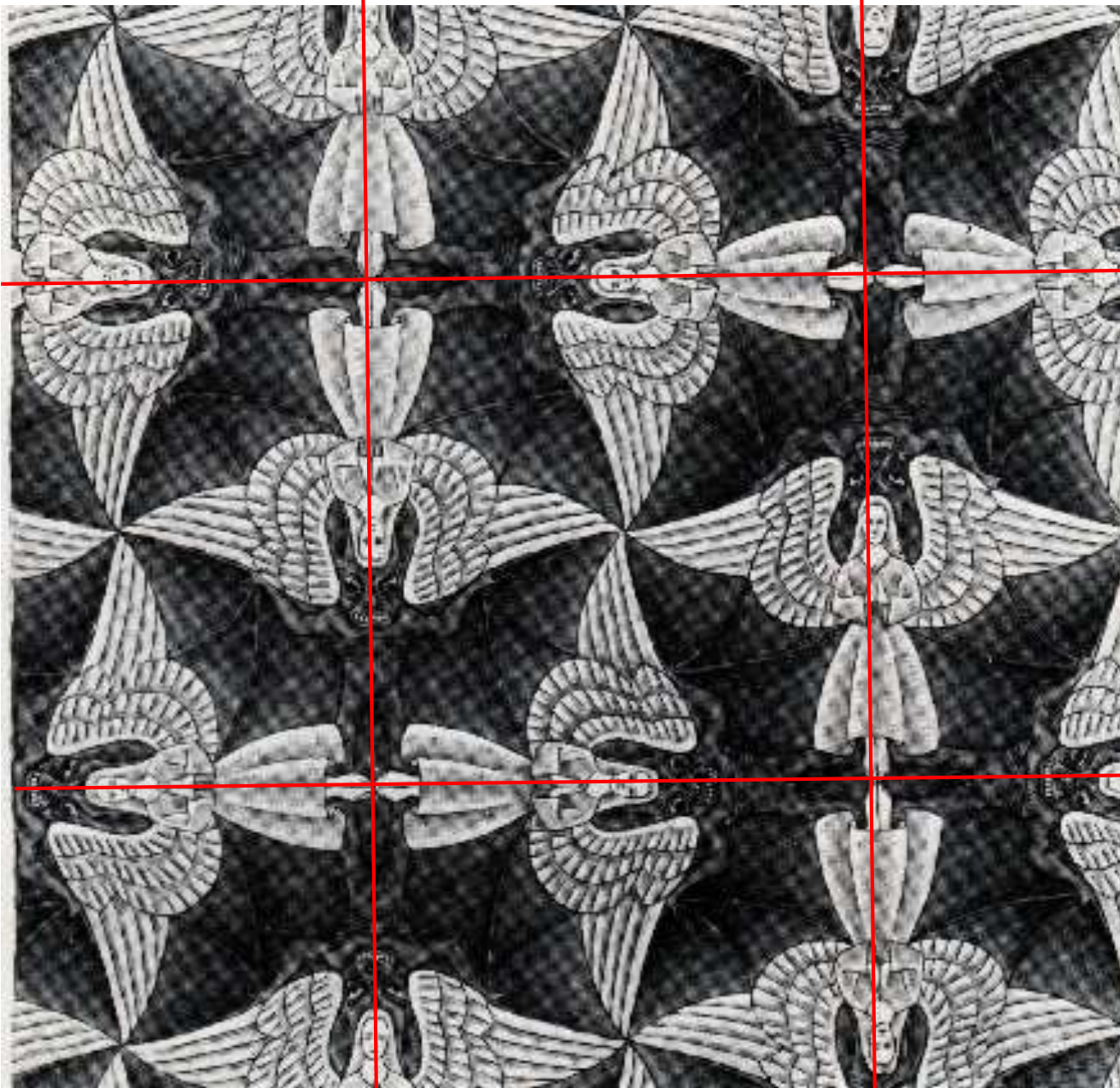


Figure 2.12 Rotational symmetry.



Inversion

In crystals with inversion or center of symmetry, any line drawn through the origin will find identical feature on the opposite side of the crystal equidistant from the center

Notation: $\bar{1}$

Rotoinversion

Symmetry produced by rotation followed by inversion

Can be 1, 2, 3, 4, or 6 fold rotoinversion axes.

For 2 fold rotoinversion: Effect same as a mirror plane perpendicular to the rotation axis

