MAC 2313 (Calculus III)
Test 1, September 21, 2016

Name: **PID:**

Remember that no documents or calculators are allowed during the exam. Be as precise as possible in your work; you must show all your work to deserve the full mark assigned to any question. Do not cheat, otherwise I will be forced to give you a zero and report your act of cheating to the University Administration. Always do your best. Total=85 points on 3 pages.

1 [10] Describe the given surface according to the values of the parameter m; if it is a sphere, state its radius and center. If it is a point, state its coordinates. $x^2 + y^2 + z^2 - 4x - 6my + 10z + 38 = 0$.

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2. [10] a) Set $\overrightarrow{u}$=$\overrightarrow{i}$-4$\overrightarrow{j}$+2$\overrightarrow{k}$, $\overrightarrow{v}$=2$\overrightarrow{i}$+$\overrightarrow{j}$+$\overrightarrow{k}$ and $\overrightarrow{z}$=-2$\overrightarrow{i}$+$\overrightarrow{j}$+3$\overrightarrow{k}$. a) Show that $\overrightarrow{u}$, $\overrightarrow{v}$ and $\overrightarrow{z}$ are pairwise orthogonal vectors. b) Let $\overrightarrow{w}$=3$\overrightarrow{i}$+2$\overrightarrow{j}$-4$\overrightarrow{k}$. Find three scalars a, b and c such that $\overrightarrow{w}$=a$\overrightarrow{u}$+b$\overrightarrow{v}$+c$\overrightarrow{z}$.

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3. [14] Let $\overrightarrow{q}$=$\overrightarrow{i}$-$\overrightarrow{j}$+4$\overrightarrow{k}$, and $\overrightarrow{r}$=-2$\overrightarrow{i}$+$\overrightarrow{j}$-$\overrightarrow{k}$. a) Find the vector component of $\overrightarrow{q}$ that is orthogonal to $\overrightarrow{r}$.

b) If θ is the angle between $\overrightarrow{r}$ and $\overrightarrow{q}$, find $\cos(\theta)$ and $\sin(\theta)$.

c) If a force $\overrightarrow{F}$=-2$\overrightarrow{q}$ is applied to move an object 4 meters in the direction of the vector $\overrightarrow{r}$, find the work done by $\overrightarrow{F}$.

4. [12] Set $\vec{u}=\vec{i}$-2$\vec{j}$+3$\vec{k}$, $\vec{v}$=2$\vec{i}$-$\vec{j}$+$\vec{k}$ and $\vec{w}$=2$\vec{i}$-$\vec{j}$-$\vec{k}$. a) Find the area of the parallelogram having $\vec{v}$ and $\vec{w}$ as adjacent sides. b) Find the volume of the parallelepiped having $\vec{u}$, $\vec{v}$ and $\vec{w}$ as adjacent edges.

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5. [20] a) Show that the two lines $L_1 : x = 1-3t, \quad y = 4+2t, \quad z = 4+3t$, and $L_2 : x = 3+t, \quad y = 4-2t, \quad z = 3-2t$ intersect, and find their point of intersection A.

   b) Find an equation for the plane $\mathcal{P}$ that contains both L_1 and L_2.

   c) Find the distance between the plane $\mathcal{P}$ and the point $C(1, -2, -3)$.

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6. [4] Find an equation and identify the surface that results when the cone $z = \sqrt{3x^2 + 3y^2}$ is reflected about the plane: i) $z = 0$, ii) $x = z$.

7 [6]. a) Convert from rectangular to spherical coordinates: i) $(3, -\sqrt{3}, -2)$. ii) Convert the equation $\theta = \frac{\pi}{4}$ from cylindrical to rectangular coordinates, and identify the surface.

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8. [9] a) Find the points of intersection of the line $L: x = 1 + t, \quad y = 2 - t, \quad z = 5$ and the paraboloid $z = x^2 + y^2$.
b) Find an equation for the plane that contains both the line L from part a) and the point $D(2, 3, 4)$.