Say which of the following are "TRUE" or "FALSE". (2 points each)

(10) 1. (a) If L is any language on \{0,1\}, then we always have \( (L^R)^* = (L^*)^R \).

(b) The set of all infinite languages on the alphabet \{a\} is countable.

c) If \( L \neq \{\lambda\} \text{ contains } L^* \), then \( L \) has to be an infinite language.

d) If a CFG \( G \) has at least two productions and no useless ones, then \( L(G) \) is infinite.

(e) If a DFA \( M \) has no inaccessible states, has a loop at a non-accepting state, and at least one accepting state, then \( L(M) \) must be infinite.

Just write down the correct answer. (3, 3, 4, 4 points respectively)

(18) 2. (a) Find a regular expression \( E \) which describes the set of all strings in \{0,1\}^* which contains at least 2 occurrences of the string 101.

Ans: \( E = \)

(b) If \( M \) is the NFA below, then \( L(M) = \)

(c) If \( G = \{S \rightarrow ASBB, \ S \rightarrow b, \ A \rightarrow aa, \ B \rightarrow b, \ B \rightarrow \lambda\} \),
then \( L(G) = \)

(d) Find a RLG \( G \) which generates the language \( a \cdot (b \cdot a)^* \cdot c^* \).

Ans: \( G = \)

(e) Find a DFA \( M \) with \( L(M) = 1^* + (1 \cdot 0^*) \).

Ans: \( M = \)

Use the back of this paper for question #3. (2, 3, 3, 4 points respectively)

(12) 3. (a) A regular expression over \{a,b,c\} is a string of characters from which alphabet?

(b) Define precisely what kinds of productions are allowed in a right linear grammar (RLG).

(c) Define what it means for two states B & C in DFA \( M \) to be indistinguishable.

(d) Define what is the transition relation \( \Delta \) of an NFA and specify its domain.
1(a) TRUE. This is a theorem from class, $(L^*)^* = (L^*)^*.$
(b) FALSE. The set of all inf. languages on $\{a\}$ is uncountable.
(c) TRUE. If $L \neq \{\lambda\} \subseteq L^*$, then $L$ has a nonempty string, so $L^*$ is inf.
(d) FALSE. $S \rightarrow aA$, $A \rightarrow b$.
(e) FALSE. $L(M) = \{1\}$

2(a) $E = (0+1)^* \cdot (10101 + 101 \cdot (0+1) \cdot 0101 \cdot (0+1)^*)$.
(b) $L(M) = (0+10)^* \cdot 0^* 1 (00^* 1)^*$.
(c) $L(G) = \{a^{2n} b b^n : n \geq 0, 0 \leq k \leq 2n\} = \{a^{2n} b : 1 \leq k \leq 2n+1\}$.
(d) $\rightarrow A$, $A \rightarrow aB$, $B \rightarrow bB$, $B \rightarrow C$, $C \rightarrow C$, $C \rightarrow \lambda$.
(e)

3(a) A regular expression over $\{a, b, c\}$ is a string of characters from the alphabet $\{a, b, c, \lambda, \phi, +, \cdot, *, (, )\}$.
(b) In a RLG only productions of the form $A \rightarrow \alpha B$ or $A \rightarrow \beta$
with $A, B \in V$ & $\alpha, \beta \in T^*$ are allowed.
(c) Two states B & C in a DFA are indistinguishable in $M$
if for each $q \in T, S^*(B, q) \in A(M) \iff S^*(C, q) \in A(M)$.
(d) The transition relation of an NFA $M$ is a binary
relation $\Delta$ from $Q \times (T \cup \lambda)$ to $Q$ such that for each $q \in Q$, $\langle \langle q \lambda \rangle, q \rangle \in \Delta$. The domain of $\Delta$ is $Q \times (T \cup \lambda)$ and the codomain of $\Delta$ is $Q$. 