Answer all 6 questions. Provide all reasoning and show all working. An unjustified answer will receive little or no credit. BEGIN EACH QUESTION ON A SEPARATE PAGE.

(15) 1. (a) Define what is the extended transition function of an NFA.

(b) Convert the NFA on the right into an equivalent DFA.

(15) 2. Find regular expressions which describe the languages below:
(a) \( L_1 = \{ \alpha \in \{0,1\}^* : \alpha \text{ contains both } 01 \text{ & } 100 \} \)
(b) \( L_2 = \{ \beta \in \{0,1\}^* : \beta \text{ has at most two occurrences of } 10 \} \)

(20) 3. (a) Find all the inaccessible states in the DFA below.
(b) Then partition the remaining states into blocks of indistinguishable states and find the reduced machine.

(15) 4. Find a DFA which accepts precisely the strings in the language
\( L_4 = \{ \omega \in \{a,b\}^* : |2n_a(\omega) - 3n_b(\omega) - 1| \mod 4 < 2 \}
and then check your DFA with \( aaba \) as input.

(20) 5. (a) Define what are inaccessible productions and non-terminating productions in a CFG \( G \).
(b) Find a context-free grammar which generates the language
\( L_5 = \{ c^k b^n : k > 2n \} \cup \{ b^k a^n : k < 2n + 3 \} \).

(15) 6. Let \( A, B \) and \( C \) be languages based on the alphabet \( \{0,1\} \).
(a) Is it always true that \( (A \cdot C') \cup (B \cdot C') \subseteq (A \cup B) \cdot C' \) ?
(b) Is it always true that \( A \cdot (B - C) \subseteq (A \cdot B) - (A \cdot C) \) ?
Justify your answers completely.
1.(a) The extended transition function $\Delta^*: Q \times \Sigma^* \to \mathcal{P}(Q)$ of an NFA is defined by $\Delta^*(p, w) = \{ q \in Q : w \text{ can lead you from } p \text{ to } q \}$

(b)

\[ R(\lambda) = \{ A, B \} \]

\[ A \xrightarrow{0} \{ B \} \quad A \xrightarrow{B} \{ A, B \} \]

DFA:

\[ \begin{array}{c}
\{A, B\} \\
\{B\}
\end{array} \]

2. (a)

\[
E_1 = (0+1)^* (01(0+1)^*100 + 100(0+1)^*01 + 0100 + 1001)(0+1)^*
\]

(b)

No 10's, one 10, two 10's

\[
E_2 = 0^*1 + 0^*100^*1 + 0^*101^*100^*101^*10^*1^* 
\]

3. (a)

D and H are inaccessible states

(b)

$P_0: \{ A, B, F \}, \{ C, E, G \}$

$P_1: \{ A, B, F \}, \{ C \}, \{ E, G \}$

$P_2: \{ A, F \}, \{ B, C \}, \{ E, G \}$

$P_3: \{ A, F \}, \{ B \}, \{ C \}, \{ E \}, \{ G \}$

$P_4: \{ A, F \}, \{ B \}, \{ C \}, \{ E \}, \{ G \} = P_3$

4. (a) Let $f(n) = 2n_0(n) - 3n_6(n) - 1 \pmod{4}$ and let $A_i \ (i = 0, 1, 2, 3)$ be the state that stores the information that $f(n) = i \pmod{4}$.
4. (a) Then \( f(\lambda) = 2n_a(\lambda) - 3n_b(\lambda) - 1 = 0 - 0 - 1 \equiv 3 \pmod{4} \). So \( A_3 \) will be the initial state. \( A_0 \) and \( A_1 \) will be the accepting states because \( 0 < 2 \) and \( 1 < 2 \). Also
\[
\begin{align*}
    f(aa) &= 2n_a(aa) - 3n_b(aa) - 1 = 2n_a(\omega) - 3n_b(\omega) - 1 + 2 = f(\omega) + 2 \pmod{4} \\
f(ab) &= 2n_a(ab) - 3n_b(ab) - 1 = 2n_a(\omega) - 3n_b(\omega) - 1 - 3 = f(\omega) + 1 \pmod{4}
\end{align*}
\]

(b) Input: \( w = a \ a \ a \ b \ a \)

States: \( A_3 \ A_1 \ A_3 \ A_0 \ A_2 \)

Check: \( 2n_a(\omega) - 3n_b(\omega) - 1 = 2(3) - 3 \cdot 1 \equiv 2 \pmod{4} \)

5. (a) An inaccessible (or unreachable) production is one which involves a variable that cannot be reached from the starting variable.

A non-terminating production is one that involves a variable that does not terminate or lead to something that eventually terminates.

6. (a) **YES.** Let \( \phi \in (A.C^*) \cup (B.C^*) \). Then \( \phi \in A.C^* \) or \( \phi \in B.C^* \).

So \( \phi = \alpha \gamma_1 \) for some \( \alpha \in A \) and \( \gamma_1 \in C^* \), or \( \phi = \beta \gamma_2 \) for some \( \beta \in B \) and \( \gamma_2 \in C^* \).

In the first case \( \phi = \alpha \gamma_1 \) with \( \alpha \in (A \cup B) \) and \( \gamma_1 \in C^* \), so \( \phi \in (A \cup B).C^* \). And in the second case \( \phi = \beta \gamma_2 \) with \( \beta \in (A \cup B) \) and \( \gamma_2 \in C^* \), so \( \phi \in (A \cup B).C^* \). So in either \( \phi \in (A \cup B).C^* \). Hence \( (A.C^*) \cup (B.C^*) \subseteq (A \cup B).C^* \).

(b) **NO.** Let \( A = \{0, 01\} \), \( B = \{1\} \), and \( C = \{11\} \). Then \( B \cdot C = \{1\} \).

So \( A.(B \cdot C) = \{0, 01\} \cdot \{1\} = \{01, 011\} \), \( A \cdot B = \{0, 01\} \cdot \{1\} = \{01, 011\} \), \( A \cdot C = \{0, 01\} \cdot \{11\} = \{01, 0111\} \), and \( \{A.B \} \cdot (A.C) = \{01, 011\} \cdot \{11, 0111\} = \{01\} \).

Hence \( A.(B \cdot C) = \{01, 011\} \neq \{01\} = (A \cdot B) \cdot (A \cdot C) \).