Notation for a six fold rotoinversion axis: $\bar{A}_6$

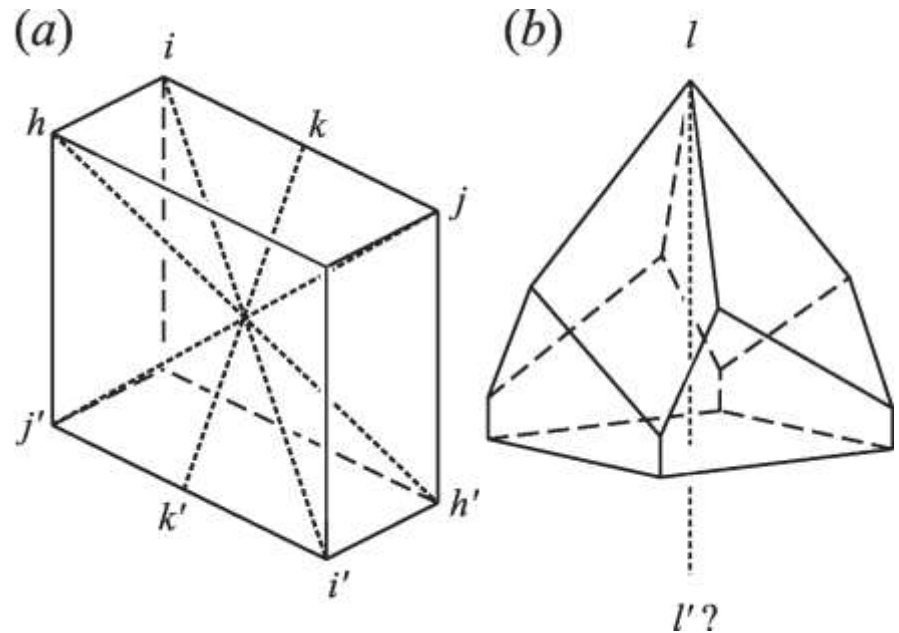


Figure 2.13 Inversion.

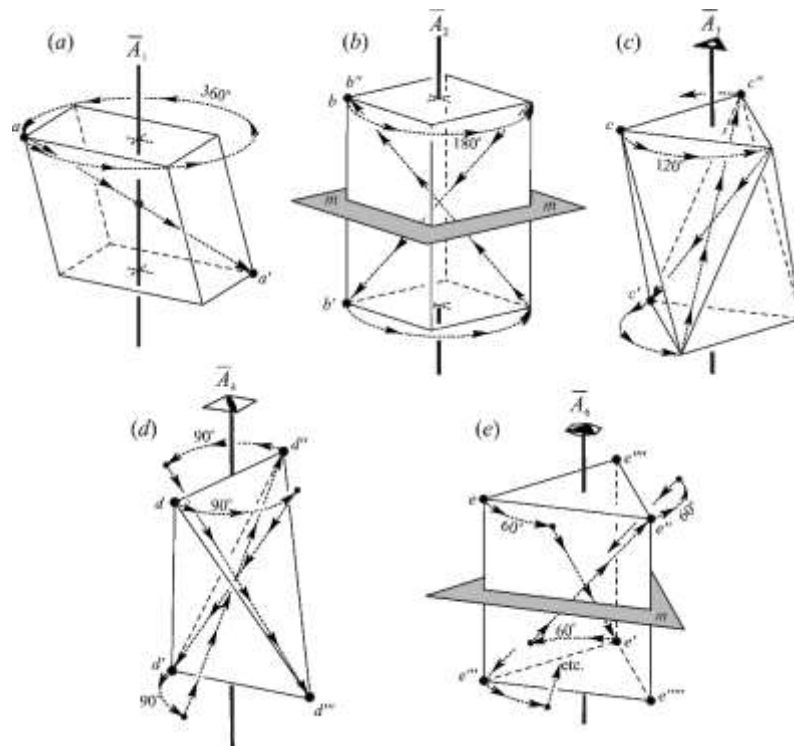
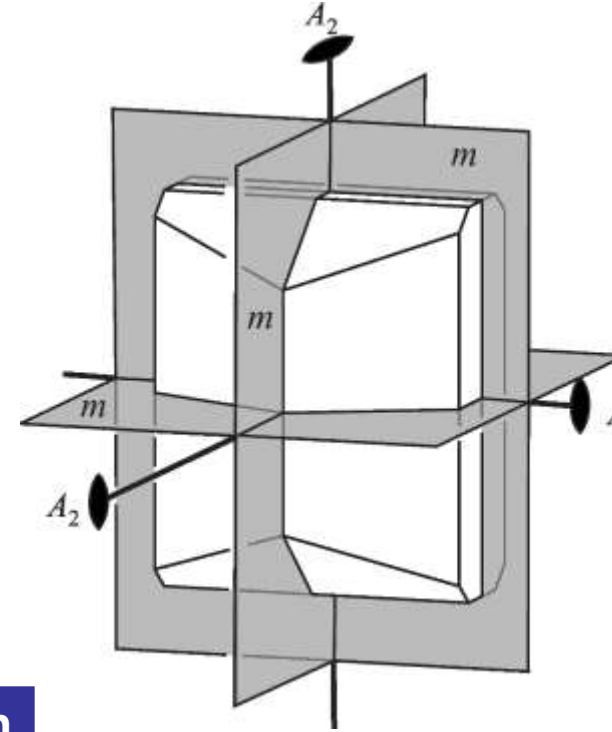


Figure 2.14 Rotoinversion.

