1. The Elements of a Test of Hypothesis

Conclusions and Consequences for a Test of Hypothesis

<table>
<thead>
<tr>
<th>Conclusion</th>
<th>True State of Nature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accept ( H_0 ) (Assume ( H_0 ) True)</td>
<td>Correct decision</td>
</tr>
<tr>
<td>Reject ( H_0 ) (Assume ( H_a ) True)</td>
<td>Type I error (probability ( \alpha ))</td>
</tr>
</tbody>
</table>

Elements of a Test of Hypothesis

1. *Null hypothesis* (\( H_0 \)): A theory about the values of one or more population parameters. The theory generally represents the status quo, which we adopt until it is proven false. By convention, the theory is stated as \( H_0 \): parameter = value.

2. *Alternative hypothesis* (\( H_a \)): A theory that contradicts the null hypothesis. The theory generally represents that which we will accept only when sufficient evidence exists to establish its truth.

3. *Test statistic*: A sample statistic used to decide whether to reject the null hypothesis.

4. *Rejection region*: The numerical values of the test statistic for which the null hypothesis will be rejected. The rejection region is chosen so that the probability is \( \alpha \) that it will contain the test statistic when the null hypothesis is true, thereby leading to a Type I error. The value of \( \alpha \) is usually chosen to be small (e.g., .01, .05, or .10) and is referred to as the **level of significance** of the test.

5. *Assumptions*: Clear statements of any assumptions made about the population(s) being sampled.

6. *Experiment and calculation of test statistic*: Performance of the sampling experiment and determination of the numerical value of the test statistic.

7. **Conclusion**:
   a. If the numerical value of the test statistic falls into the rejection region, we reject the null hypothesis and conclude that the alternative hypothesis is true. We know that the hypothesis-testing process will lead to this conclusion incorrectly (a Type I error) only \( 100\alpha \% \) of the time when \( H_0 \) is true.
   b. If the test statistic does not fall into the rejection region, we do not reject \( H_0 \). Thus, we reserve judgment about which hypothesis is true. We do not conclude that the null hypothesis is true because we do not (in general) know that the probability \( \beta \) that our test procedure will lead to an incorrect acceptance of \( H_0 \) (a Type II error).
Determining the Target Parameter

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Key Words or Phases</th>
<th>Type of Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>Mean; average</td>
<td>Quantitative</td>
</tr>
<tr>
<td>$p$</td>
<td>Proportion; percentage; fraction; rate</td>
<td>Qualitative</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Variance; variability; spread</td>
<td>Quantitative</td>
</tr>
</tbody>
</table>

2. Large-Sample Test of Hypothesis about a Population Mean

**Steps for Selecting the Null and Alternative Hypotheses**

1. Select the alternative hypothesis as that which the sampling experiment is intended to establish. The alternative hypothesis will assume one of three forms:
   a. **One tailed, upper tailed**
      
      Example: $H_a: \mu > 2,400$
   b. **One tailed, lower tailed**
      
      Example: $H_a: \mu < 2,400$
   c. **Two tailed**
      
      Example: $H_a: \mu \neq 2,400$

2. Select the null hypothesis as the status quo – that which will be presumed true unless the sampling experiment conclusively establishes the alternative hypothesis. The null hypothesis will be specified as the parameter value closest to the alternative in one-tailed tests and as the complementary (or only unspecified) value in two-tailed test.

**Rejection Regions for Common Values of $\alpha$**

<table>
<thead>
<tr>
<th>Alternative Hypotheses</th>
<th>Lower Tailed</th>
<th>Upper tailed</th>
<th>Two Tailed</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = .10$</td>
<td>$z &lt; -1.28$</td>
<td>$z &gt; 1.28$</td>
<td>$z &lt; -1.645$ or $z &gt; 1.645$</td>
</tr>
<tr>
<td>$\alpha = .05$</td>
<td>$z &lt; -1.645$</td>
<td>$z &gt; 1.645$</td>
<td>$z &lt; -1.96$ or $z &gt; 1.96$</td>
</tr>
<tr>
<td>$\alpha = .01$</td>
<td>$z &lt; -2.33$</td>
<td>$z &gt; 2.33$</td>
<td>$z &lt; -2.575$ or $z &gt; 2.575$</td>
</tr>
</tbody>
</table>
### Large-Sample Test of Hypothesis about $\mu$

#### One-Tailed Test

$H_0 : \mu = \mu_0$

$H_a : \mu < \mu_0$ (or $H_a : \mu > \mu_0$)

**Test statistic:** $z = \frac{\bar{x} - \mu_0}{\sigma_x}$

**Rejection region:** $z < -z_{\alpha}$

(Or $z > z_{\alpha}$ when $H_a : \mu > \mu_0$)

Where $z_{\alpha}$ is chosen so that $P(z < -z_{\alpha}) = \alpha$

#### Two-Tailed Test

$H_0 : \mu = \mu_0$

$H_a : \mu \neq \mu_0$

**Test statistic:** $z = \frac{\bar{x} - \mu_0}{\sigma_x}$

**Rejection region:** $z < -z_{\alpha/2}$ or $z > z_{\alpha/2}$

Where $z_{\alpha/2}$ is chosen so that $P(z > z_{\alpha/2}) = \alpha / 2$

**Note:** $\mu_0$ is the symbol for the numerical value assigned to $\mu$ under the null hypothesis

### Conditions Required for a Valid Large-Sample Hypothesis Test for $\mu$

1. A random sample is selected from the target population.
2. The sample size $n$ is large (i.e., $n \geq 30$). (Due to the central limit theorem, this condition guarantees that the test statistic will by approximately normal regardless or the shape of the underlying probability distribution of the population).

