MAC 2311: Worksheet #12 2/24/16 (Chain Rule)

1. Compute \( f''(x) \) for \( \frac{x^2-1}{(4x+7)^6} \)

\[
\frac{f''(x)}{(4x+7)^6} = \frac{2x(4x+7)^9 - (x^2-1)(4x+7)^8 \cdot 4}{(4x+7)^10} = \frac{(4x+7)^8}{(4x+7)^8} \frac{2x(4x+7) - 36(x^2-1)}{(4x+7)^10} = \frac{-28x^2+14x+36}{(4x+7)^10}
\]

2) Suppose that the energy used by a factory is given (in megawatt-hours, MWh) by

\[
E(t) = 200t + \frac{1200}{\pi} \sin \left( \frac{\pi t}{12} \right)
\]

where \( 0 \leq t \leq 24 \) is measured in hours after noon.

a) Calculate the power, \( P(t) = E'(t) \), consumed by this factory. What are the units in this case?

\[
E'(t) = 200 + \frac{1200}{\pi} \cdot \frac{\pi}{12} \cos \left( \frac{\pi t}{12} \right) = 200 + 100 \cos \left( \frac{\pi t}{12} \right)
\]

b) When is the power consumption highest?

\[
\cos(x) \text{ maximized at } x = 0 \text{ or } x = 2\pi,
\]

i.e. at beginning of period.

So \( E'(t) \) maximized at beginning of period: \( t = 0, t = 24 \text{ etc.} \)

c) When is the power consumption lowest?

\[
\cos(x) \text{ minimized at } x = \pi, \text{ so } E'(t) \text{ minimized at } t = 12 \text{ set } \frac{\pi}{12}t = \pi. \text{ Ans: } t = 12, 36, \text{ etc.}
\]
3. Suppose the $x$-axis represents a road along the Sahara (units kilometers), and suppose the temperature (units degrees Celsius) along the road is given be $h(x) = 40 + 4 \sin(x)$. Suppose the position of a car driving along the road is 80 km/hr. What is the rate of change of temperature after 4 hours?

We cannot set initial position: $x(0)=0$. Then $x(t) = 80t$.

$$h(x(t)) = 40 + 4 \sin(80t)$$

$$h'(x(t)) = \frac{dh}{dt} (40 + 4 \sin(80t)) = 320 \cos(80t) \text{ after 4 hours}$$

b) Suppose the car's speed changed to 10 meters per second. What now is the rate of change of temperature?

10 meters per second $\Rightarrow$ 10 $\frac{3600}{1000}$ km/hour $= 36 \text{ km/hr}$ 1 mile per hour $= 1.6$ km/hr

$\Rightarrow \quad 3.6 \frac{1}{1.6}$ miles per hour $= 2.25$ miles/hr

$c) \quad x(t) = 22.5$

$\Rightarrow \quad x'(t) = \frac{dx}{dt} = 90 \cos(22.5t)$

$c)$ Now suppose the speed is given by $t^2$, where $t$ is time in hours.

$x(t) = t^2 \Rightarrow x(t) = \frac{t^2}{3} + C$

$x(0) = 0 \Rightarrow C = 0 \Rightarrow x = \frac{t^3}{3}$

$$h = h(x(t)) = 40 + 4 \sin \left( \frac{t^3}{3} \right)$$

$$\frac{dh}{dt} = 4 \cdot \frac{3t^2}{3} \cos \left( \frac{t^3}{3} \right) = 4t \cos \left( \frac{t^3}{3} \right)$$

$$\left. \frac{dh}{dt} \right|_{t=4} = 64 \cos \left( \frac{64}{3} \right)$$