MAC 2311: Worksheet #1

Panther ID: ___________________________  NAME: ___________________________

1) Consider the function \( f(x) = x^2 - 4x \).

a) Sketch the graph of \( y = f(x) \).

b) Find the average rate of change of \( f(x) \) with respect to \( x \) over the interval \([2, 4]\).

c) On your graph in part (a), sketch the line between the points \((2, f(2))\) and \((4, f(4))\). What is the slope of this line?
3) Compute the limits, if the graph of $y = g(x)$ is given as follows,

![Graph of $y = g(x)$]

a) $\lim_{x \to -1} g(x)$  

b) $\lim_{x \to 1} g(x)$  

c) $\lim_{x \to 2} g(x)$

4) Define

$$f(x) = \begin{cases} 
  x^2 - 1 & \text{if } x \leq 2 \\
  x + 1 & \text{if } x > 2 
\end{cases}$$

Compute the following limits or explain why they don’t exist:

a) $\lim_{x \to 2^-} f(x)$

b) $\lim_{x \to 2^+} f(x)$

c) $\lim_{x \to 2} f(x)$
5) Compute \( \lim_{x \to 1} (x - 1)/|x - 1| \) or explain why it doesn’t exist.

6a) Sketch the graph \( y = \frac{1}{x^2-1} \). (Hint: first sketch \( y = x^2 - 1 \).)

Use this graph to compute:

b) \( \lim_{x \to 1^-} \frac{1}{x^2-1} \)

c) \( \lim_{x \to 1^+} \frac{1}{x^2-1} \)

d) \( \lim_{x \to 1} \frac{1}{x^2-1} \)

7) Compute \( \lim_{x \to 1} \frac{x-1}{x^2-1} \)
Problem 8 The graph of a function $f$ is given below. Use the graph to find the limits below. Specify if a limit does not exist or is infinite.

\[
\begin{align*}
\lim_{x \to -3^-} f(x) &= \quad \lim_{x \to -3^+} f(x) &= \quad \lim_{x \to 3^-} f(x) &= \quad \lim_{x \to 3^+} f(x) = \\
\lim_{x \to 2^-} f(x) &= \quad \lim_{x \to 2^+} f(x) &= \quad \lim_{x \to 2} f(x) = \\
\lim_{x \to 1^-} f(x) &= \quad \lim_{x \to \infty} f(x) = \quad \lim_{x \to \infty} f(x) =
\end{align*}
\]

Problem 9 Sketch the graph of a function $y = f(x)$ which satisfies all of the following conditions:

(i) the domain of $f$ is $(0, +\infty)$; 

(ii) $f(2) = f(4) = 0$;

(iii) $\lim_{x \to 0^-} f(x) = -\infty$; 

(iv) $\lim_{x \to 2} f(x) = +\infty$;

(v) $\lim_{x \to 4^-} f(x) = 0$ and $\lim_{x \to 4^+} f(x) = 1$;

(vi) $\lim_{x \to \infty} f(x) = 3$. 