Herman-Mauguin system:

- This crystal contains three 2-fold axis, three mirror planes and a center of symmetry.
- It's symmetry symbol is: i , $3A_2$, $3m$
- In Herman-Mauguin system: more simplified
 $2/m, 2/m, 2/m$ i.e., three 2 fold axes perpendicular to three mirror planes



Symmetry Operation	Symmetry Symbol	Herman-Mauguin Symbol
Mirror	m	m
Rotation Axis	A_1, A_2, A_3, A_4, A_6	$1, 2, 3, 4, 6$
Rotoinversion Axis	$\bar{A}_1 = i, \bar{A}_2, \bar{A}_3, \bar{A}_4, \bar{A}_6$	$\bar{1}, m, \bar{3}, \bar{4}, \bar{6}$

Figure 2.15 This crystal has center symmetry (i), three 2-fold axes ($3A_2$), and three mirrors ($3m$).

Point Groups:

- In 2D: By various combination of rotation and mirror, a total of 10 different point groups can be produced; note that inversion is not possible in two dimension
- In 3D: the number of different combination of symmetry is 32 and these symmetry combinations are known as the **32 point groups** also known as **32 crystal classes**

- 32 point groups are grouped in six crystal systems based on common elements
- These six systems are same as those generated by Bravais Lattice (see Table 2.2)

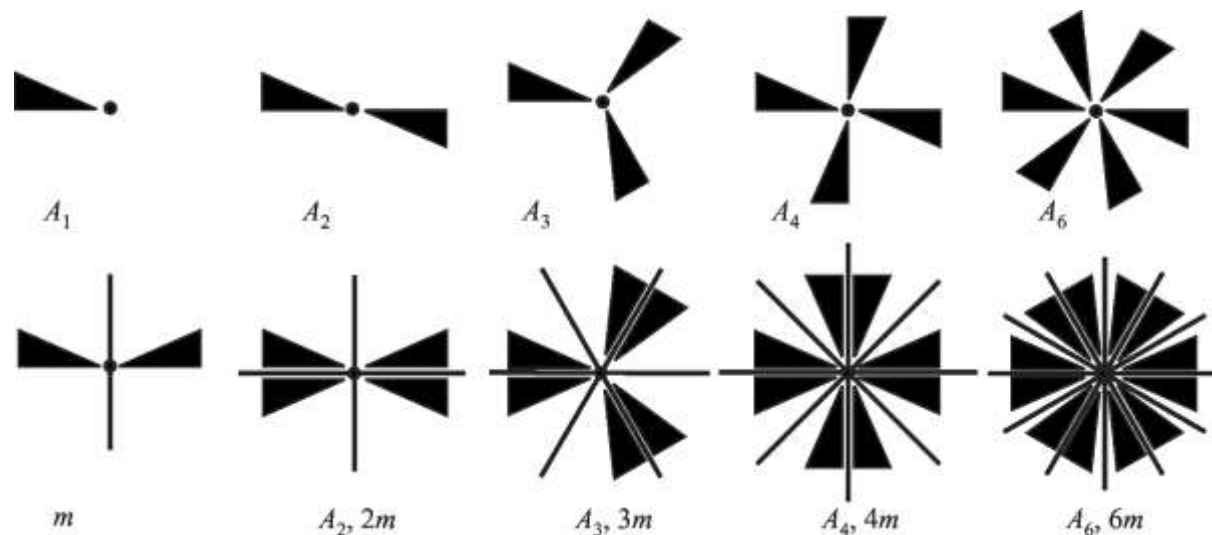


Figure 2.16 The ten two-dimension point groups.

32 point Groups arranged in six crystal systems

System	Class		Symmetry elements
	Inter- national	Schön- fließ	
triclinic	1	C_1	E
	$\bar{1}$	C_i	$E i$
monoclinic	m	C_s	$E \sigma_h$
	2	C_2	$E C_2$
	$2/m$	C_{2h}	$E C_2 i \sigma_h$
orthorhombic	$2mm$	C_{2v}	$E C_2 \sigma'_v \sigma''_v$
	222	D_2	$E C_2 C'_2 C''_2$
	mmm	D_{2h}	$E C_2 C'_2 C''_2 i \sigma_h \sigma'_v \sigma''_v$
tetragonal	4	C_4	$E 2C_4 C_2$
	$\bar{4}$	S_4	$E 2S_4 C_2$
	$4/m$	C_{4h}	$E 2C_4 C_2 i 2S_4 \sigma_h$
	$4mm$	C_{4v}	$E 2C_4 C_2 2\sigma'_v 2\sigma_d$
	$\bar{4}2m$	D_{2d}	$E C_2 C'_2 C''_2 2\sigma_d 2S_4$
	422	D_4	$E 2C_4 C_2 2C'_2 2C''_2$
	$4/mmm$	D_{4h}	$E 2C_4 C_2 2C'_2 2C''_2 i 2S_4 \sigma_h 2\sigma'_v 2\sigma_d$
trigonal (rhombohedral)	3	C_3	$E 2C_3$
	$\bar{3}$	S_6	$E 2C_3 i 2S_6$
	$3m$	C_{3v}	$E 2C_3 3\sigma_v$
	32	D_3	$E 2C_3 3C_2$
	$\bar{3}m$	D_{3d}	$E 2C_3 3C_2 i 2S_6 3\sigma_d$
hexagonal	$\bar{6}$	C_{3h}	$E 2C_3 \sigma_h 2S_3$
	6	C_6	$E 2C_6 2C_3 C_2$
	$6/m$	C_{6h}	$E 2C_6 2C_3 C_2 i 2S_3 2S_6 \sigma_h$
	$\bar{6}m2$	D_{3h}	$E 2C_3 3C_2 \sigma_h 2S_3 3\sigma_v$
	$6mm$	C_{6v}	$E 2C_6 2C_3 C_2 3\sigma_v 3\sigma_d$
	622	D_6	$E 2C_6 2C_3 C_2 3C'_2 3C''_2$
	$6/mmm$	D_{6h}	$E 2C_6 2C_3 C_2 3C'_2 3C''_2 i 2S_3 2S_6 \sigma_h 3\sigma_d 3\sigma_v$
cubic	23	T	$E 4C_3 4C_3^2 3C_2$
	$m\bar{3}$	T_h	$E 4C_3 4C_3^2 3C_2 i 8S_6 3\sigma_h$
	$\bar{4}3m$	T_d	$E 8C_3 3C_2 6\sigma_d 6S_4$
	432	O	$E 8C_3 3C_2 6C'_2 6C_4$
	$m\bar{3}m$	O_h	$E 8C_3 3C_2 6C_2 6C_4 i 8S_6 3\sigma_h 6\sigma_d 6S_4$

