Worksheet on the limit definition - MAC 2311, Fall 2015

1. Find the distance between:
   (a) 2 and 5;  (b) 5 and 2;  (c) -3 and 2;
   (d) 5.2 and -3.7;  (e) a and b, with a and b real numbers

2. (a) Sketch the set of points \((x, 0)\) on the \(x\)-axis such that \(|x - 3| < 0.7\). Then sketch the set of points \((x, 0)\) on the \(x\)-axis such that \(0 < |x - 3| < 0.7\). How are these two sets different?
   (b) Describe the set of points on the \(x\) axis whose distance from \((5, 0)\) is less than 0.1 in two ways:
   (i) Sketch this set;
   (ii) Write an inequality characterizing this set (hint: look at (2a)).

3. Describe the set of points on the \(y\) axis whose distance from \((0, 3)\) is less than 0.2 in two ways:
   (i) Sketch this set;
   (ii) Write an inequality characterizing this set (hint: look at (2a)).
4. In this problem, we will use the $\varepsilon$-$\delta$ definition to prove that $\lim_{x \to 5} (2x + 3) = 13$.

(a) Identify $f(x)$, $a$, $L$ in this case.

(b) Compute $|f(x) - L|$ in this case (you want in your result to see the expression $|x - a|$).

(c) Using your computation in (b), show that if $|x - 5| < 0.1$ then $|f(x) - 13| < 0.2$.

More generally, show that if $\delta > 0$ is a positive number (not yet specified) then if $|x - 5| < \delta$ then it follows, in this case, that $|f(x) - 13| < 2\delta$.

(d) Based on part (c), if $\epsilon > 0$ is given, how would you choose $\delta > 0$ in this case?

(e) With your choice from part (d), show that if $|x - 5| < \delta$ then $|f(x) - 13| < \epsilon$.

5. Repeat the steps in the previous problem to show that $\lim_{x \to 3} (100x - 1) = 299$. 
6. Prove that \( \lim_{x \to -3} (2x - 7) = -13 \).

7. True or false. Answer and justify your answer.
   (a) If \( \lim_{x \to 2} f(x) = f(2) = 5 \), then \( 4.9 < f(x) < 5.1 \) for all \( x \) in a small enough interval around 2.

   (b) If \( \lim_{x \to 2} f(x) = f(2) = 5 \), then \( f(x) \neq 4.99 \) for all \( x \) in a small enough interval around 2.

7. (Challenge problem) Prove \( \lim_{x \to 2} x^2 = 4 \).