Worksheet week 12 - MAC2312  
NAME: ___________________________  

1. Decide if each of the following series is convergent or divergent. Specify which test you are using and show how the test applies.

(a) \[ \sum_{k=0}^{\infty} \frac{1}{(2k+1)^2} \]

(b) \[ \sum_{k=1}^{\infty} \frac{k}{\sqrt{k^3+1}} \]

(c) \[ \sum_{k=1}^{\infty} \frac{(k!)^2}{(2k)!} \]

2. Find the values of \( p \) for which the series is convergent \[ \sum_{k=2}^{\infty} \frac{1}{k \ln(k)^p} \]

3. True or False. Answer and briefly justify in each case.

(a) If \( S_n = \sum_{k=1}^{n} a_k \) and \( \lim_{n \to \infty} S_n \) does not exist or is not finite, then \( \sum_{k=1}^{\infty} a_k \) is a divergent series.

(b) If \( \{a_k\} \) is a convergent sequence then the series \( \sum_{k=1}^{\infty} a_k \) is also convergent.

(c) If \( \{a_k\} \) is a divergent sequence then the series \( \sum_{k=1}^{\infty} a_k \) is also divergent.

(d) If \( \sum_{k=1}^{\infty} a_k \) converges to \( A \) and \( \sum_{k=1}^{\infty} b_k \) converges to \( B \), then \( \sum_{k=1}^{\infty} (a_k-b_k) \) converges to \( A - B \).

(e) If \( a_k > 0 \) for all \( k \) and \( \sum_{k=1}^{\infty} a_k \) converges, then \( \sum_{k=1}^{\infty} (a_k)^2 \) also converges.