Besides estimating the value of population parameter, we are also interested in making an inference about how the value of a parameter relates to a _______________. Is it less than, equal to, or greater than the specific value?
This type of inference is called a ________________.

### 8.1 The elements of a test of hypothesis

A statistical hypothesis is a _________ about the numerical value of a population parameter.

**Summary of elements of a test of hypothesis:**

1. _______________ (\( H_0 \)): represents the hypothesis that will be accepted unless the data provide convincing evidence that it is false. This generally represents the status quo, which we adopt until it is proven false.
   \[ H_0 : \________________________\]

2. _______________ (\( H_a \)): represents the hypothesis that will be accepted only if the data provide convincing evidence of its truth.
   \[ H_a : \]

3. _______________ : a sample statistic used to decide between the null and alternative hypothesis.

4. _______________ :
   a. If the numerical value of the test statistic _________ the rejection region, we _________ the null hypothesis and conclude that the alternative hypothesis is true.
   b. If the numerical value of the test statistic _______________ the rejection region, we _______________ the null hypothesis and conclude that there is insufficient evidence to conclude that the alternative hypothesis is true.
• Basic logic of hypothesis testing:

1. Take a _______________ from the population of interest;
2. If the sample data ____________ sufficient evidence to conclude that ______________ is false, we reject \( H_0 \) and assert the alternative hypothesis.
3. If the sample data ____________ provide sufficient evidence to conclude that \( H_0 \) is false, we fail to reject \( H_0 \) and conclude there is not enough evidence to assert the alternative hypothesis.

• Significance level \( \alpha \)

Conclusions and consequences for a test of hypothesis

<table>
<thead>
<tr>
<th>conclusion</th>
<th>True state of Nature (( H_0 ) in fact is)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Court example: \( H_0 \): a person is innocent

Type I error:
Type II error:
Correct decision:
Correct decision:

Type I Error = _______ reject a null hypothesis when in fact it is _______

_______ = The probability of committing a Type I error

Type II Error = _______ reject a null hypothesis when in fact it is _______

_______ = The probability of committing a Type II error

_______ = 1 - \( \beta \) = P(rejecting a null hypothesis when in fact it is false)

The relationship between \( \alpha \) and \( \beta \): If \( \alpha \) is ________, then \( \beta \) will be ________: vice verse.

When we do the hypothesis test, _________ will be controlled.
Typical values for $\alpha$ are: __________________.

Example: To investigate if the mean Statistics test score is greater than 75 in a college, $H_0: \mu = 75$ $H_a: \mu > 75$

then, Type I Error = We __________ , when in fact the mean score is ___.$( H_0 : \mu = 75 \text{ is true. } )$

Type II Error = We __________ , when in fact the mean score _____75 $( H_0 : \mu = 75 \text{ is false}$.)

Note: when conclusion goes to ______$H_0$, there will be only chance to make _____ error.

When conclusion goes to ______$H_0$, there will be only chance to make _____ error.

- How to formulate the null hypothesis and alternative hypothesis:

  Three points: 1. What is the target ______ (mean $\mu$, proportion $p$ or variance $\sigma^2$)?

  2. What is the ___________ the parameter will be compared to?

  3. What is the ____________ between the parameter and the specific value you are interested in comparing (not equal, less than or greater than)?

Examples: Set up null hypothesis and alternative hypothesis and specify Type I, II Error

Example1: SSHA: The Survey of Study Habits and Attitudes is a psychological test that measures the motivation, attitude, and study habits of college students. The mean score is expected 115. A survey based on 81 incoming freshmen result in mean score is 116.2 and standard deviation is 25. Do the data provide evidence that the mean SSHA score is different from 115?

