MAD 3512: Quiz #1 - Spring 09

TIME: 25 min.

Just write "TRUE" or "FALSE".

(10) 1(a) For any language A on \(\{0,1\}\), we always have \((A^c)^r = (A^r)^c\). _____
(b) The set of all infinite languages on \(\{a\}\) is countable. _____
(c) If a DFA, M has no inaccessible states then \(L(M) \neq \emptyset\). _____
(d) The DFA obtained from an NFA will always have more states. _____
(e) If G is a CFG with a production of the form A-bA and G has no useless production, then \(L(G)\) is infinite. _____

Just write down the correct answer.

(18) 2(a) Find a regular expression E for the set of all strings in \(\{0,1\}\) * which contains at least two occurrences of the string 10.

Ans: \(E = \)

(b) If G = \(\{S-SAASa, S-a, A-b, A-\lambda\}\), then

\(L(G) = \)

(c) If M is the NFA below, then

\[L(M) = \]

(d) Find a RLG G for \(0.1^* 0^* 1\)

Ans: \(G = \)

(e) Find a DFA M with \(L(M) = (a^*b) + b^*\)

Ans: \(M = \)

Use the back of this paper for question #3.

(12) 3(a) Define what is a regular expression over the alphabet \(\{0,1,2\}\).
(b) Define what is an inherently ambiguous context-free language L.
(c) Define when two states of a DFA M are indistinguishable.
(d) Define what is the extended transition function of an DFA M.
1. (a) TRUE \( \varphi \in (A^c)^c \Rightarrow \varphi \in A^c \Rightarrow \varphi \in A \Rightarrow \varphi \in A^c \Rightarrow \varphi \in (A^c)^c \), etc.
(b) FALSE \( \mathcal{L}(a^3) \) is uncountable by a Theorem in class.
(c) FALSE Consider the DFA
(d) FALSE Consider the NFA
(e) TRUE \( A \rightarrow bA \) will generate \( b^nA \) & the A will terminate.

2. (a) \( E = (0 + 1)^*101(01)^*00\( (0 + 1)^* \)
(b) \( L(G) = \{ a^k b^{n+1} : 0 \leq k \leq 2n, n \geq 0 \} \)
(c) \( L(M) = a \cdot (b + ba)^* \)
(d) \( S \rightarrow 0A, A \rightarrow 1A, A \rightarrow B, B \rightarrow 0B, B \rightarrow 1 \)
(e) 

3. (a) A regular expression over \( \{0,1,2\} \) is defined recursively as follows. (i) \( 0, 1, 2, \lambda \) and \( \lambda \) are regular expressions,
(ii) If \( E \) & \( F \) are reg. expr. then so are \( (E+F), (E\cdot F) \) & \( (E^*) \).
(b) An inherently ambiguous context-free language is a language that can be generated by an ambiguous CFG but cannot be generated by an unambiguous CFG.
(c) Two states \( p \) & \( q \) in a DFA are indistinguishable if for each string \( w \) in \( \Sigma^* \), \( s^*(p, w) \in A \iff s^*(q, w) \in A \).
   Here \( \Sigma = \) input alphabet & \( A = \) set of accepting states of DFA.
(d) The extended transition function of a DFA \( M \) is defined recursively as follows. \( s^*: Q \times \Sigma^* \rightarrow Q \) and (i) \( s^*(q, \lambda) = q \),
   (ii) \( s^*(q, wa) = s(s^*(q, w), a) \) for any \( a \in \Sigma \) & \( w \in \Sigma^* \).
1(a) **TRUE**  
(b) **FALSE**, Set of all inf. lang. on \{a\} is uncountable  
(c) **FALSE**, Consider a DFA with no accepting states  
(d) **FALSE**, Let the NFA be a DFA  
(e) **TRUE**

2(a) \( E = (0+1)^*10 \cdot (0+1)^* \cdot 10 \cdot (0+1)^* \)  
(b) \( L(G) = \{ b^k a^{n+1} : 0 \leq k \leq 2n, n \geq 0 \} \)  
(c) \( L(M) = a_1( b + ba )^* = \{ a_1, \{ b, ba \}^* \} \)  
(d) \( S \rightarrow 0A, A \rightarrow 1A|B, B \rightarrow 0B|1 \)  

![Diagram of automaton](attachment:image.png)

3(a) A regular expression over \{0,1,2\} is defined recursively as follows:  
(a) 0, 1, 2, \( \lambda \) and \( \emptyset \) are regular expressions  
(b) If \( \text{E} \& \text{F} \) are reg. expr. then so are \( \text{E} + \text{F} \), \( \text{E} \cdot \text{F} \) & \( \text{E}^* \).  

(b) An inherently ambiguous context-free language is a context-free language that cannot be generated by an unambiguous context-free grammar.

(c) Two states \( p \) and \( q \) in a DFA are indistinguishable if for each \( w \in \Sigma^* \), \( S^*(p, w) \in A^* \) if and only if \( S^*(q, w) \in A^* \).

(d) The extended transition function of a DFA is the function \( S^*: Q \times \Sigma^* \rightarrow Q \) that is recursively defined as follows:  
\( S^*(q, \lambda) = q \) and \( S^*(q, wa) = S( S^*(q, w), a ) \) for any \( a \in \Sigma \) and any \( w \in \Sigma^* \).