Derivatives of Inverse Trigonometric Functions. Logarithmic Differentiation.

LECTURE: Definition of \( \arcsin(x) \), \( \arctan(x) \). Derivative of \( \arcsin(x) \).

1) Without using a calculator, compute:

   a) \( \arcsin(1/2) \)  
   b) \( \arctan(1) \)  
   c) \( \sin(\arcsin(1/5)) \)  
   d) \( \arcsin(\sin(\pi/5)) \)  
   e) \( \arctan(\tan(-3\pi/4)) \)  
   f) \( \arcsin(\sin(3\pi/4)) \)

2) Compute the following derivatives:

   a) \( \frac{d}{dx}(x^3 \arcsin(3x)) \)
   b) \( \frac{d}{dx}\left(\frac{\sqrt{x}}{\arcsin(x)}\right) \)
   c) \( \frac{d}{dx}[\ln(\arcsin(e^x))] \)
   d) \( \frac{d}{dx}[\arcsin(\cos x)] \)

   The result of part d) might be surprising, but there is a reason for it. If you find it, it will also lead you to a simple proof for the derivative of \( \arccos x \)!

3) In this problem, you will compute \( \frac{d}{dx} \arctan(x) \)

   a) Using the chain rule, differentiate both sides of the equality \( \tan(\arctan(x)) = x \) and solve the resulting equation for \( \frac{d}{dx} \arctan(x) \).

   b) Let \( \theta = \arctan(x) \) so \( \tan(\theta) = x \). Draw a right triangle with vertices \( A, B, \) and \( C \) and angles \( \angle ABC = \pi/2 \) and \( \angle BAC = \theta \). If the length of the side \( AB \) is \( |AB| = 1 \), find the lengths \( |BC| \) and \( |AC| \) in terms of \( x \).

   c) Using the triangle you drew in (b), find \( \sec(\arctan(x)) \).

   d) Combine your answers for (c) and (a) to get \( \frac{d}{dx} \arctan(x) \).
4) Compute the following derivatives:

\[ \frac{d}{dx} \arctan(e^x) \]  \hspace{1cm} \frac{d}{dx} [e^x \arctan(x)]

\[ \frac{d}{dx} \sin(\arctan(x)) \]  \hspace{1cm} \frac{d}{dx} [\arctan(\arcsin(x^2))] 

LECTURE BREAK: Logarithmic differentiation. Show the example \((x^x)’\)

5) Use logarithmic differentiation to find the derivative of each of the following functions:

(a) \( y = x^{\sin x} \)  \hspace{1cm} (b) \( y = \frac{x^{\frac{3}{2}}}{(x+2)^5} \)

6) (a) We proved the power rule \((x^n)’ = nx^{n-1}\) for the case when \(n\) was a positive integer and in some other special cases. Now use logarithmic differentiation to show that the power rule \((x^r)’ = rx^{r-1}\) holds for any real constant \(r\).

(b) Use logarithmic differentiation to prove the product rule.

(c) Use logarithmic differentiation to prove the quotient rule.

LECTURE BREAK: Implicit differentiation; Show one or two examples.

7) For each of the following implicitly defined functions, find \(\frac{dy}{dx}\):

(a) \( y^4 - 3y^3 - x = 3 \)  \hspace{1cm} (b) \( \cos(xy) = x - y \)

8) Consider the function implicitly defined by \(y^4 = x + y\).

(a) Find an expression for the derivative \(\frac{dy}{dx}\).

(b) Find the equation of the line tangent to this function at the point (0,1).

(c) Find where the tangent line is vertical.

Practice: (Don’t turn these in.) 3.3 # 43-53 odd, 65 – Inverse trig differentiation problems. 3.1 # 1-13odd, 19, 25, 27, 29*, 33* – Implicit diff problems.
Logarithmic Differentiation problems were recorded on the previous worksheet (in 3.2).