Type I Error: we ____________, when in fact the mean SSHA score _____115 $( H_0 : \mu = 115 \text{ is } ______).$

Type II Error: we ____________, when in fact the mean SSHA score _____ 115. $( H_0 : \mu = 115 \text{ is } ______).$

Example2. Gun Control: In a survey conducted by Louis Harris of LH research, 1250 U.S. adults were polled regarding their views on banning handgun sales, 650 favored a ban. At 5% significance level, do the data provide sufficient evidence to conclude that more than 50% of U.S. adults favor banning handgun sales?

Type I Error: we ______, when in fact the proportion of favoring banning gun sales ____ 0.5.

Type II Error: we______, when in fact the proportion of favoring banning gun sales ____ 0.5.
8.2 Large-sample test of hypothesis about one population mean

- Condition required for a valid large-sample hypothesis test for population mean $\mu$:
  1. A _______ sample is selected from the target population.
  2. The sample size $n$ is _______ (_______).

Under the conditions, by Central Limit Theorem, the Sampling distribution of $\bar{x}$ is approximately normal with:

Mean: 

Standard error: 

- Large sample $Z$-test of hypothesis for a population mean $\mu$:
  1. Set up null hypothesis and alternative hypothesis
     
     $H_0: \__________________________$
     
     $H_a: (two – tailed)$
     
     or $H_a: (lower – tailed)$
     
     or $H_a: (upper – tailed)$
  2. Significance level $\alpha$
  3. Test statistic: $z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \approx$
  4. Rejection region:__________ when $H_a: \mu \neq \mu_0$

          __________ when $H_a: \mu < \mu_0$

          __________ when $H_a: \mu > \mu_0$

![Figure 8.4 Rejection regions corresponding to one- and two-tailed tests](image)

5. Conclusion: if the value of test statistic falls in rejection region, _______ $H_0$, and conclude that

at $\alpha$ level, there is sufficient evidence to conclude $H_a$ is true.

if the value of test statistic does not fall in rejection region, _____________ $H_0$, and

conclude that at $\alpha$ level, there is insufficient evidence to conclude $H_a$ is true.
Examples for Large-sample Z-test of hypothesis about a population mean $\mu$

**Problem** The effect of drugs and alcohol on the nervous system has been the subject of considerable research. Suppose a research neurologist is testing the effect of a drug on response time by injecting 100 rats with a unit dose of the drug, subjecting each rat to a neurological stimulus, and recording its response time. The neurologist knows that the mean response time for rats not injected with the drug (the “control” mean) is 1.2 seconds. She wishes to test whether the mean response time for drug-injected rats differs from 1.2 seconds.

The sample of 100 drug-injected rats yielded mean response time is 1.05 seconds and standard deviation is 0.5 second, using $\alpha = 0.01$ to conduct this test.

At $\alpha = 0.01$, there is sufficient evidence to conclude the mean response time for drug-injected rats differs from the control mean of 1.2 second.

**Step1:** (State the null and alternative hypotheses mathematically and in words of the problem.)

**Step2:** (calculate the value of test statistic.)

**Step3:** (Find the rejection region.)

**Step4:** (Make a decision)

**Step5:** (state a complete conclusion.)

**Example2, SSHA:** The Survey of Study Habits and Attitudes is a psychological test that measures the motivation, attitude, and study habits of college students. The mean score is expected 115. A survey based on 81 incoming freshmen result in mean score is 116.2 and standard deviation is 25. Do the data provide evidence that the mean SSHA score is higher from 115? Use $\alpha = 0.05$.