Steno's Law or the Law of Constancy of Interfacial Angle:

- The angle between the same two equivalent crystal faces of the same mineral is constant no matter where the crystal has been collected from or its size
- The interfacial angle is measured with a contact goniometer

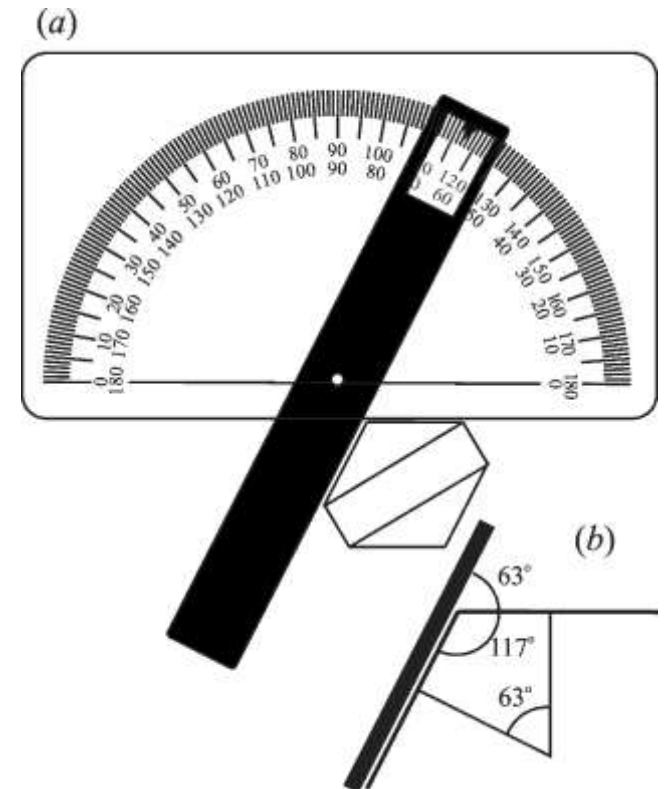


Figure 2.17 Contact goniometer

Determining Crystal Symmetry

- Determine if the crystal has a center of symmetry
- Identify mirror planes
- Identify rotation axes
- Compile the combination of symmetry

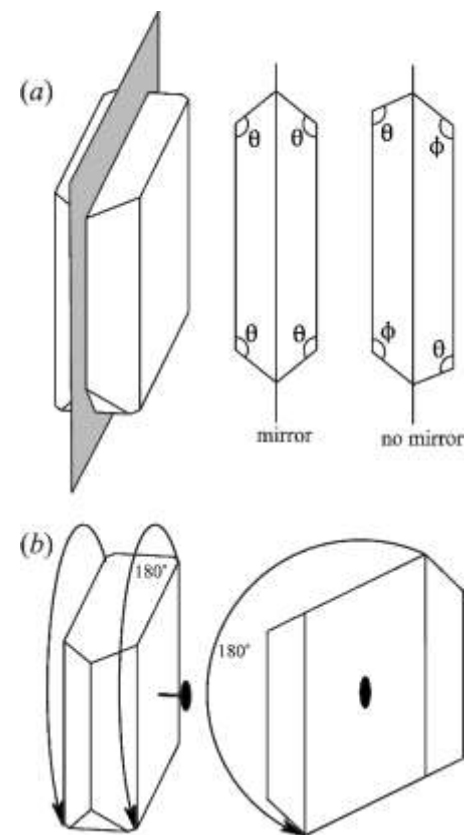


Figure 2.18 Determining crystal symmetry.

