MAC 2313 (Calculus III)-U04
Test 2, Tuesday March 04, 2008

Name: **PID:**

Remember that no documents or graphing calculators are allowed during the test. Be as precise as possible in your work; you shall show all your work to deserve the full mark assigned to any question. Do not cheat, otherwise I will be forced to give you a zero and report your act of cheating to the University Administration. Good luck.

1. [26] Evaluate each integral.

a) $\int_0^1 \int_z^{z^2} \int_0^{\ln y} e^x \, dxdydz =$

b) $\int_0^2 \int_0^1 y \sin z \, dydz =$

c) $\int_0^{\frac{1}{2}} \int_x^{4x} x \, dydx =$

d) $\int_{-1}^1 \int_0^{\sqrt{1-y^2}} \int_0^{\sqrt{1-x^2-y^2}} (x^2+y^2+z^2)^{\frac{-1}{2}} \, dzdxdy =$

2. [15] Use polar coordinates to evaluate $\int_0^1 \int_x^{\sqrt{x}} \sqrt{x^2+y^2}\, dydx$. (Hint. First sketch the region of integration.)

3. [18] Let $x = u - v$, $y = u + v - w$, $z = w - u$. Find the jacobians $\frac{\partial(x,y,z)}{\partial(u,v,w)}$ and $\frac{\partial(u,v,w)}{\partial(x,y,z)}$

4. [15] Let $\vec{u}=\vec{i}-2\vec{j}+2\vec{k}$, and $\vec{v}=3\vec{i}-3\vec{j}-\vec{k}$. a) Find the angle θ between $\vec{u}$ and $\vec{v}$ to the nearest degree. b) Find the component of $\vec{u}$ that is parallel to $\vec{v}$, and the component of $\vec{v}$ that is orthogonal to $\vec{u}$. c) Find the direction cosines of $\vec{u}$, and approximate the direction angles to the nearest degree.

5. [10] Use an appropriate order of integration to evaluate $\int\int_{\mathcal{R}} x\cos(xy)\,dA$, where $\mathcal{R}=\{(x,y); 0\le x\le \pi/4,\ \ 0\le y\le 1$

6. [16] Let $f(x, y) = x - y + z$. Use the Lagrange multipliers to find the maximum and minimum values of f subject to the constraint: $x^2 + y^2 + z^2 = 1$.