We now turn to a discussion of two-party electoral competition in representative democracy. The underlying policy question addressed in this chapter, as well as the remaining chapters of this part, is what determines the size of government spending. To make things as transparent as possible, we abstain from the difficulties caused by multidimensional policy conflict discussed in chapter 2. Instead, we deal with a very simple policy example, where conflict among the voters is unidimensional. The policy to be determined concerns the size of a program, supplying a publicly provided good, that benefits all the voters alike and is financed by proportional income taxes, income being the only dimension of heterogeneity among the voters. The same economic model is used in all the remaining chapters of part 1, with slight variations only.

Throughout the chapter we retain two key assumptions. First, political candidates are opportunistic. More precisely, their only motivation is to hold office. Candidates thus do not care what policy is implemented: they do not have partisan preferences, and they do not benefit directly from the policy because the rents from holding office are exogenously given and independent of policy. Alternative assumptions about candidate motivation are discussed in the next two chapters, in which we deal with agency problems and with partisan policy preferences, respectively. Second, throughout the chapter we assume that candidates commit to a well-defined policy ahead of the elections, sticking to the realm of pre-election politics. Thus, electoral competition is viewed as a choice of location by two competing parties. The parties announce a policy platform, so as to maximize the probability of victory, and voters select the preferred policy. The policy announced by the winning candidate gets implemented.

This model of pre-election politics with opportunistic politicians is widely used in the literature. It naturally directs the attention to the conflict among voters over alternative policies and to the question of which groups of voters are more influential. As we shall see, equilibrium policy here reflects features of the voters themselves, such as the distribution of their policy preferences, the ability of different groups to organize as a lobby, or the likelihood that different groups reward policy favors with a vote. Even under the maintained assumptions of this chapter, however, there is not a single model of electoral competition. On the contrary, depending on the specific assumptions about the likelihood that voters reward policy favors with a vote, different forces influence electoral competition and the resulting equilibrium policy.

In section 3.1, we formulate the underlying policy problem. Sections 3.2 and 3.3 discuss the simplest possible model of electoral competition, due to Downs’s classical study, in which the competing parties are identical in all respects and voters care only about economic policy. Here, both parties converge to the median voter optimum. Section 3.4 illustrates the equilibrium under probabilistic voting, while section 3.5 adds lobbying. In these sections different political forces are at play and policy diverges from the median voter’s bliss point.
3.1 A Simple Model of Public Finance

Consider a society inhabited by a large number (formally a continuum) of citizens, where we normalize the size (mass) of the population to unity. These citizens are of different types indexed by $i$. Each type $i$ has the same basic and quasi-linear preferences over private consumption $c$ and publicly provided goods $g$, which is given by

$$w^i = c^i + H(g),$$

where $H(\cdot)$ is a concave and increasing function. Implicit in (3.1) is also the (unrealistic) assumption that government spending cannot be targeted to specific groups but instead must be provided in the same, nonnegative, amount to everyone: $g^i = g \geq 0$. We can interpret $g$ in different ways, as publicly provided private goods, or traditional public goods. In either case, we let $g$ measure spending per capita. Nor can taxes be targeted, so government spending is financed by taxing the income of every individual at a common rate $\tau$, bounded by $0 \leq \tau \leq 1$.

Income differs across individuals, however, implying that their consumption differs according to

$$c^i = (1 - \tau) y^i.$$  \hspace{1cm} (3.2)

We assume that $y^i$ is distributed in the population according to a cumulative distribution function (c.d.f.) $F(\cdot)$. The expected (average) value of a variable such as $y^i$ is always denoted by a symbol without superscript, that is, $E(y^i) = y$, where $E$ denotes an expected value. Finally, the median value of $y^i$, labeled $y^m$, is implicitly defined by $F(y^m) = \frac{1}{2}$. We assume that $y^m \leq y$, so that the income distribution is skewed to the right, in accordance with evidence from virtually every country. The government budget constraint is then simply

$$\tau y = g.$$

Given these preliminaries, we can easily write down the policy preferences of citizen $i$ as

$$W^i(g) = (y - g) \frac{y^i}{y} + H(g).$$  \hspace{1cm} (3.3)

