

Syllabus for MAS 4302/5311 Topics in Algebraic Structures/Graduate Algebra Section U01, Course Numbers 20069/20070, Spring'24

Book: Paolo Aluffi, Algebra: Chapter 0; ISBN-10: 0821847813 ISBN-13: 978-0821847817

Synopsis: This course is a natural continuation of the Algebraic Structures taught at FIU. It is formally divided in four parts. Part One covers the basic theorems there is on the structure of finite groups (Abelian or non-Abelian), namely Sylow theorems. Some of the techniques needed to prove these theorems were partially introduced in the course Algebraic Structures, and will be fully developed and used here. Part Two of the course is devoted to the classification of the finite Abelian groups, a much easier object to deal with. The classification mentioned will serve as a motivation for the considerations in the second half of the course where modules over Principal Ideal Domains (PIDs) will be studied. In Part Three of the course, the general concepts of ring and module over a ring are introduced and briefly discussed, and the attention is quickly focused on the theory of Unique Factorization Domains (UFDs), important examples of which are Euclidean Domains and PIDs. The main objects of interest here are, as mentioned above, PIDs and modules over them. The principal ideal domains worked with here are fields K , the ring of integers \mathbb{Z} , and the ring of polynomials of one generator over a field $K[X]$. The particularly important examples here are Abelian groups, as modules over the ring of integers, and the vector spaces, as modules over fields. The structure theorem of finite Abelian groups serves, in Part Four, as a toy-model for a similar theorem about finitely generated modules over PIDs. Proving the latter, at least two notable things are achieved: a structure theorem about finitely generated (rather than just finite) Abelian groups (a straightforward thing), and two results about canonical forms of linear operators of finite dimensional vector spaces (over a field K). The first of these forms, called Rational Canonical Form, exists for linear operators over any field, while the second, called Jordan Canonical Form, exists only when the eigenvalues of the operator belong to the field K . In particular, when $K = \mathbb{C}$, the field of complex numbers, any operator has a Jordan Canonical Form (a fact of great importance in many areas of math, in particular ODE, Lie groups and algebras, etc.). For this part of the course, we will need to review some notions on rings, vector spaces, and linear operators. We will do that in a timely manner.

Here is the list of sections of the book which will be covered in the course:

- (i) 4.1, 4.2, 4.3.3, 4.4, 4.5.1, and 4.6. For this part, the knowledge of what a group is, as well as the rudiments of group action (on a set) would be helpful.
- (ii) 5.1, 5.2, 5.3.2, 5.4.2, 5.5.3, and 5.6.1. The definition of the ring of polynomials $R[X]$ over a commutative ring R , from chapter 3, will be discussed as well.
- (iii) 6.1, 6.2.1, 6.2.2, 6.3.2, 6.3.3, 6.5.1, 6.5.3, 6.6, and 6.7.

Course Policy: The overall grade will be based on the results on several Turn-in Homework Assignments, Midterm, and Final Exam. All the problems for the exams will be taken from the ones given for work at home during the Semester. The novelty here is in two specific features of the course. It is designed for both under- and graduate students. On the exams, the students will have the opportunity to work on problems for their respective levels. In particular, the students will be asked to give proofs of selected theorems from the course. Those will reflect the level of the students as well. The overall grade will be based on 30% of the Turn-in homework's total score, 30% of the total of the Midterms' scores, and 40% of the Final Exam score. The scale for the overall grades follows:

Example: Suppose a student has A points total on the HW, B points total on the Midterm Exams, and C points on the Final Exam. Suppose further that the maximal possible points one can get on these are A' , B' , C' , and D' respectively. Then, one can compute a number S by the formula

$$S = [30*A + 30*B + 40*C] / [30*A' + 30*B' + 40*C'].$$

The overall grade of the student above is determined now by the scale:

0.92 < S : A	0.89 < S < 0.92 : A-	
0.86 < S < 0.89 : B+	0.78 < S < 0.86 : B	0.75 < S < 0.78 : B-
0.71 < S < 0.75 : C+	0.62 < S < 0.71 : C	0.58 < S < 0.62 : C-
0.55 < S < 0.58 : D+	0.49 < S < 0.55 : D	0.46 < S < 0.49 : D-
S < 0.46 : F		

No make-up exams will be scheduled.

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Academic Misconduct includes: Cheating – The unauthorized use of books, notes, aids, electronic sources; or assistance from another person with respect to examinations, course assignments, field service reports, class recitations; or the unauthorized possession of examination papers or course materials, whether originally authorized or not. Plagiarism – The use and appropriation of another’s work without any indication of the source and the representation of such work as the student’s own. Any student who fails to give credit for ideas, expressions or materials taken from another source, including internet sources, is responsible for plagiarism. To learn more about the academic integrity policies and procedures visit integrity.fiu.edu (Links to an external site.)

Accessibility and Accommodation

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Remark: The Instructor reserves the right to make any changes he considers academically advisable. Any such changes will be announced in advanced in class or by posting them to the e-mail accounts of the students. The students are responsible to be aware of the changes announced this way.