Glide symmetry

- Translation and then mirror

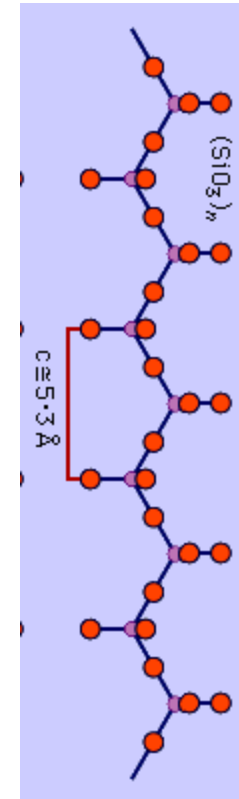
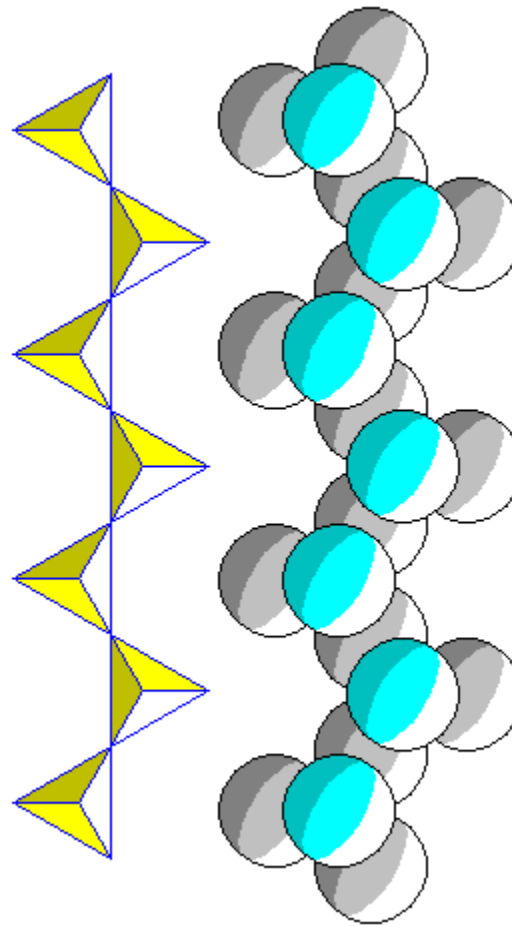
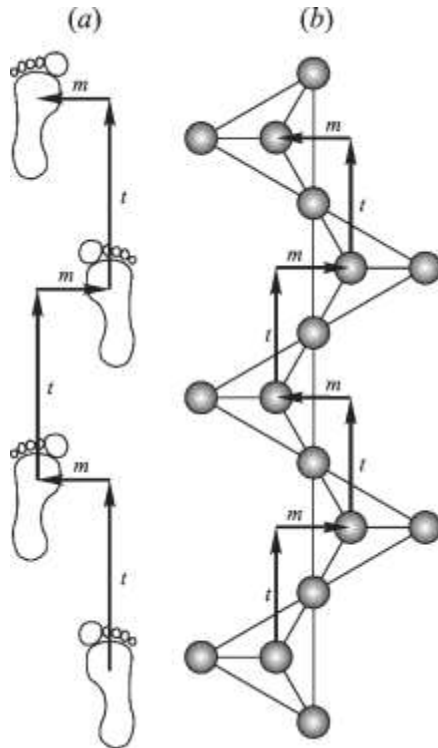


Figure 2.19 Glide symmetry.