These preferences are concave in policy, implying that every citizen has a uniquely preferred policy. It is easy to see that this policy satisfies

$$g^i = H^{-1}_g(y^i / y).$$  \hspace{1cm} (3.4)

Clearly, the policy conflict between different citizens is quite smooth in this model. This smoothness, of course, reflects the restrictive assumptions about the policy space, namely
that neither government spending nor taxes can be targeted to specific voters or groups of
evoters and that politicians cannot appropriate tax revenues as rents for themselves. Policy
preferences therefore become monotonic in the one parameter that distinguishes individuals,
namely their relative income, \( y^i / y \). Richer individuals want a smaller government because,
with taxes proportional to income, they pay a larger share of the tax burden. By concavity
of \( H(\cdot) \), (3.4) implies that \( g^i \) is decreasing in \( y^i \).

It is easy to see that these policy preferences fulfill the single-crossing condition (2.4),
introduced in chapter 2. Here, the corresponding condition is

If \( g > g' \) and \( y^{ii} < y^i \), or \( g < g' \) and \( y^{ii} > y^i \), then

\[
W^i(g) \geq W^i(g') \Rightarrow W^{ii}(g) \geq W^{ii}(g').
\]

These properties of policy preferences considerably simplify the analysis to follow.

Let us also formulate a normative benchmark. As a basis for this benchmark, consider a
utilitarian social welfare function that simply sums up (integrates over) the welfare of all
individual citizens:

\[
w = \int W^i(g) dF = W(g),
\]

where the last term is just the utility of the average individual, namely the individual
with average income. The second equality follows from the definition of \( W^i(\cdot) \) and the
fact that \( \mathbb{E}(y^i) = y \). Even though a utilitarian objective is often quite restrictive, it is not
very restrictive in conjunction with quasi-linear preferences, as these rule out meaningful
distributional considerations anyway. According to the utilitarian objective, the socially
optimal policy coincides with the policy desired by the average citizen:

\[
g^* = H^{-1}_g(1).
\]

Problem 2 of this chapter formulates an alternative simple model of public finance, in
which agents are heterogenous in their preferences for public versus private goods, rather
than in their income. Problems 2–5 deal with equilibria in this alternative model, under the
same assumptions about the political environment as those we will make in sections 3.2–3.5.

### 3.2 Downsian Electoral Competition

Throughout the chapter, we maintain a number of assumptions about the nature of
political competition and candidates that are akin to those in Downs’s classical study.
Like Downs, we postulate two candidates—or parties, as the two here boil down to the
same thing—indexed by \( P = A, B \). Each of these maximizes the expected value of some
exogeuous rents, \( R \). These exogenous rents reflect the value attached to winning the
elections
and holding office, but they do not appear in the government budget. Candidate $P$ thus sets his policy so as to maximize $p_P R$, where $p_P$ is the probability of winning the election, given the other candidate’s policy. If we use $\pi_P$ to denote the vote share of candidate $P$, we can write $p_P = \operatorname{Prob} [\pi_P \geq \frac{1}{2}]$.

The timing of events is as follows: (1) The two candidates, simultaneously and noncooperatively, announce their electoral platforms: $g_A, g_B$. (2) Elections are held, in which voters choose between the two candidates. (3) The elected candidate implements his announced policy platform. The candidates’ commitments to their electoral platforms are thus assumed to be binding.

To see how the model works, we start with a very simple case. Assume that the income distribution is degenerate so that every citizen has the same income $y^i = y$. Voters thus face a very simple problem, and they just vote for the candidate whose platform gives them the highest utility. If indifferent, a voter tosses a coin to decide for whom to vote. This implies the following probability of winning for candidate $A$:

$$p_A = \begin{cases} 0 & \text{if } W(g_A) < W(g_B) \\ \frac{1}{2} & \text{if } W(g_A) = W(g_B) \\ 1 & \text{if } W(g_A) > W(g_B), \end{cases}$$

whereas $p_B$ is just given by $1 - p_A$.

Suppose now that candidate $A$’s announcement $g_A$ is further away, utility-wise, from the unanimously preferred policy $g^*$ than candidate $B$’s announcement $g_B$. Obviously, $A$ can then discontinuously increase his probability of winning by announcing a policy closer to $g^*$. As the same holds true for candidate $B$, there is a unique subgame-perfect equilibrium $g_A = g_B = g^*$.

