1. (a) Use IBP to derive a reduction formula for

\[ \int x^n e^{-wx} \, dx, \] where \( n \) is a nonnegative integer and \( w \) is a positive constant.

Next, you will use your reduction formula in part (a) to show that

\[ \int_0^{+\infty} x^n e^{-wx} \, dx = \frac{n!}{w^{n+1}}. \]

Follow these steps:

(b) Denote \( I_n = \int_0^{+\infty} x^n e^{-wx} \, dx \). Compute directly \( I_0 = \int_0^{+\infty} e^{-wx} \, dx \).

(c) Use l'Hôpital to show that \( \lim_{x \to +\infty} x^n e^{-wx} = 0 \).

(d) Use the reduction formula from part (a) and the observation in (c), to get the recursive formula

\[ I_n = \frac{n}{w} I_{n-1}, \text{ for all } n \geq 1. \]

(e) From (d) and (b), conclude that \( I_n = \frac{n!}{w^{n+1}} \).

Note: You have to trust me that the improper integral you computed is an important one. Hence, it was worth the effort!