**Step1:** (State the null and alternative hypotheses mathematically and in words of the problem.)

**Step2:** (calculate the value of test statistic.)
**Step3:** Find the rejection region

**Step4:** (Make a decision.)

**Step5:** (state a complete conclusion.)

**Example 3. HOSPLOS,** The length of stay (in days) in hospital for 100 randomly selected hospital patients are presented in the table. Suppose we want to test the hypothesis that the true mean length of stay (LOS) at the hospital is less than 5 days. Use \( \alpha = 0.05 \).

| LOS for 100 hospital patients | 2, 3, 8, 6, 4, 4, 6, ........., 10, 2, 4, 2 |

**Step1:** (State the null and alternative hypotheses mathematically and in words of the problem.)

**Step2:** (calculate the value of test statistic.)

**Step3:** Find the rejection region

**Step4:** (Make a decision.)

**Step5:** (state a complete conclusion.)
SPSS output for HOSPLOS:
One-Sample Statistics

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>Std. Error Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOS</td>
<td>100</td>
<td>4.53</td>
<td>3.678</td>
<td>.368</td>
</tr>
</tbody>
</table>

One-Sample Test

<table>
<thead>
<tr>
<th></th>
<th>t</th>
<th>df</th>
<th>Sig. (2-tailed)</th>
<th>Mean Difference</th>
<th>95% Confidence Interval of the Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOS</td>
<td>-1.278</td>
<td>99</td>
<td>2.04</td>
<td>-.470</td>
<td>-1.20 - .26</td>
</tr>
</tbody>
</table>

8.3 Observed significance levels: p-values

__________ (OSL): the _________ (assuming $H_0$ is true) of observing a value of the test statistic that is at least contradictory to the null hypothesis, and supportive of the alternative hypothesis, as the actual one computed from the sample data. This measure the disagreement with the null hypothesis based on the sample data.

- How to calculate the p-value: (the value of test statistic and $H_a$)
  1. Determine _______________________ based on the sample data;
  2. p-value = ____________ when $H_a: \mu \neq \mu_0$

     p-value = ____________ when $H_a: \mu < \mu_0$

     p-value = ____________ when $H_a: \mu > \mu_0$
Examples for p-value calculation:

Recall three examples in section 8.2 (using p-value to make decision)

**Example1**, The sample of 100 drug-injected rats yielded mean response time is 1.05 seconds and standard deviation is 0.5 second, using $\alpha = 0.01$ to test whether the mean response time for drug-injected rats differs from control mean of 1.2 second.

**Step1**: (State the null and alternative hypotheses mathematically and in words of the problem.)

**Step2**: (calculate the value of test statistic.)

**Step3**: Calculate p-value.
Step 4: (Make a decision.)

Step 5: (state a complete conclusion.)

Example 2, SSHA: A survey based on 81 incoming freshmen result in mean SSHA score is 116.2 and standard deviation is 25. Do the data provide evidence that the mean SSHA score is higher than 115? Use $\alpha = 0.05$.

Step 1: (State the null and alternative hypotheses mathematically and in words of the problem.)

Step 2: (Calculate the value of test statistic.)

Step 3: Calculate p-value.

Step 4: (Make a decision.)

Step 5: (state a complete conclusion.)
Example 3. HOSPLOS, The length of stay (in days) in hospital for 100 randomly selected hospital patients are presented in the table. Suppose we want to test the hypothesis that the true mean length of stay (LOS) at the hospital is less than 5 days. Use $\alpha = 0.05$.

| LOS for 100 hospital patients | 2, 3, 8, 6, 4, 4, 6, ………, 10, 2, 4, 2 |

Step 1: (State the null and alternative hypotheses mathematically and in words of the problem.)

Step 2: (Calculate the value of test statistic.)

Step 3: Calculate p-value.

Step 4: (Make a decision.)

Step 5: (State a complete conclusion.)

SPSS output for HOSPLOS,

One-Sample Statistics

<table>
<thead>
<tr>
<th>N</th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>Std. Error Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOS</td>
<td>100</td>
<td>4.53</td>
<td>3.678</td>
</tr>
</tbody>
</table>

One-Sample Test

| Test Value = 5 |
|---|---|---|---|
| LOS | 1.278 | 99 | 204 |
| Mean Difference | -.470 | 95% Confidence Interval of the Difference |
| Lower | .20 | Upper | .26 |
Note: compare the examples in sec8.2 and sec8.3, you can find out either using rejection region or p-value approach to make decision always leads to the same conclusion.