Both candidates thus converge to the socially optimal policy.

The normative implications are thus consistent with the claim made by the Chicago school: political competition indeed leads to an optimal outcome for society. The positive implications—thinking about variation in the size of government across countries or time—are also straightforward. Observed differences are entirely driven by voters’ policy preferences. For example, trendwise growth in government could be consistent with Wagner’s law if the marginal benefits of $g$ are positively correlated with average income $y$. Similarly, it could be consistent with Baumol’s disease if the relative cost of
government versus private goods had an upward trend, because of an adverse productivity development due to the nature of government production. This could be formally shown by adding a parameter capturing the relative cost of public goods to the government budget constraint, as is done in the next chapter.

3.3 Median-Voter Equilibria

When voters disagree over the desired fiscal policy, the candidates must decide which voters to please, in order to enhance their chances of winning the election. To study this question, we assume that the income distribution is no longer degenerate and that the c.d.f. \( F(\cdot) \) is indeed a continuous function. The equilibrium we will study in this setting is an application of the median-voter theorem, proposed by Black for voting in committees and applied to electoral competition by Downs.

Voter \( i \) now votes for candidate A with certainty only if \( W^i(g_A) > W^i(g_B) \). Under the other assumptions of the model, we have

\[
p_A = \begin{cases} 
0 & \text{if } W^m(g_A) < W^m(g_B) \\
\frac{1}{2} & \text{if } W^m(g_A) = W^m(g_B) \\
1 & \text{if } W^m(g_A) > W^m(g_B). 
\end{cases}
\] (3.6)

The pivotal role played by the voter with median income \( y^m \) is easy to establish. Recall from (3.4) that \( g^i \) is decreasing in \( y^i \). This fact and the monotonicity of preferences (3.5) imply that whenever the median voter prefers one platform over the other, at least half of the electorate agrees. To see the logic behind this separation argument, suppose, for example, that \( y^m \) considers \( g_B \) too low relative to \( g_A \). Then so does everyone with \( y^i < y^m \), as they prefer an even larger government, \( g^i > g^m \). More than half the electorate would thus vote for A. Given this, the only situation in which neither of the candidates can increase his probability of winning is when they have both converged to the policy preferred by the median voter: \( g_A = g_B = g^m \). In the jargon of chapter 2, \( g^m \) is the unique Condorcet winner, that is, a policy capable of beating any alternative policy in a pairwise vote. Individuals with median income become pivotal, and both candidates converge to those individuals’ bliss point.

The median-voter equilibrium suggests a new set of determinants to the size of the public sector. By (3.4), the first-order condition describing the equilibrium is

\[
g^m = H_{g^{-1}}(y^m/y). 
\] (3.7)

Thus as \( y^m \) drops relative to \( y \), \( g^m \) rises: a relatively poorer median voter prefers a larger government. Thus (3.7) says that larger governments (a higher \( \tau \) and \( g \)) are associated with
a more skewed income distribution in the specific sense of a higher percentage gap between median and mean income. Furthermore, what matters for the political equilibrium is median income in the electorate, whereas, by the government budget constraint, average income refers to the population as a whole. An extension of the franchise, extending voting rights to poorer segments of the population, should therefore also raise the equilibrium size of government, since it widens the gap between the median voter’s income and that of the average citizen. The normative properties are also simple to state. If the income distribution is symmetric, so that \( y^m = y \), then electoral competition still implements a socially optimal allocation. But an income distribution skewed to the right implies overspending and overtaxation, at least relative to the utilitarian benchmark. The inequality predictions, in particular, have been studied quite extensively. To summarize very briefly, however, it has been hard to find compelling empirical evidence supporting the predictions.

We rely on these kinds of median-voter equilibria in several other parts. Such equilibria are useful, for they are so simple to characterize; as a result, one can add much more structure to the model’s economic side. We use them to discuss the determinants of pensions, unemployment insurance, and regional redistribution. We also analyze capital taxation and its implications for growth in a median-voter model. But existence of a Condorcet winner does not mean this policy will be implemented.