Screw

- Translation and then rotation

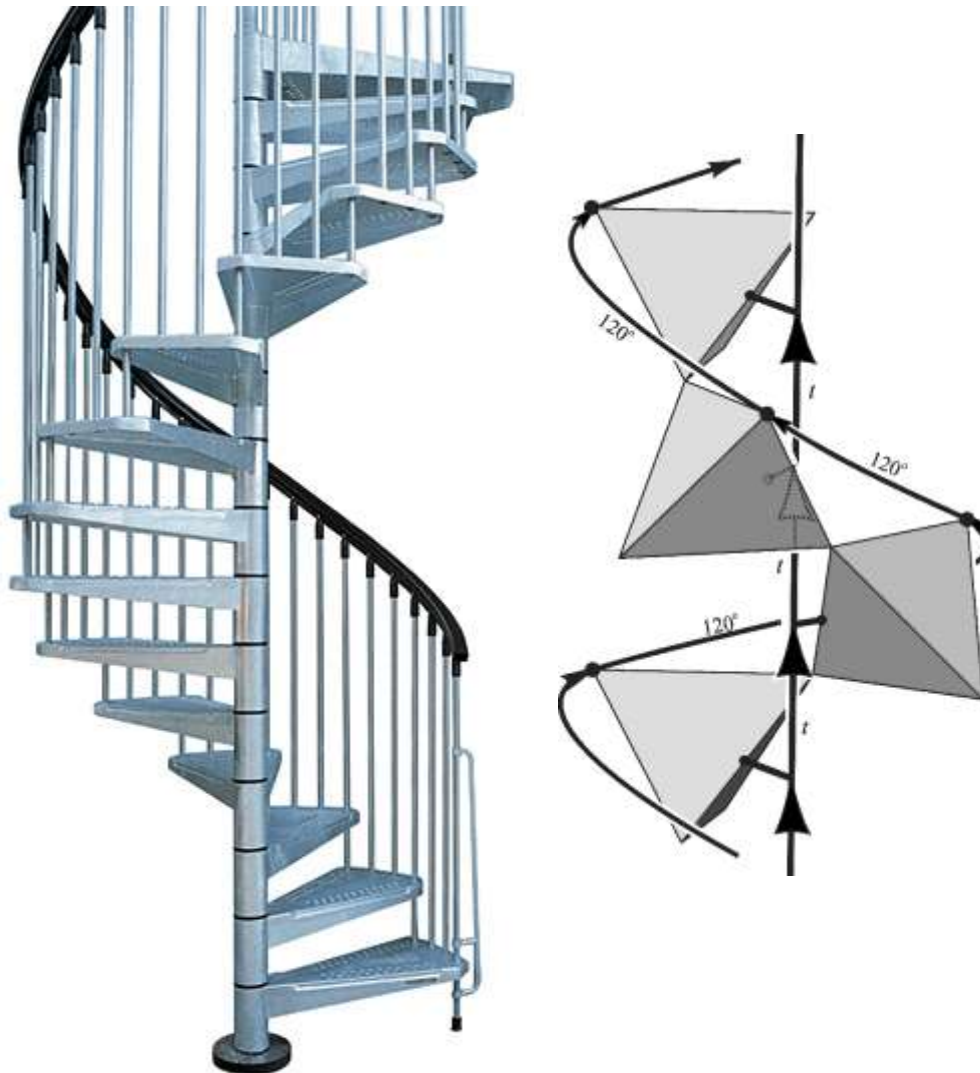


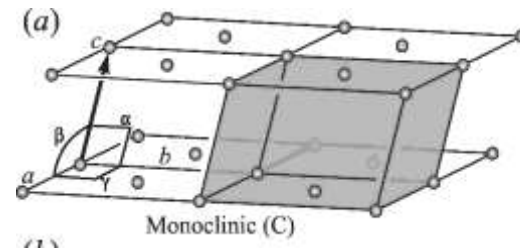
Figure 2.20 Screw axis.

230 Space Groups

- 32 Point Groups (10 2D point groups + inversion yields 32 point groups)
- Combined with simple translation yields 73 Space Groups
- With Glide and Screw thrown in – we end up with 230 Space Groups
- 230 Space Groups represent all the continuation of point symmetry with translational symmetry

Crystal faces

- The most common crystal faces are often parallel to the surface of unit cells, often grow as simple diagonal to the lattice and in general have simple, rational orientation with respect to the crystal lattice:
- **Law of Haüy** (pronounced aa-wee)
 - Crystal faces make simple rational intercept on crystal axes
- **Law of Bravais**
 - Common crystal faces are parallel to lattice planes that have high lattice node density.



- Monoclinic lattice
- Planes A, B and C are parallel to the unit cell and have high node density: most likely crystal faces
- Plane T has moderate lattice density: likely crystal face
- Plane Q has low lattice density: not a likely crystal face

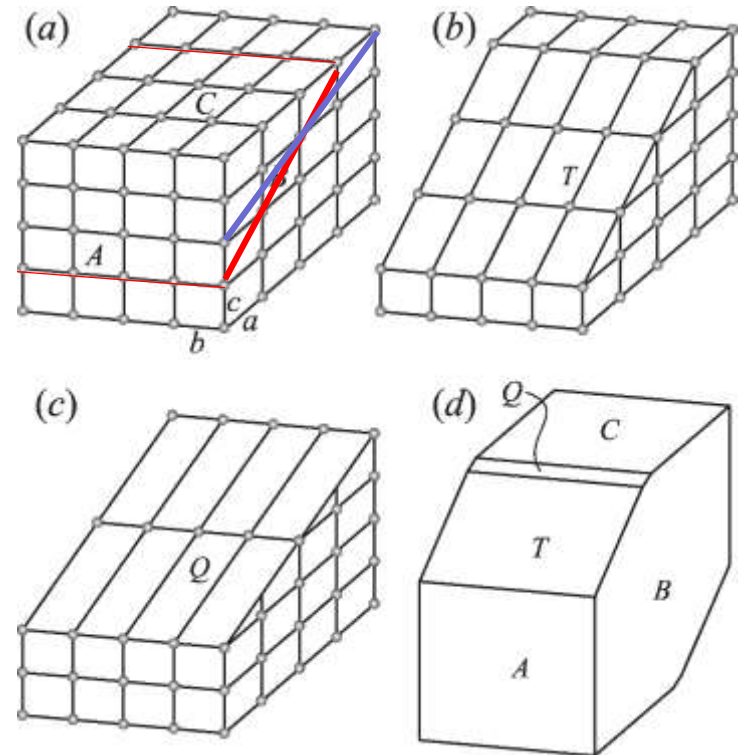


Figure 2.21 Primitive monoclinic lattice. Prominent crystal faces are parallel to rational planes in the lattice that have high density of lattice nodes.