- **Two approaches to do hypothesis test:**
  
  **A. _______________ Approach:**
  1. set up null hypothesis and alternative hypothesis,
  2. calculate test statistic,
  3. find appropriate rejection region,
  4. make conclusion. (if test statistic falls in R. R, reject null hypothesis.)

  **B. ________________ Approach:**
  1. set up null hypothesis and alternative hypothesis,
  2. calculate test statistic,
  3. find p-value for the test,
  4. make conclusion. (if p-value is smaller than \( \alpha \), reject null hypothesis.)

### 8.4 Small-Sample test of hypothesis about a population mean

- **Condition required for a valid small-sample hypothesis test for \( \mu \):**
  1. A __________________ is selected from the target population;
  2. The population from which the sample is selected has a distribution that is approximately______________.
  3. Sample size is small (_________).

- **Small sample t- test of hypothesis for a population mean \( \mu \):**
  1. set up null hypothesis and alternative hypothesis
    
    \[
    H_0 : \mu = \mu_0
    \]
    
    \[
    H_a : \mu \neq \mu_0 \text{ (two-tailed)}
    \]
    
    or \( H_a : \mu < \mu_0 \text{ (lower-tailed)} \)
    
    or \( H_a : \mu > \mu_0 \text{ (upper-tailed)} \)

  2. significance level \( \alpha \)

  3. test statistic: \( t = \) __________with \( \text{df} = \) __________

  4. rejection region: __________when \( H_a : \mu \neq \mu_0 \)
5. conclusion: if the value of test statistic falls in R.R, reject $H_0$, and conclude that at $\alpha$ level, there is sufficient evidence to conclude $H_a$ is true.

if the value of test statistic does not fall in R.R, do not reject $H_0$, and conclude that at $\alpha$ level, there is insufficient evidence to conclude $H_a$ is true.

• How to find the p-value: (the value of test statistic and $H_a$)

1. Determine the value of test statistic based on the sample data;

2. p-value = ____________ when $H_a: \mu \neq \mu_0$

   p-value = ____________ when $H_a: \mu < \mu_0$

   p-value = ____________ when $H_a: \mu > \mu_0$

Note: since t-table in our textbook only provides some critical t-values, we can’t use t-table to find exact p-value. We can use statistical software to calculate the appropriate p-value.
Examples for Small-sample t-test of hypothesis about a population mean $\mu$

**Example1. EMISSION,**

A major car manufacturer wants to test a new engine to determine whether it meets new air-pollution standards. The mean emission of this type engines must be **less than 20** parts per million of carbon. Ten engines are manufactured for testing purposes. The data is listed below.

| Emissions | 15.6 | 16.2 | 22.5 | 20.5 | 16.4 | 19.4 | 19.6 | 17.9 | 12.7 | 14.9 |

Do the data supply sufficient evidence to allow the manufacturer to conclude that the type of engine **meets the pollution standard**? Assume to risk a type I error with probability $\alpha = 0.01$.

**Step1:** (State the null and alternative hypotheses mathematically and in words of the problem.)

**Step2:** (calculate the value of test statistic.)

**Step3:** (Find the rejection region.)

**Step4:** (Make a decision)

**Step5:** (state a complete conclusion.)
SPSS output for EMISSION,

<table>
<thead>
<tr>
<th>N</th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>Std. Error Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>17.5700</td>
<td>2.95223</td>
<td>.93358</td>
</tr>
</tbody>
</table>

One-Sample Test

<table>
<thead>
<tr>
<th>t</th>
<th>df</th>
<th>Sig. (2-tailed)</th>
<th>Mean Difference</th>
<th>95% Confidence Interval of the Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2.603</td>
<td>9</td>
<td>.029</td>
<td>-2.43000</td>
<td>-4.5419 - .3181</td>
</tr>
</tbody>
</table>

Example2,

Most water-treatment facilities monitor the quality of their drinking water on an hourly basis. One variable monitored is pH, which measures the degree of alkalinity or acidity in the water. A pH below 7.0 is acidic, one above 7.0 is alkaline, and a pH of 7.0 is neutral. One water-treatment plant has a target pH of 8.5. (Most try to maintain a slightly alkaline level.) The mean and standard deviation of 1 hour’s test results, based on 17 water samples at this plant, are

$\bar{x} = 8.42 \quad s = .16$

Does this sample provide sufficient evidence that the mean pH level in the water differs from 8.5?