### 3.4 Probabilistic Voting

Up to this point, voters have cared only about the economic policy platforms announced by the two candidates. But candidates, or parties, may also differ in some other dimension unrelated to this policy, \( g \). We shall refer to this other dimension as “ideology,” but it could also involve other attributes such as the personal characteristics of the party leadership. This ideological dimension is a permanent feature in that it cannot credibly be modified as part of the electoral platform.

Furthermore, we assume that voters differ in their evaluation of these features. One way to motivate this assumption is to think about a second policy dimension, orthogonal to fiscal policy, in which candidates cannot make credible commitments but set an optimal policy after the election according to their ideology. Voters’ preferences over the alternative policy dimension imply derived preferences over the candidates themselves.

In this setting, some groups of voters may become more attractive prey for office-seeking politicians, who are willing to modify policy in the direction of the favored groups. To
To illustrate this mechanism, we build on a version of the probabilistic voting model introduced in chapter 2.

To make the point in a transparent way, we now assume that the population consists of three distinct groups, \( J = R, M, P \), representing the rich, the middle class, and the poor, respectively. Everyone in group \( J \) has the same income \( y^J \), with the obvious ranking: \( y^R > y^M > y^P \). The population share of group \( J \) is \( \alpha^J \), with \( \sum_J \alpha^J = 1 \). (Formally, \( F(\cdot) \) is now a step function with \( F(\cdot) = 0 \), for \( y^i < y^P \), \( F(\cdot) = \alpha^P \), for \( y^P \leq y^i < y^M \), and so on).

Naturally, by definition of \( y \), \( \sum_J \alpha^J y^J = y \). At the time of the elections, voters base their voting decision both on the economic policy announcements and on the two candidates’ ideologies. Specifically, voter \( i \) in group \( J \) prefers candidate \( A \) if

\[
W^J(g_A) > W^J(g_B) + \sigma^{iJ} + \delta. \tag{3.8}
\]

Here, \( \sigma^{iJ} \) is an individual-specific parameter that can take on negative as well as positive values. It measures voter \( i \)'s individual ideological bias toward candidate \( B \). A positive value of \( \sigma^{iJ} \) implies that voter \( i \) has a bias in favor of party \( B \), whereas voters with \( \sigma^{iJ} = 0 \) are ideologically neutral, that is, they care only about economic policy. We assume that this parameter has group-specific uniform distributions on

\[
\left[-\frac{1}{2\phi^J}, \frac{1}{2\phi^J}\right].
\]

These distributions thus have density \( \phi^J \), and each group has members inherently biased toward both candidates. The parameter \( \delta \), which measures the average (relative) popularity of candidate \( B \) in the population as a whole, can also be positive or negative. Here, too, we assume a uniform distribution on

\[
\left[-\frac{1}{2\psi}, \frac{1}{2\psi}\right].
\]

Although the distributional assumptions regarding \( \sigma^{iJ} \) and \( \delta \) are special, they facilitate a simple closed-form solution. We discuss below how to generalize these distributional assumptions.

The timing of events is as follows. (1) The two candidates, simultaneously and noncooperatively, announce their electoral platforms: \( g_A, g_B \). At this stage, they know the voters’ policy preferences. They also know the distributions for \( \sigma^{iJ} \) and \( \delta \), but not yet their realized values. (2) The actual value of \( \delta \) is realized and all uncertainty is resolved. (3) Elections are held. (4) The elected candidate implements his announced policy platform.

To formally study the candidates’ decisions at stage (2), let us identify the “swing voter” in group \( J \), a voter whose ideological bias, given the candidates’ platforms, makes him
indifferent between the two parties:

\[ \sigma^J = W^J(g_A) - W^J(g_B) - \delta. \]  \hfill (3.9)

All voters \( i \) in group \( J \) with \( \sigma^{iJ} \leq \sigma^J \) prefer party \( A \). Hence, given our distributional assumptions, candidate \( A \)'s actual vote share is

\[ \pi_A = \sum J \alpha^J \phi^J \left( \sigma^J + \frac{1}{2\phi^J} \right). \]

Since \( \sigma^J \) depends on the realized value of \( \delta \), the vote share \( \pi_A \) is also a random variable. From the perspective of both candidates, the electoral outcome is thus a random event, related to the realization of \( \delta \). Specifically, candidate \( A \)'s probability of winning, given (3.9), becomes

\[ p_A = \text{Prob}_{\delta} \left[ \pi_A \geq \frac{1}{2} \right] = \frac{1}{2} + \frac{\psi}{\phi} \left[ \sum J \alpha^J \phi^J \left( W^J(g_A) - W^J(g_B) \right) \right], \]  \hfill (3.10)

where \( \phi = \sum J \alpha^J \phi^J \) is the average density across groups. Obviously, candidate \( B \) wins with probability \( 1 - p_A \).

