MAC 2313 (Calculus III)
Test 2, Friday April 13, 2012

Name: **PID:**

Remember that no documents or calculators are allowed during the test. Be as precise as possible in your work; you shall show all your work to deserve the full mark assigned to any question; answers without any explanation won't get any credits. Do not cheat, otherwise I will be forced to give you a zero and report your act of cheating to the University Administration. Good luck.

1. [20] Evaluate each integral.

a) $\displaystyle\int_1^2 \int_y^2 \int_0^{\sqrt{3}z} \frac{z}{z^2+x^2}\, dxdzdy =$

b) $\displaystyle\int_{\frac{\pi}{2}}^{\pi} \int_1^2 y\sin(xy)\, dxdy =$

c) I=$\displaystyle\int\int_R e^{(x^4)}\, dA$, where R is the region bounded by the curves $y = 0$, $y = x^3$, $x = 2$.

d) Evaluate the line integral along $\mathcal{C}$ given by $\mathcal{C} : x = t,\ y = 3\sin t,\ z = 3\cos t,\ 0 \leq t \leq \pi$.

$\displaystyle\int_{\mathcal{C}} \frac{e^x}{y^2+z^2}\, ds =$

2. [10] Use the change of variables $u = x - y$, $v = x + y$ to evaluate $\displaystyle\int\int_R (x+y)e^{x^2-y^2}\, dA$, where R is the rectangular region enclosed by the lines $x + y = 0$, $x + y = 1$, $x - y = 1$, $x - y = 3$.

3. [20] a) Set up, but do not evaluate, an iterated integral equal to the surface integral $\int\int_{\sigma} y^2 z dS$, where σ is the portion of the cylinder $x^2 + z^2 = 4$ in the first octant between the planes $y = 0$, $y = 6$, $x = z$, and $x = 2z$.

b) Use cylindrical coordinates to find the volume and the centroid of the solid bounded below by the paraboloid $z = x^2 + y^2$ and above by the plane $z = 3$.

c) Reverse the order of integration in the following integral $\displaystyle\int_0^2 \int_{e^y}^{e^2} f(x, y)\, dx dy =$

4. [10] a) State Green's Theorem.

b) Can we use it to evaluate the line integral $\int_{\mathcal{C}} (e^y + \cos(1 + e^{\sin x}))\, dx + xe^y\, dy$, where $\mathcal{C}$ is the portion of the circle $x^2 + y^2 = 4$ going from (2,0) to (0,-2) in the counterclockwise direction? Clearly explain your answer, but do not evaluate the line integral.

5. [15] Find the maximum and minimum values of the function $f(x, y, z) = x + 2y + 3z$ on the ellipsoid $x^2 + 2y^2 + 2z^2 = 1$.

6. [12] Let G be the solid in the first octant bounded below by the cone $z = \sqrt{x^2 + y^2}$ and above by the cone $z = 4 - \sqrt{x^2 + y^2}$. Express the volume of G (Do not evaluate any of the integrals involved, but include all integration limits) using:

a) rectangular coordinates: $V =$

b) cylindrical coordinates: $V =$

c) spherical coordinates: $V =$

7. [15] Let $F(x, y) = (2xy + \sin x)\vec{i} + (x^2 + \cos y)\vec{j}$. a) Show that F is conservative. b) Find a potential function φ for F. c) Evaluate the line integral $\int_{\mathcal{C}} (2xy + \sin x)\, dx + (x^2 + \cos y)\, dy$ along the curve $\mathcal{C}$ parametrized by $\vec{r}(t) = \sqrt{1+t}\,\vec{i} + \sin^{-1} t\,\vec{j}, \quad 0 \leq t \leq 1.$