Miller Index:

- General Form (hkl) where h, k, l are whole numbers
- (001) spoken as zero,zero, one
- $(00\bar{1})$: zero, zero, bar one
- (224) will be represented as (112)
- In the diagram: the shaded face intersects a and b axis at a distance of 12 lattice nodes and c axes at a distance of six lattice nodes
- Or the ratio of the three intercepts are 1,1,1/2
- Reciprocal of the ratio of intercept on the crystallographic axes = 1,1,2
- So the face is (112)
- A face parallel to a and b axes and that cuts c axes = (001)

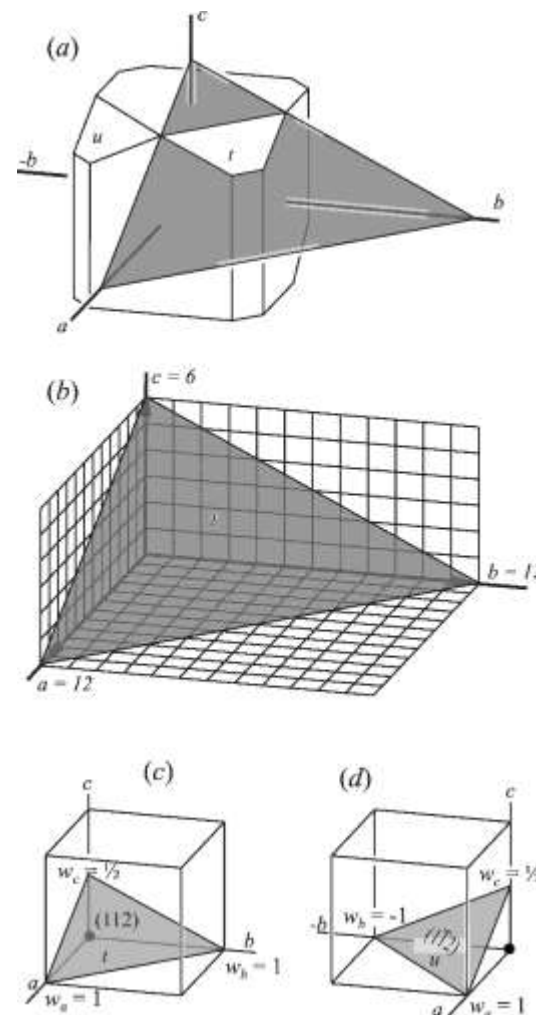


Figure 2.22 Miller index.

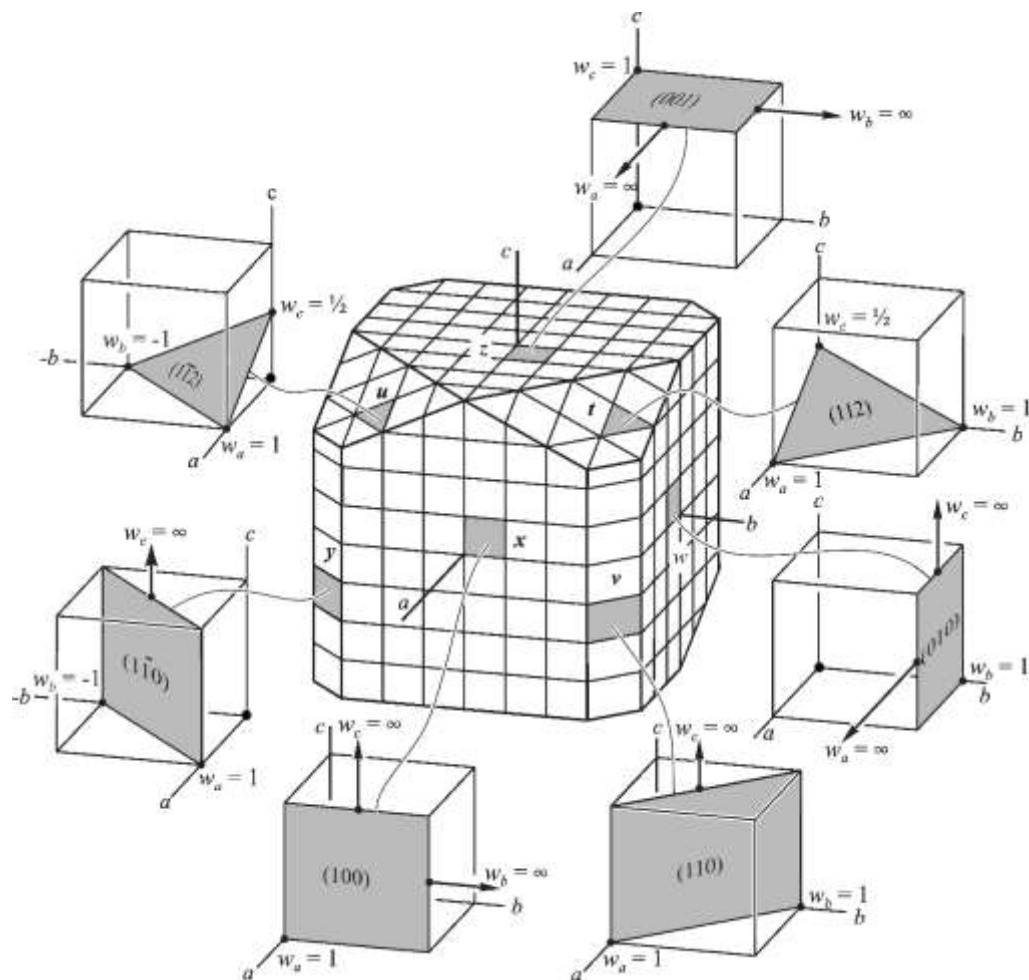


Figure 2.23 Miller indices for all visible faces on the crystal shown in Figure 2.22. The traces of unit cells are sketched on the crystal. See text for discussion.