**Step1:** (State the null and alternative hypotheses mathematically and in words of the problem.)

**Step2:** (calculate the value of test statistic.)

**Step3:** (Find the rejection region.)

**Step4:** (Make a decision)

**Step5:** (state a complete conclusion.)
8.5 Large-sample test of hypotheses about a population proportion

- **Condition required for a valid large-sample hypothesis test for \( P \):**
  1. A ______ sample is elected from a binomial population;
  2. The sample size \( n \) is _____ (This condition will be satisfied if ___________________)

- **Large-sample test for a population proportion \( P \):**
  1. **set up null hypothesis and alternative hypothesis**
     \[
     H_0 : \\
     H_a : \quad \text{(two-tailed)} \\
     \text{or } H_a : \quad \text{(lower-tailed)} \\
     \text{or } H_a : \quad \text{(upper-tailed)}
     \]
  2. **significance level** \( \alpha \)
  3. **test statistic:** \( z = \)
  4. **rejection region:** _____________ when \( H_a : p \neq p_0 \)
     _____________ when \( H_a : p < p_0 \)
     _____________ when \( H_a : p > p_0 \)
  5. **conclusion:** if the value of test statistic falls in R.R, reject \( H_0 \), and conclude that at \( \alpha \) level, there is sufficient evidence to conclude \( H_a \) is true.

      if the value of test statistic does not fall in R.R, do not reject \( H_0 \), and conclude that at \( \alpha \) level, there is insufficient evidence to conclude \( H_a \) is true.
Examples for Large-sample test for a population proportion $P$:

Example 1. Shipment Defectives.
The reputations of many businesses can be severely damaged by shipments of manufactured items that contain a large percentage of defectives. A manufacturer of alkaline batteries wants to be reasonably certain that fewer than 5% of its batteries are defective. Suppose 300 batteries are randomly selected from a large shipment, 10 defective batteries are found. Does this provide sufficient evidence for the manufacturer to conclude that the fraction defective in the entire shipment is less than 0.05? Use $\alpha = 0.01$.

Step 1: (State the null and alternative hypotheses mathematically and in words of the problem.)

Step 2: (Calculate the value of test statistic.)

Step 3: (Find the rejection region.)

Step 4: (Make a decision)

Step 5: (State a complete conclusion.)
Example 2. Gun Control: In a survey conducted by Louis Harris of LH research, 1250 U.S. adults were polled regarding their views on banning handgun sales, 650 favored a ban. At 10% significance level, do the data provide sufficient evidence to conclude that more than half of U.S. adults favor banning handgun sales?

Step 1: (State the null and alternative hypotheses mathematically and in words of the problem.)

Step 2: (Calculate the value of the test statistic.)

Step 3: (Find the rejection region.)

Step 4: (Make a decision)

Step 5: (State a complete conclusion.)

Learning objective of Chapter 8:
1. describe the basic elements of a test of hypothesis
2. Understand type I, type II error; significance level $\alpha$; set up $H_0$, $H_a$; specify rejection region.
3. conduct a compete z-test of population mean (5 steps, critical value approach)
4. conduct a compete t-test of population mean (5 steps, critical value approach)
5. conduct a compete z-test of population proportion (5 steps, critical value approach)
6. know how to use p-value to make decision (if p-value < $\alpha$, reject $H_0$)