1. Decide if each of the following series is convergent or divergent. Specify which test you are using and show how the test applies.

(a) \( \sum_{k=0}^{\infty} \frac{1}{(2k+1)^2} \)
Convergent series
by the comparison test with \( \sum_{k=1}^{\infty} \frac{1}{k^2} \)

(b) \( \sum_{k=1}^{\infty} \frac{k}{\sqrt{k^3+1}} \)
Divergent series
Limit comparison test with \( \sum_{k=1}^{\infty} \frac{1}{k^{3/2}} \)

(c) \( \sum_{k=1}^{\infty} \frac{(k!)^2}{(2k)!} \)
Convergent series
by Ratio Test
\[ P = \lim_{k \to \infty} \frac{(k+1)(k)!}{(2k+1)!} = \frac{1}{4} \]

2. Find the values of \( p \) for which the series is convergent.
\( \sum_{k=2}^{\infty} \frac{1}{k^{1/p}(k+1)^p} \)
If \( p > 1 \), integral test succeeds.
Otherwise, diverges.

3. True or False. Answer and briefly justify in each case.

(a) If \( S_n = \sum_{k=1}^{n} a_k \) and \( \lim_{n \to \infty} S_n \) does not exist or is not finite, then \( \sum_{k=1}^{\infty} a_k \) is a divergent series.
True. By definition, \( \sum_{k=1}^{\infty} a_k = \lim_{n \to \infty} S_n \)

(b) If \( (a_k)_k \) is a convergent sequence then the series \( \sum_{k=1}^{\infty} a_k \) is also convergent.
False, if \( a_k = \frac{1}{k} \), then \( a_k \to 0 \), but \( \sum_{k=1}^{\infty} a_k = \sum_{k=1}^{\infty} \frac{1}{k} = +\infty \)

(c) If \( (a_k)_k \) is a divergent sequence then the series \( \sum_{k=1}^{\infty} a_k \) is also divergent.
True, if \( \lim_{k \to \infty} a_k \) is diverged \( \Rightarrow \lim_{k \to \infty} a_k \neq 0 \), so \( \sum_{k=1}^{\infty} a_k \) diverges by divergence test.

(d) If \( \sum_{k=1}^{\infty} a_k \) converges to \( A \) and \( \sum_{k=1}^{\infty} b_k \) converges to \( B \), then \( \sum_{k=1}^{\infty} (a_k - b_k) \) converges to \( A - B \).
True, \( \sum_{k=1}^{\infty} (a_k - b_k) = (\sum_{k=1}^{\infty} a_k) - (\sum_{k=1}^{\infty} b_k) \) and take the limit as \( n \to +\infty \).

(c) If \( a_k > 0 \) for all \( k \) and \( \sum_{k=1}^{\infty} a_k \) converges, then \( \sum_{k=1}^{\infty} (a_k)^2 \) also converges.
True. If \( \sum_{k=1}^{\infty} a_k \) converges \( \Rightarrow \lim_{n \to +\infty} a_k = 0 \) (div. test)
So, for a suf. large \( k \), \( a_k < 1 \)
Thus \( 0 < a_k^2 < a_k \), for \( k \geq k_0 \).
By simple comparison, since \( \sum_{k=1}^{\infty} a_k \) converges,
\[ \sum_{k=1}^{\infty} a_k^2 \] converges too.