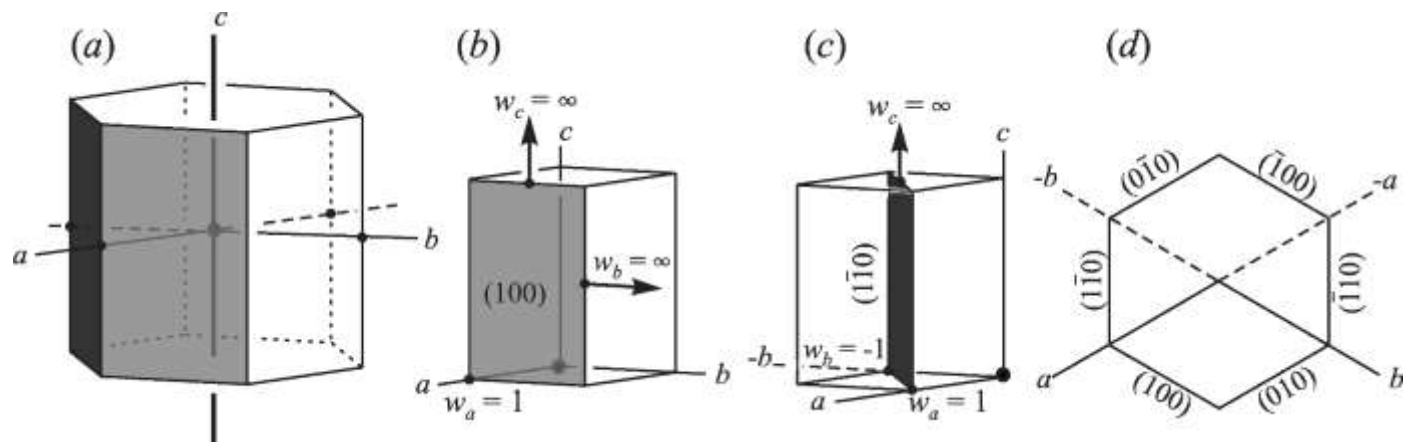


Figure 2.24 Miller indices in the hexagonal crystal system.

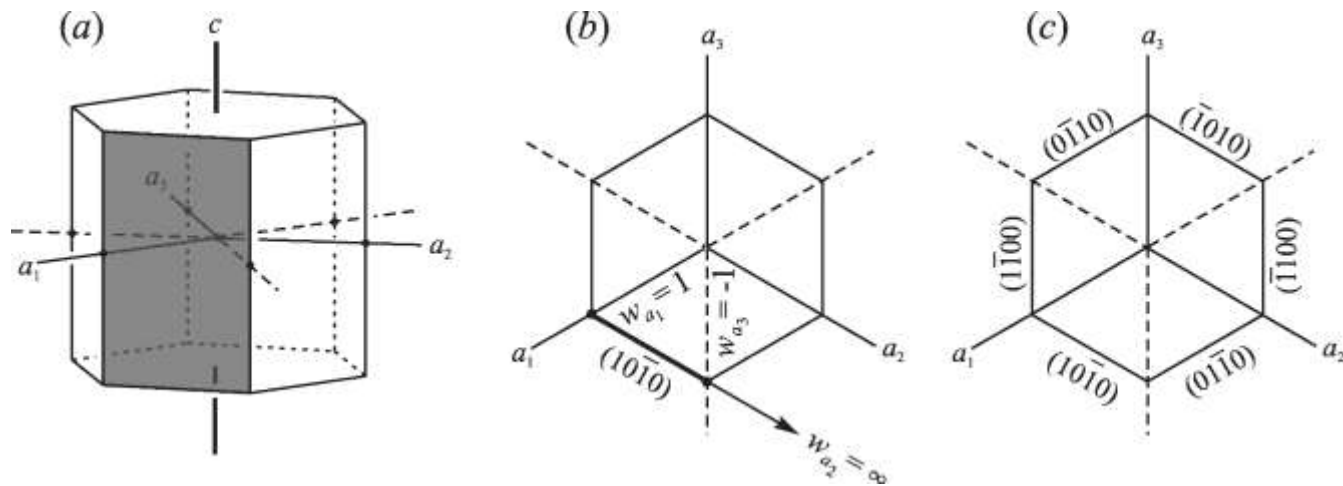


Figure 2.25 Miller-Bravais indices in the hexagonal crystal system.

Crystallographic planes are assigned the same Miller index as the crystal face to which they are parallel.

