Syllabus for the course Linear Algebra, Lie Algebras, and Differential Geometry, Summer A, 2012

The course comprises the teaching part, the first three weeks, of the Summer Research Program with the same title. It will give the background from several disciplines in Math needed for successful research activities during the last three weeks of the Program. The schedule of the class is as follows.

Monday through Thursday

Morning Session:

10:00 a.m. – 11:15 a.m. - Class lecture

11:15 a.m. – 11:45 a.m. – Coffee break

11:45 a.m. – 1:00 p.m. – Class lecture

Lunch Break:

1:00 p.m. – 2:30 p.m.

Afternoon Session:

2:30 p.m. – 3:45 p.m. – Problem solving session

3:45 p.m. – 4:00 p.m. – Coffee break

4:00 p.m. – 5:00 p.m. – Problem solving session

Friday

Morning session only:

10:00 a.m. – 1:00 p.m. – Discussion on the material taught during the week.

Note: The Afternoon Sessions are compulsory only for the students associated with the Summer Research program, but are open for all students who want to participate

During the teaching morning sessions the students will be given the theoretical material in the form and the amount they will need for the research component of the project. The afternoon sessions are for discussing the problems given to the students for independent work and on the material taught so far. The Friday’s morning session will be devoted to presentation by the students of solutions to selected problems from the week. During the presentations, the students will have to answer questions from the mentors of the project.

Books to be used: The main book on Linear Algebra is "Linear Algebra and Geometry" by Kostrikin and Manin (the participants will be provided with the relevant parts of the book). A complementary book will be the "Exercises in Algebra: A Collection of Exercises, in Algebra, Linear Algebra and Geometry". Later on, we will be using books on Graph Theory, Lie algebras, and Differential Geometry. The information about these will be given in due time.

Assessment of student achievements in class: The overall grade will be determined on the student participation in class, the Friday sessions (to be attended by all students), as well as the afternoon problem solving sessions (for those students who want to be there). After the three weeks of theory learning, the students will be given problems and elements of the theory to prepare for presentation to the mentors of the project. This work should be completed by the end (sixth week) of the project. The overall grade will depend significantly on the quality of this work.

Topics to be covered during the course:

Linear Algebra

Vector spaces, subspaces; linear maps, epi-, mono-, iso-morphisms; the dual space; quotient spaces;

Bases (in general) and their universal property; existence of such (in general); Finite dimensional vector spaces, coordinates arithmetic vector spaces.

Sum and direct sum of vector spaces (dimensions);

(Tensor) product of vector spaces – universal properties (dimensions); the tensor algebra of a vector space; bilinear forms on a vector space and the induced identifications (symmetric and skew-symmetric forms, symplectic). Symmetric products and wedge products of a vector space with itself. Volume forms (determinants) and star operators. Applications to physics.

Primer on Categories

Most basic stuff with examples from Linear Algebra

Lie algebras

Basic linear groups. Lie algebras: definitions; examples (Kaplanski), matrices, vectors in R^3; structural constants; nilpotent Lie algebras; the category of Lie algebras; automorphisms of Lie algebras

Graph Theory

Basic definitions, basic types of graphs, isomorphisms of graphs; more generally - morphisms of graphs; the category of (directed) graphs

Lie algebras v/s graphs

The constructions; categorical considerations; other types of Lie algebras defined this/similar way

Diff. Geometry stuff

(Smooth) Manifolds; maps between manifolds, the category of smooth manifolds; Differential of a map between manifolds, tangent and cotangent bundles; vector fields, differential forms; tensor fields; index notations and change of bases; metric tensor, symplectic form tensor, almost complex structure tensor; Einstein convention; differential and gradient of a function. Electrodynamics, classical mechanics.