Syllabus for Algebraic Structures: MAS 4301, Class Number 10699, Spring 2011

Book: J. Rotman "Advanced Modern Algebra" Revised Printing; Prentice Hall 2002.
(The book is out of print, but a pdf version of it is easily obtainable.)

In this course, the basic algebraic notions of groups, rings and fields are introduced and studied.

1) Groups. The main source for examples will be the (symmetric) group of permutations of n elements. Historically, this is the example the theory of groups started from. Moreover, any finite group can be realized as a subgroup of this group. We will study in particular the structure of cyclic, dihedral, alternating groups as well as the famous Klein's V-group. We will prove that almost all alternating groups are simple – a fact needed for the theory of algebraic equations later in the Galois Theory course. To do all that, we will develop the theory of group morphisms and quotient groups, as well as group actions on finite sets. To mention some names, basic fundamental results such as Lagrange's theorem, Cauchy's theorem, the Class Equation will be proved. Based on them, some structure results about finite groups will be established. Applications to Number Theory will be given: Fermat's Little theorem, Wilson's Theorem etc. will be reproved using the group theory techniques developed in the course. More detailed study of the finite groups will be postponed for the Topics in Algebraic Structures class in Fall'10.
2) Rings and Fields. We will be working with commutative rings only. Fields are a special type of commutative rings. The main example will be the ring of polynomials of one variable over a field. We will study ring homomorphisms and quotient rings, will study the arithmetic of polynomial rings (greatest common divisor etc.), will introduce some more special rings (Euclidean rings), and will use them to give some applications to Number Theory (most notably, the sum-of-two-squares theorem of Fermat). In the theory of fields, we will consider field extensions, and will prove Kronecker's theorem that every polynomial has a splitting field (where all its roots live). Finally, the fields with finitely many elements (Galois fields) will be introduced and their basic properties studied. A more detailed investigation of the structure of the fields will be given in the Galois Theory course.

The course of Algebraic Structures introduces the language and studies the basic concepts of Algebra. It is a basis for the next algebraic courses taught at FIU (Topics in Algebraic Structures, Graduate Algebra, and Galois Theory) as well as the courses on Algebraic Geometry I and II.
It is important to note that we will be proving every statement we make in this course. The students are expected to have knowledge about proofs (as explained in the course Introduction to Advanced Mathematics), as well as some background in Number Theory, Set Theory, real, and complex numbers. To refresh the memory of the participants, we will make a quick review of the needed facts and methods as they are exposed in Chapter 1 (Things Past) of the book.

Assessment of students' knowledge: The overall grade will be based on the results on several Quizzes, Turn-in Homework Assignments, Midterm, and Final Exam. All the problems for the exams will be taken from the ones given for work at home during the Semester. **The novelty here is that, on the exams, the students might be asked to give proofs of selected theorems from the course.** The overall grade will be based on 15% of the Turn-in homework's total score, 15% of the total Quizzes' score, 30% of the total of the Midterms' scores, and 40% of the Final Exam score.

The scale for the overall grades follows:

Example: Suppose a student has A points total on the HW, B points total on the Quizzes, C points total on the Midterm Exams, and D points on the Final Exam. Suppose further that the maximal possible points one can get on these are A', B', C', and D' respectively. Then, one can compute a

number S by the formula
$$S = [15*A + 15*B + 30*C + 40*D] / [15*A' + 15*B' + 30*C' + 40*D'].$$

The overall grade of the student above is determined now by the scale:

$0.92 < S$	: A	$0.89 < S < 0.92$	: A-	$0.86 < S < 0.89$	: B+
$0.78 < S < 0.86$	: B	$0.75 < S < 0.78$	: B-	$0.71 < S < 0.75$	: C+
$0.62 < S < 0.71$	: C	$0.58 < S < 0.62$	: C-	$0.55 < S < 0.58$	: D+
$0.49< S < 0.55$	: D	$0.46 < S < 0.49$	: D-	$S < 0.46$	: F

Note: No make-up exams will be scheduled.

Important remark: The Instructor reserves the right to make any changes he considers academically advisable. Any such changes will be announced in advanced in class or by posting them to the e-mail accounts of the students. The students are responsible to be aware of the changes announced this way.