### Possible Conclusions for a Test of Hypothesis

1. If the calculated test statistic falls into the rejection region, reject $H_0$ and conclude that the alternative hypothesis $H_a$ is true. State that you are rejecting $H_0$ at the $\alpha$ level of significance. Remember that the confidence is in the testing process, not the particular results of a single test.
2. If the test statistic does not fall into the rejection region, conclude that the sampling experiment does not provide sufficient evidence to reject $H_0$ at the $\alpha$ level of significance. [Generally, we will not “accept” the null hypothesis unless the probability $\beta$ of a Type II error has been calculated. (See optional Section 8.6.)]

### 3. Observed Significance Levels: $p$-values

**Definition 8.1**

The **observed significance level**, or **$p$-value**, for a specific statistical test is the probability (assuming that $H_0$ is true) of observing a value of the test that is at least as contradictory to the null hypothesis, and supportive of the alternative hypothesis, as the actual one computed from the sample data.
Steps for Calculating the p-value for a Test of Hypothesis

1. Determine the value of the test statistic $z$ corresponding to the result of the sampling experiment.
   a. If the test is one-tailed, the $p$-value is equal to the tail area beyond $z$ in the same direction as the alternative hypothesis. Thus, if the alternative hypothesis is of the form $>$, the $p$-value is the area to the right of, or above, the observed $z$-value. Conversely, if the alternative is of the form $<$, the $p$-value is the area to the left or, or below, the observed $z$-value. (See Figure 8.8.)
   b. If the test is two tailed, the $p$-value is equal to twice the tail area beyond the observed $z$-value in the direction of the sign of $z$. That is, if $z$ is positive, the $p$-value is twice the area to the right, or above, the observed $z$-value. Conversely, if $z$ is negative, the $p$-value is twice the area to the left of, or below, the observed $z$-value. (See Figure 8.9.)

Reporting Test Results as p-Values: How to Decide Whether to Reject $H_0$

1. Choose the maximum value of $\alpha$ that you are willing to tolerate.
2. If the observed significance level ($p$-value) of the test is less that the chosen value of $\alpha$, reject the null hypothesis. Otherwise, do not reject the null hypothesis.

Converting a Two-Tailed p-Value from a Printout to a One-Tailed p-Value

\[
p = \frac{\text{Reported } p - \text{value}}{2} \quad \text{if } \begin{cases} H_a : \text{is of form } > \text{ and } z \text{ is positive} \\ H_a : \text{is of form } < \text{ and } z \text{ is negative} \end{cases}
\]

\[
p = 1 - \frac{\text{Reported } p - \text{value}}{2} \quad \text{if } \begin{cases} H_a : \text{is of form } > \text{ and } z \text{ is negative} \\ H_a : \text{is of form } < \text{ and } z \text{ is positive} \end{cases}
\]

4. Small-Sample Test of Hypothesis about a Population Mean

**Small-Sample Test of Hypothesis About $\mu$**

<table>
<thead>
<tr>
<th>One-Tailed Test</th>
<th>Two-Tailed Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0 : \mu = \mu_0$</td>
<td>$H_0 : \mu = \mu_0$</td>
</tr>
<tr>
<td>$H_a : \mu &lt; \mu_0$ (or $H_a : \mu &gt; \mu_0$)</td>
<td>$H_a : \mu \neq \mu_0$</td>
</tr>
</tbody>
</table>

Test statistic: $t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$

Rejection region: $t < -t_{\alpha}$

( or $t > t_{\alpha}$ when $H_a : \mu > \mu_0$)

Where $t_{\alpha}$ and $t_{\alpha/2}$ are based on $(n - 1)$ degrees of freedom
What Can Be Done if the Population Relative Frequency Distribution Departs Greatly from Normal?

Answer: Use one of the nonparametric statistical methods of Chapter 14.

5. Large-Sample Test of Hypothesis about a Population Proportion

Large-Sample Test of Hypothesis about \( p \)

One-Tailed Test

\[ H_0 : p = p_0 \]

\[ H_a : p < p_0 \text{ (or } H_a : p > p_0 \text{) } \]

Test statistic: \( z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} \)

Rejection region: \( z < -z_\alpha \)

( or \( z > z_\alpha \) when \( H_a : p > p_0 \) )

Two-Tailed Test

\[ H_0 : p = p_0 \]

\[ H_a : p \neq p_0 \]

Test statistic: \( z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} \)

Rejection region: \( z < -z_{\alpha/2} \) or \( z > z_{\alpha/2} \)

Where \( \hat{p} \) = hypothesized value of \( p \), \( \sigma_{\hat{p}} = \sqrt{\frac{p_0 q_0}{n}} \), and \( q_0 = 1 - p_0 \)

Conditions Required for a Valid Large-Sample Hypothesis Test for \( p \)

1. A random sample is selected from a binomial population
2. The sample size \( n \) is large. (This condition will be satisfied if \( np_0 \) and \( nq_0 \) are both at least 15.)