This objective illustrates a general property of probabilistic voting models. As both individual utility and the distribution of ideological preferences are continuous functions, the probability of winning now becomes a smooth function of the distance between the two electoral platforms. In this sense, competition between the two candidates becomes less stiff than in the traditional model of the previous section. Hence, we should also expect equilibrium behavior to differ.

The unique equilibrium has both candidates converging to the same platform. Convergence follows from the two candidates’ facing exactly the same optimization problem. To see this formally, notice that \( g_A \) and \( g_B \) enter the expression in (3.10) with opposite signs and that \( p_A R = (1 - p_A)R \). Intuitively, the two candidates share the same concave preferences and the same technology for converting tax dollars into expected votes, so they end up finding the same policy announcements optimal.

In equilibrium, therefore, both candidates maximize a weighted social welfare function. The weights, \( \phi^J \alpha^J \), correspond to group size, \( \alpha^J \), as in the utilitarian optimum, but also to
Figure 3.1

the group densities, $\phi^J$, because these densities summarize how responsive are the voters in each group to economic policy, that is, how each group rewards policy with votes at the elections.

To illustrate the equilibrium, consider first figure 3.1, which depicts the distribution of $\sigma^{iJ}$ in the three groups. All three distributions are symmetric around a mean of zero. The height of the distribution corresponds to the density $\phi^J$ and measures how many voters are gained in that group per marginal increase in economic welfare in the group.

As the figure is drawn, group $R$ has the highest density and group $P$ the lowest. In equilibrium, all candidates announce the same policy. Thus the equilibrium swing voter in each group is the individual with parameter $\sigma^J = -\delta$. Voters with $\sigma^{iJ}$ to the left of $-\delta$ in each group vote for party $A$; the others vote for $B$.

Now consider a small unilateral deviation from the equilibrium policy by candidate $A$, promising a smaller government policy and lower taxes. The utility functions (3.3) of the three groups of voters are illustrated in figure 3.2, where $g^*$ denotes the equilibrium policy.

A smaller government benefits group $R$, for whom equilibrium spending is too high, but hurts groups $M$ and $L$, for whom the opposite is true. As a result of the deviation, party $A$ thus gains votes in group $R$ and loses votes in the other two groups. Going back to figure 3.1, the new swing voter in group $R$ is individual $\sigma^R$, whereas the new swing voters in the other two groups correspond to the points $\sigma^M$ and $\sigma^L$. Note that the horizontal distance between $\sigma^J$ and $-\delta$ in each group is proportional to the utility gain or loss from
the deviation. As the figure is drawn, the equilibrium is closest to the bliss point of group \( R \), and the horizontal shift of the swing voter is accordingly smallest in that group, since at the margin the deviation affects him least. There are no incentives to deviate from the equilibrium when the shaded area to the right (the gain in votes) is equal to the shaded areas to the left (the loss in votes). Figure 3.2 illustrates the implication: equilibrium policy must be especially catered in favor of group \( R \). As this group has many swing voters, it must have less-intense policy preferences, which implies an equilibrium policy relatively close to its bliss point. The opposite is true for group \( P \).

To characterize the equilibrium algebraically, take the derivative of \( p_A \) with respect to \( g_A \), using the definition of voter utility in (3.3). Setting the resulting expression equal to 0 and rearranging, we obtain

\[
\sum_j \alpha^J \phi^J H_g(g) = \frac{1}{y} \sum_j \alpha^J \phi^J y^J.
\]

