1. Use FTC or geometry to evaluate each integral:

\[(a) \int_0^3 (2x-1) \, dx = A_1 + A_2 \]

\[A_1 = \frac{1 + \frac{1}{3}}{2} = \frac{1}{2} \]

\[A_2 = \frac{5 - \frac{5}{3}}{2} = \frac{5}{6} \]

Thus \( \int_0^3 (2x-1) \, dx = \frac{1}{2} \cdot \frac{5}{6} = \frac{5}{12} \)

\[(b) \int_1^2 \frac{x^2 + 1}{x} \, dx = \int_1^l \left( x + \frac{1}{x} \right) \, dx \]

\[= \int_1^l x \, dx + \int_1^l \frac{1}{x} \, dx \]

\[= \left[ \frac{x^2}{2} + \ln x \right]_1^l \]

\[= \frac{3}{2} + \ln 2 \]

\[(c) \int_0^{\pi/3} \sec^2 x \, dx = \int_0^{\pi/3} \frac{1}{\cos^2 x} \, dx \]

\[= \left[ \tan x \right]_0^{\pi/3} \]

\[= \sqrt{3} \]

2. Find the average value of \( f(x) = \frac{1}{x+1} \) on the interval \([-1, 1]\) and find all values of \( x^* \in [-1, 1] \) so that \( f(x^*) \)
equals the average value of \( f \) on \([-1, 1] \). Why are such values of \( x^* \) guaranteed to exist?

\[f_{\text{ave}} = \frac{\int_{-1}^1 \frac{1}{x+1} \, dx}{2} = \left( \frac{\ln (x + 1)}{x+1} \right)_{x=-1}^{x=1} = \frac{\ln -\frac{2}{3} - \ln -\frac{1}{2}}{2} = \frac{\ln 2}{2} \]

As \( f(x) = \frac{1}{x+1} \) is continuous on \([-1, 1]\), \( x^* \in [-1, 1] \) so that \( f(x^*) = f_{\text{ave}} \) is guaranteed to exist by MVT for Integrals.

To find \( x^* \), solve \( \frac{1}{x^* + 1} = \frac{1}{2} \Rightarrow x^* + 1 = \frac{2}{3} \Rightarrow x^* = \frac{2}{3} - 1 \Rightarrow x^* = \pm \sqrt{2} \)

Both values work as \( \pm \sqrt{2} \in [-1, 1] \)

3. Use substitution to compute each integral:

\[(a) \int_{\ln 2}^\infty \frac{1}{x \sqrt{\ln x}} \, dx = \int_{\frac{\ln 2}{2}}^{\infty} \frac{1}{w} \, dw \]

Subst. \( w = \ln x \quad \text{(Note, } x > e \Rightarrow dw = \frac{1}{x} \, dx) \)

\[= \int_{\frac{\ln 2}{2}}^{\infty} \frac{1}{w} \, dw \]

\[= \frac{1}{2} \ln w \bigg|_{w=2}^{w=\infty} \]

\[= \frac{1}{2} \ln 2 \]
4. Given that \( F(x) = \int_0^x \sqrt{8t - t^2} \, dt \), for \( x \in [0, 8] \), do the following:

(a) Determine the values of \( F(0) \), \( F(4) \), \( F(8) \). Hint: Complete the square and use geometry.
(b) Determine \( F'(x) \) and \( F''(x) \).
(c) Based on parts (a) and (b), sketch the graph of the function \( y = F(x) \), for \( x \in [0, 8] \). What kind of point is \( x = 4 \) for the graph of \( y = F(x) \)?

(a) Completion of the square: based on \((A \pm B)^2 = A^2 \pm 2AB + B^2\)

\[
8t - t^2 = -(t^2 - 8t) = -(t^2 - 2 \cdot t \cdot 4) = \\
= -(t^2 - 2 \cdot t \cdot 4 + 4^2 - 4^2) = -[(t - 4)^2 - 16]
\]

Thus \( 8t - t^2 = 16 - (t - 4)^2 \) so

\[
F(x) = \int_0^x \sqrt{16 - (t - 4)^2} \, dt
\]

\[
F(0) = 0, \quad F(4) = \int_0^4 \sqrt{16 - (t - 4)^2} \, dt = \frac{1}{4} \cdot \pi \cdot 4^2 = 4\pi
\]

\[
F(8) = \int_0^8 \sqrt{16 - (t - 4)^2} \, dt = \frac{1}{2} \cdot 8 \cdot \sqrt{8^2 - 4^2} = 8\pi
\]

(b) By FTC (vi)

\[
F'(x) = \frac{d}{dx} \left( \int_0^x \sqrt{8t - t^2} \, dt \right) = \sqrt{16 - x^2} = \sqrt{16 - (x - 4)^2}
\]

\[
F''(x) = \frac{d}{dx} \left( \sqrt{16 - (x - 4)^2} \right) = \frac{1}{2} \left( 16 - (x - 4)^2 \right)^{-\frac{1}{2}}, \quad (x - 4) = -\frac{x - 4}{\sqrt{16 - (x - 4)^2}}
\]

(c) Graph of \( y = F(x) \)

\( F(x) \) is increasing, since \( F'(x) > 0 \) on \([0, 8]\)

\( F(x) \) is concave up on \([0, 4]\), as \( F''(x) > 0 \) on \([0, 4]\)

\( F(x) \) is concave down on \([4, 8]\), as \( F''(x) < 0 \) on \([4, 8]\)

\( x = 4 \) is an inflection point for \( F(x) \).