MAP 2302 July 12, 2013
Quiz II and Key Prof. S. Hudson

This should take about 15 minutes (but it took 20-25), with a lecture afterwards.

1) [50 pts] Given that \( y = (x + 1) \) solves the DE \( (x + 1)^2 y'' - 3(x + 1)y' + 3y = 0 \), find a second L.I. solution.

2) [50 pts] Find the general solution using variation of parameters: \( y'' + y = \tan^2(x) \).

Bonus [5 points]: Check that your answer to part (2) solves the D.E.

Remarks and Answers: The average was about 65 / 100 based on the top half, with high scores of 98, 95 and 85. This is a bit low, but in the normal range. The unofficial scale is

- A’s 77 to 100
- B’s 65 to 76
- C’s 55 to 64
- D’s 45 to 54

1) You are given one solution, and are expected to use reduction of order to find a second one. So, \( y = v(x + 1) \) leads to \((x + 1)v'' - v' = 0\) or \((x + 1)w' - w = 0\) which is separable. Get \(|w| = C|x + 1|\) or (safely ignoring absolute values and constants) just \( w = x + 1 \), so \( v = (x + 1)^2 \) (it’s also OK to use \( v = x^2 / 2 + x \), etc). So \( y = (x + 1)^3 \). It is easy to check this is LI and is a solution.

The above explanation follows the method of Ex 4.16. A couple of people used the formula from Thm 4.7 Conclusion 2 instead, and got the same \( v \).

You could treat this like a Cauchy-Euler DE, instead, and set \( x + 1 = e^t \), etc.

There is a slight problem with the textbook here (see Conclusion 3 and answers to the exercises). The general solution is NOT \( c_1(x + 1) + c_2(x + 1)^3 \). For example, \( y(x) = |x + 1|^3 \) is a solution to this problem that does not fit that pattern. This is probably not a very serious issue and may be understood by simply graphing the solutions.

2) \( y_p = v_1(x) \sin(x) + v_2(x) \cos(x) = \sin(x) \ln|\sec x + \tan x| - 2 \). You should get \( y_c \) easily from memory and add it in at the end. You can plug into the usual formulas to get

\[
v_1 = - \int \frac{F \cdot y_2}{a_0 W} \, dx = - \int \frac{\tan^2(x) \cos(x)}{1} \, dx = \ln|\sec x + \tan x| - \sin(x)
\]

\[
v_2 = \int \frac{F \cdot y_1}{a_0 W} \, dx = \int \frac{\tan^2(x) \sin(x)}{1} \, dx = -\sec x - \cos x
\]

If you got stuck here, you may need to practice integration or basic trig identities. This was exercise 4.4.2 and was done in class.

B) From \( y = \sin x \ln|\sec x + \tan x| - 2 \), get \( y' = \sin x \sec x + \cos x \ln|\sec x + \tan x| \) and \( y'' = \sec^2 x - \sin x \ln|\sec x + \tan x| + 1 \). So \( y'' + y = \sec^2 x - 1 = \tan^2 x \). Check!