MAC 2311: Worksheet 0901

Panther ID: ______________________   NAME: ______________________

1. Evaluate the following limits:
   
   (a) \( \lim_{x \to 3^+} \frac{2}{x - 3} \)   
   
   (b) \( \lim_{x \to 3^-} \frac{2}{x - 3} \)   
   
   (c) \( \lim_{x \to 3} \frac{2}{x - 3} \)

2. Find the limit as \( x \to 2 \) (if it exists) and then sketch the graph of each of the following:
   
   (a) \( f(x) = x + 2 \)   
   
   (b) \( g(x) = \frac{x^2 - 4}{x - 2} \)

3. If \( \lim_{x \to 1} f(x) = 3 \), \( \lim_{x \to 1} g(x) = -2 \), and \( \lim_{x \to 1} h(x) = 7 \), find:
   
   (a) \( \lim_{x \to 1} (f(x) + g(x)) \)   
   
   (b) \( \lim_{x \to 1} (g(x) - h(x)) \)
   
   (c) \( \lim_{x \to 1} (f(x)h(x)) \)
   
   (d) \( \lim_{x \to 1} (3f(x))^2 \)
   
   (e) \( \lim_{x \to 1} \left( \frac{g(x)}{h(x)} \right) \)
4. Find the following limits, provided they exist:

(a) \( \lim_{{x \to 0}} \frac{x^2 - 3x}{x^2 - 4x + 3} \)

(b) \( \lim_{{x \to 3}} \frac{x^2 - 3x}{x^2 - 4x + 3} \)

(c) \( \lim_{{x \to 2}} \frac{x^2 + x - 6}{2 - x} \)

(d) \( \lim_{{x \to 2}} \frac{x^2 + x - 6}{|2 - x|} \)

5. Find the following limits, provided they exist:

(a) \( \lim_{{x \to 4}} \frac{x - 4}{\sqrt{x} - 2} \)

(b) \( \lim_{{x \to -1}} \frac{\sqrt{x^2 + 8} - 3}{x + 1} \)

(c) \( \lim_{{x \to 1}} \frac{x^3 - 1}{x^3 + 6x^2 - 8x + 1} \)

(d) \( \lim_{{x \to 2}} \frac{8 - x^3}{x^3 - 5x + 2} \)
6. For each of the following functions compute \( \lim_{x \to +\infty} f(x) \) and \( \lim_{x \to -\infty} f(x) \):

(a) \( f(x) = 3x^3 - x^2 + 2x - 7 \)

(b) \( f(x) = \frac{2x + 1}{3x^4 - 2} \)

(c) \( f(x) = \frac{40x^5 + x^2}{16x^4 - 2} \)

(d) \( f(x) = \frac{3x^7 - 4x^4 + 1}{2x^7 + 2x} \)

(e) \( f(x) = \frac{2x}{x^2 + 4} \)

Which of the functions above have horizontal asymptotes and what are the asymptotes?
7. Find the following limits, provided they exist:

(a) \( \lim_{x \to +\infty} \frac{2x}{\sqrt{x^2 + 4}} \)  
(b) \( \lim_{x \to -\infty} \frac{2x}{\sqrt{x^2 + 4}} \)

Does the function \( f(x) = \frac{2x}{\sqrt{x^2 + 4}} \) have horizontal asymptotes? If yes, describe them?

8. Consider the function \( f(x) = \frac{3 - x}{x^2 - 9} \).

(a) Does this function have horizontal asymptotes? Justify your answer with limits.

(b) Does this function have vertical asymptotes? Justify your answer with limits.

(c) Graph this function.