Recalling the definition \( \phi = \sum_j \alpha^J \phi^J \), we can write

\[
\hat{g} = H_g^{-1}(\bar{y}/y),
\] (3.11)
where
\[ \tilde{y} = \frac{\sum J \alpha^J \phi^J y^J}{\phi} \]
and the $S$ superscript stands for swing-voter equilibrium. This equilibrium can be socially optimal: if the density $\phi^J$ is the same in all groups $\phi^J = \phi$, we have $\tilde{y} = y$, and $g_S = g^*$. Both parties are trying to maximize their expected vote and are therefore appealing to the expected swing voters in each group. If the number of swing voters (slightly abusing the fact that within-group distributions are continuous) is the same, all groups get equal weight in the candidates’ decision, which makes them maximize the average voter’s utility. Generally, groups will differ, however, in how easily their votes can be swayed. Office-seeking candidates will therefore not give them equal weight. If $\phi^J$ is high, because the group is ideologically homogeneous, it has a large number of swing voters. In line with the discussion around figures 3.1 and 3.2, this makes the group attractive for the candidates, who thus tilt their policy in the direction desired by the group.

In drawing figures 3.1 and 3.2, we assumed $\phi^R > \phi > \phi^P$, so that the poor group had the least number of swing voters. In this case, obviously, $\tilde{y} > y$, implying a size of government that is below its benchmark: $g_S < g^*$. For given $y$, the bias is larger, the larger is the rich group ($\alpha^R$ higher)—as more votes can be gained—and the richer it is relative to the middle class ($y^R - y^M$ larger)—as the rich then have a higher stake in the policy. Furthermore, the bias is larger, the larger is the poor group relative to the middle class (lower $\phi$ given $\phi^R$). We see that the conclusion runs exactly counter to the prediction of the median-voter model of the previous section, namely that more-skewed income distributions are associated with larger governments. The previous conclusion is salvaged only if we make the opposite assumption about ranking, namely $\phi^R < \phi < \phi^P$, so that the poor have the most political clout and $\tilde{y} < y$. In this event, more concentration of income at the top, or more poverty at the bottom, for a given $y$, indeed lowers $\tilde{y}$. This in turn leads to a larger government and a positive association between inequality and the size of government.

How does the equilibrium change for more general distributions of voters’ ideological preferences, namely if the group distributions of the parameter $\sigma^{i,J}$ are not uniform, but unimodal? Not much. The properties of the equilibrium are dictated by the group density of $\sigma^{i,J}$ in a neighborhood of the equilibrium policy. But the interpretation remains the same: ideologically neutral groups are more responsive to policy (and hence care less about ideology) in a neighborhood of equilibrium; they are thus more likely to reward politicians with votes and get a policy closer to their bliss point. Naturally, the second-order conditions for an optimum impose additional restrictions on the distributional form, to guarantee existence of equilibrium. Problem 4 of this chapter deals formally with the extension to more general distributions in a similar model of probabilistic voting.
The general lesson from the probabilistic voting model is that ideological shifts in the population systematically alter the political power of different groups. Ideologically neutral groups with many mobile voters—those willing to swing their vote for small changes in economic policy—become an attractive target for office-seeking politicians. A seemingly small and trivial change in the underlying model of electoral competition (the competing parties are not identical but instead differ somewhat in voters’ eyes) changes dramatically the implications for the equilibrium. Rather than trying to please the median individual, both parties now seek to please the more mobile voters. We know of no attempts in the literature to try to discriminate empirically between this model of electoral competition and the median-voter model. Direct tests of the probabilistic voting model would have to combine disaggregated historical electoral data, proxying for mobility in different voter groups, with data on economic policy.

We will use similar probabilistic voting models repeatedly in the following chapters. As discussed in chapter 2, these models have unique equilibria even when the policy conflict is multidimensional. As we have seen in (3.10), the voters’ ideological preferences smooth the candidates’ problems by eliminating sharp discontinuities in their probability of election, such as the one appearing in (3.6). We will use this property of the equilibrium with probabilistic voting in chapter 7, when dealing with special-interest politics, and in chapter 8, when contrasting alternative electoral rules. Probabilistic voting models are also relevant when politicians care about policy and not only about winning office. We thus use a probabilistic voting model in chapter 14, when studying public debt issued by partisan candidates. We also use such a model in chapter 4, where candidates care about economic policies not because of their ideology, but because they want to appropriate rents for themselves.

3.5 Lobbying

In the probabilistic voting model, groups derive their political power solely from their attributes as voters. But well-defined interest groups may also exert influence on the policy process through other forms of political action. Lobbying is a prime example. In this section, we extend the probabilistic voting model to encompass campaign contributions by interest groups.

