Syllabus for MAS 5311, Graduate Algebra, Fall Term 2009

Book: J. J. Rotman "Advanced Modern Algebra", Revised Printing;
Prentice Hall, 2002, ISBN 0-13-087868-5.

Synopsis: The course is a continuation of the Linear Algebra courses taught at the Department of Mathematics. It provides also a new perspective toward the theory of Abelian groups from the course Topics in Algebraic Structures. It is designed for students with interests in graduate studies in Algebra, Analysis, Differential and Algebraic Geometry, and related areas of math and sciences. More advanced undergraduate students are also encouraged and welcomed to the course.
Background in Linear Algebra and/or Algebraic Structures would be useful but not necessary – the main notions and facts will be reviewed along the way of lectures.

The main objects in Linear Algebra course are fields, vector spaces, and linear mappings. When the fields are replaced by (commutative with an identity element) rings, the vector spaces and the linear mappings are replaced by modules over the rings and morphisms between them.
In the Linear Algebra case, the vector spaces are easy to describe and study: they are determined by their dimension only (the cardinality of any their basis, which always exists). The characterization of the linear mappings, although a little bit harder, is also straightforward.
The case of modules over a ring and their morphisms is much more interesting and hard in general. (Thus, the notion of a basis and dimension of a module are much more subtle and depend on the ring under consideration.) The course is devoted to laying down the basics of the theory of these objects.

The course will begin with the general definitions and properties of rings and modules. The basic types of rings (integral domains, PIDs, polynomial rings, Euclidean rings) will be introduced as well. This occupies Sections 3.1-3.6 of the book above.
Among all possible modules over a ring, there are some which are very important and useful in the modern Mathematics. We will introduce some of those, namely – projective and injective modules, and will be using them extensively throughout the course. See Sections 7.1 and 7.4. Along the way, we will be needing some knowledge about Noetherian rings which we will get from Section 6.3.

There is a very important type of rings, Principal Ideal Domains (PID), with wide range of applications, for which the theory of modules is comparatively easy, and can be developed with the help of the very basic notions of the general theory of modules over rings. Examples of such rings are the ring of integers **Z,** the ring of polynomials of one variable over a field, k[X], and more generally – Euclidean rings. Note that modules over **Z** are also known as Abelian groups.
A substantial part of the course will be devoted to PIDs and modules over them (with emphasis on, but not restricted only to, finitely generated modules). See Section 9.1. The theory of these modules can be investigated in a unified way, and the structure theorems thereof have applications in particular back to Linear Algebra. We will prove, as an illustration, and after reviewing the needed facts from the standard Linear Algebra course facts in Section 3.7, that every linear operator over the field of complex numbers has a canonical form called Jordan Normal Form (a result with far going applications to ODE, Lie Groups and Lie Algebras etc.). See Sections 9.2 and 9.3.

Finally, in Sections 8.1 and 8.2, we will go further into the general case, and will introduce non-commutative rings and modules over them. This generalization is pretty formal and harmless. The goal is to characterize, in this general set-up, all the modules to which the natural generalization of dimension of vector spaces can be realized. This result is known as The Jordan-Holder Theorem.

Due to the active help of Leonard Forret, a former student in this class, there are notes of the course. Those will be distributed to the students in a timely manner.

There will be Take-Home and In-Class Exams, several Quizzes, and Turn-in Homework assignments.

The Instructor reserves the right to make changes, whenever needed, in the content of the lectures which are academically acceptable.