# Syllabus for "Introduction to Advanced Mathematics", Spring 2020,
# MAA 3200, Section U01, Class Number 11904

**Book:** The Lecture Notes are available for reading/downloading on the web site of the Instructor at faculty.fiu.edu/~yotovm

**Description of the course**
The course presents a foundation of mathematics based on math logic and naïve set theory. The presentation is motivated by the slogan: ***every object in mathematics is a set***. Many important techniques and constructions from set theory are discussed in detail. The goal is to prepare the students for the whole spectrum of upper division math courses taught at FIU, with emphasis on Set Theory, Advanced Calculus, and Point-Set Topology.

**Main topics covered in the course are.**
1) **Elements of Logic, and most important Methods of Proof** are discussed in considerable detail in the beginning of the course.
2) **Axiomatic introduction to Set Theory**. The most important set we construct is the set of natural numbers.
3) **The Principle of Math Induction** (PMI), a.k.a. the Principle of Finite Induction. This is a method of proof that we show is a way to interpret the Least Element Principle satisfied by the natural numbers. Several versions of this method of proof are considered, including the method of Complete Induction, and the method of Reverse Induction. Numerous important examples and applications are discussed;
4) **Relations and Orders on sets**: product of sets and relations on sets are among the most important tools for defining other concepts in math; different types of orders are introduced;
5) **Functions**: one of the most important types relations in Set Theory, and hence in math, are functions (a.k.a. maps); one-to-one, onto, and bijective functions are studied in detail; the concept of quotient set is used to characterize maps between sets;
6) **The basic number systems**: the integer numbers, the rational numbers, and the real numbers are introduced, and constructed as sets; real numbers are viewed as Dedekind cuts.
7) **Intro to metric spaces and advanced calculus**: based on the properties of the absolute value in the sets of rational and real numbers, the general concept of a metric space is introduced, the "topology" on such spaces, as well as the notion of accumulation point of a subset of a metric space, are defined; the theory of sequences of rational and of real numbers is developed; complete metric spaces are defined, and Cantor's method (using Cauchy sequences of rational numbers) is sketched to give a new construction of the real numbers; limits and continuity of real valued functions are studied in detail comparing the sequential (via sequences) and Cauchy (epsilon-delta) approaches to these; the concept of compactness is studied: sequential compactness and compactness via open covers (Borel-Lebesgue) are compared; the theorems of Weierstrass and Bolzano-Weierstrass pertaining to compact subsets of metric spaces and continuous functions thereon are proved.

**A word of warning**: The course in general and the Lecture Notes in particular are not easy to understand without active learning. These are indeed very different from what the students might have experienced taking lower level math courses. The students need to read carefully the Notes, do regularly the exercises given for work at home, and ask for help if they need it.

**Homework Assignments:** After every Section covered in class, sets of exercises will be given for work at home. Specified number of them will be given as "turn-in" homework. These latter will be graded and taken in consideration when forming the overall grade in the end of the course.

**Quizzes:** There will be five Quizzes performed in class (for about 15 minutes each).

**Midterm tests:** There will be two Midterm tests. They will cover logic, methods of proof, intro to set theory (Midterm 1), and PMI, relations and orders, and functions (Midterm 2).

**Grading policy:** The lowest graded Quiz will be dropped when forming the overall grade of each student. The overall grade of the students will be formed by taking 10% of the HW grades, 20% of the Quizzes' grades, 30% of the Midterms' grades, and 40% of the Final Exam grade.

**Example:** Suppose a student has A points total on the HW, B points total on the Quizzes, C points total on the Midterm Exams, and D points on the Final Exam. Suppose further that the maximal possible points one can get on these are A', B', C', and D' respectively. Then, one can compute a number S by the formula

$$S = [10*A + 20*B + 30*C + 40*D] / [10*A' + 20*B' + 30*C' + 40*D']$$

The overall grade of the student above is determined now by the scale:

| | | |
|---|---|---|
| | $0.92 < S$ : A | $0.89 < S < 0.92$ : A- |
| $0.86 < S < 0.89$ : B+ | $0.78 < S < 0.86$ : B | $0.75 < S < 0.78$ : B- |
| $0.71 < S < 0.75$ : C+ | $0.62 < S < 0.71$ : C | $0.58 < S < 0.62$ : C- |
| $0.55 < S < 0.58$ : D+ | $0.49 < S < 0.55$ : D | $0.46 < S < 0.49$ : D- |
| | $S < 0.46$ : F | |

**Make-up exams:** No make-up exams will be given.

**Remark:** The Instructor reserves the right to make any changes he considers academically advisable. Any such changes will be announced in advance in class, posted to the web page of the course and to the email accounts of the students. The students are responsible to be aware of the changes announced these ways.