Consider again the model in the previous section, but assume that all groups have the same density, \( \phi^J = \phi \), making the members of each group, in their capacity as voters, equally attractive for office-seeking politicians. In the pure swing-voter model this assumption made the equilibrium policy socially optimal (recall (3.11)). Any departure from that benchmark will thus be due to lobbying activity.

We assume that groups may or may not be organized in a lobby; the indicator variable \( O^J \) takes a value of one if group \( J \) is indeed organized, zero otherwise. Organized groups have
the capacity to contribute to the campaign of either of the two candidates: let $C^J_P$ denote the contribution per member of group $J$ to candidate $P$, constrained to be nonnegative. As we develop below, these contributions can be interpreted both as in cash and in kind. The total contributions collected by candidate $P$ can thus be expressed as

$$C_P = \sum_J O^J \alpha^J C^J_P.$$  \hspace{1cm} (3.12)

These contributions are given between stages (1) and (2) of the model, simultaneously by all lobbies, after the parties have announced their platforms, but before the elections and before $\delta$ is realized. We assume that candidates exploit these contributions in their campaign and that campaign spending affects their popularity. Specifically, the average relative popularity of party $B$, what we called $\delta$ in (3.8), now has two components:

$$\delta = \delta + h(C_B - C_A).$$  \hspace{1cm} (3.13)

In (3.13), $\delta$ is again distributed uniformly with density $\psi$. According to the second term, a candidate who outspends the other becomes more popular, where the parameter $h$ measures the effectiveness of campaign spending. This formulation is equivalent to assuming that some voters, but not others, are informed about the candidates’ ideological attributes. But the present formulation is simple and serves our purpose.

The new formulation changes the definition of the swing voter in group $J$ to

$$\sigma^J = W^J(g_A) - W^J(g_B) + h(C_A - C_B) - \delta.$$  \hspace{1cm} (3.14)

Following the same approach as in the last section and exploiting that $\phi^J = \phi$, we can then write candidate $A$’s probability of election as

$$p_A = \frac{1}{2} + \psi [W(g_A) - W(g_B) + h(C_A - C_B)],$$  \hspace{1cm} (3.14)

where $W(g_P) = \sum_J \alpha^J W^J(g_P)$ is the utilitarian social welfare function (recall that we are now assuming that all groups have the same density $\phi$, so that in the probabilistic voting model they all receive the same weight). The last term reflects campaign spending’s influence on the expected vote share.

Next, consider how lobbies choose their campaign contributions. If organized, group $J$ chooses contributions with the objective of maximizing the expected utility its members derive from the election, minus the cost of contributions:

$$p_A W^J(g_A) + (1 - p_A) W^J(g_B) - \frac{1}{2} \left((C^J_A)^2 + (C^J_B)^2\right).$$  \hspace{1cm} (3.15)

The first two terms in the expression are obvious. The negative third term can be interpreted in two ways. If transfers are made in cash, different members of the group may differ in their willingness to give, which makes the costs convex. If transfers are made in
kind—by working in the campaign—the increasing marginal cost may just reflect disutility of effort. Note that here, ideology plays no role in the lobby’s objective function. Under our assumption that the lobby maximizes the average utility of its members, the opposite ideologies of its members exactly cancel out. Thus intrinsic party preferences matter when voting, but not in the lobbying decision (unless the interest group has an ideological bias in favor of one of the parties).

In view of (3.14) and of (3.15), group J’s optimal contributions are easily derived:

\[
C_J^A = \text{Max} \left[ 0, \psi h(W_J^A(g_A) - W_J^B(g_B)) \right]
\]

\[
C_J^B = -\text{Min} \left[ 0, \psi h(W_J^A(g_A) - W_J^B(g_B)) \right].
\]

(3.16)

Thus the group contributes only to the candidate whose platform gives the group the highest utility, and never to more than one.

We now return to the candidates and their optimal platform choice at stage (1). When making this choice, the candidates anticipate that organized groups will make contributions according to (3.16). But the symmetry of (3.16) preserves the symmetry of the two candidates’ problems. Thus they once more converge on the same equilibrium policy. To characterize this policy, we substitute (3.16) into (3.14) and simplify. Candidate A, taking \(g_B\) as given, maximizes

\[
\sum_J \alpha_J \left[ \psi + O_J^J(\psi h)^2 \right] W_J^A(g_A).
\]

(3.17)

We immediately see that if all groups are organized \((O_J^J = 1\) for all \(J)\), or no groups are organized \((O_J^J = 0\) for all \(J)\), the equilibrium coincides with the utilitarian optimum. In these cases, we have returned to the previous probabilistic voting model, and optimality follows from our symmetry assumption \(\phi_J^J = \phi\). It may appear surprising, at first sight, that the equilibrium is socially optimal when all groups are organized. The intuition is that the groups are all prepared to contribute in proportion to the marginal benefits and costs of \(g\) for their members. As a result, the candidates internalize all groups with the appropriate social weight, just paying attention to group size.

4. By (3.15) the first-order condition of the lobby \(J\) with respect to \(C_J^A\) is

\[
\frac{\partial p_A}{\partial C_J^A} [W_J^A(g_A) - W_J^B(g_B)] - C_J^A \leq 0,
\]

and by (3.14),

\[
\frac{\partial p_A}{\partial C_J^A} = h\psi.
\]

Repeating the same steps for \(C_J^B\), we get (3.16).
Departures from the benchmark arise when only a subset of the groups is organized. To see this, take the first-order condition of (3.17) with respect to $g_A$. After some transformations we get:

$$g^L = H_g^{-1}(\hat{y}/y),$$

where the $L$ superscript stands for lobbying equilibrium and where

$$\hat{y} = \frac{\sum_J \alpha_J [1 + O_J \psi h^2] y^J}{\sum_J \alpha_J [1 + O_J \psi h^2]}.$$  \hspace{1cm} (3.18)

Thus, $\hat{y}$ is a weighted average of $y^J$, with weights reflecting whether or not the group is organized. As stated above, when all groups are organized (or no group is), the expression on the right-hand side of (3.18) reduces to $y$ and the equilibrium is socially optimal: $g^L = g^*$. Otherwise, the organized groups receive greater weights and the equilibrium is tilted in their favor. Suppose for instance that only the richest group is organized. Then they receive greater weight in the computation of $\hat{y}$, so that $\hat{y} > y$: the size of government becomes smaller than the benchmark, $g^L < g^*$. Furthermore, spending is smaller the larger is group $R$’s stake in the policy (the higher is $y^R$ relative to $y$) and the more effective is campaign spending in swaying the vote (the higher is $h$). As in the probabilistic voting model, more income inequality can thus be associated with a smaller government.

Intuitively, the candidates seek only election victory, and the organized lobbies can help them achieve this goal by financing their campaigns. Both candidates thus bias their policy platforms in the direction desired by the lobbies. This illustrates a well-known point from the traditional public choice literature. Groups that have overcome the collective action problem and organized themselves have more influence on policy than nonorganized groups. This point goes back all the way to Olson and has been formalized in a growing literature on lobbying.

The public choice literature has also emphasized that groups with the largest stake in a particular policy are more likely to become organized. Applying the logic to this model, the rich or the poor would be more likely to form organized lobbies than the middle class. Strong stakes in policy are thus complementary: they make a group of citizens more likely to get organized and more willing to lobby hard once they are organized. Perhaps it is not so plausible, however, to think about lobby groups forming over general economic policies as the fiscal programs considered here. If organizations like trade unions or industrial associations already exist for other reasons, however, they are likely to use their political power to influence general economic policies. Strong organizations of this type can thus bias policy significantly to the left or to the right, even though neither the candidates themselves nor the general electorate has a corresponding bias. The specific predictions regarding the size of government from our simple model could be taken to the data.
More generally, many authors have suggested reasons why the size of government might be related to the number and orientation of interest groups. Empirical work has failed to find robust evidence of a tight link between interest group activity and the size of government.

The model also illustrates a more subtle point. In equilibrium no contributions are being paid, according to (3.16), as the candidates converge to the same policy. Obviously, this feature does not allow us to conclude that lobbying is unimportant for the policy outcome. The common argument that lobbying cannot be very important as observed contributions are so small relative to the policy benefits at stake should thus be treated with caution.