TEST #2 - Spring 2010

Answer all 6 questions. Provide all reasoning and show all working. An unjustified answer will receive little or no credit. Begin each of the 6 questions on a separate page.

16.1. Let $L$ be the language accepted by the NFA shown on the right. Find NFAs which accept
(a) $L^*$
(b) $(L^*)^*$.

16.2. (a) Find an NFA which is equivalent to the RLG given below.

$$G: \rightarrow A, A \rightarrow 01A, A \rightarrow 1B, B \rightarrow 100, B \rightarrow 0C, B \rightarrow \lambda,$$
$$C \rightarrow E, C \rightarrow 10D, D \rightarrow 00D, D \rightarrow 101, E \rightarrow \lambda, E \rightarrow 1A.$$

(b) Convert the NFA shown below on the right in Qu.#3 into an equivalent RLG.

16.3 (a) Find a regular expression for the language accepted by the NFA shown on the right.

(b) Suppose $F$ is a finite language and $A \cup F$ is regular, does it follow that $A$ must also be regular?

16.4 (a) Define what are the initial functions and what are the primitive recursive functions.

(b) Show that $f(x,y) = 3x + 4y + 2$ is a primitive recursive function by finding primitive recursive functions $g$ and $h$ such that $f = \text{prim.rec.}(g,h)$ and showing that $g \& h$ are primitive recursive.

16.5 (a) Write down what the Halting Problem says.

(b) Show what happens at each step if 10101 is the input for the TM, $M$ shown on the right.

(c) Find the language accepted by $M$.

16.6. Determine which of the following languages are regular and which are not.

(a) $L_1 = \{a^k b^n : k + l = 2n^2 \text{ (mod 3)}\}$

(b) $L_2 = \{a^k c^n : k + l > n^2\}$

[If you say that it is regular, you must find a regular expression; if you say it is non-regular, you must give a complete proof.]
1. 

\[ R(\lambda) = \{A\} \rightarrow \{A, B\} \rightarrow \{A, B, \lambda\} \]

DFA for \( L \)

DFA for \( L^c \)

OAS - NFA for \( L^c \)

NFA for \( (L^c)^R \)

2. 

\( a \rightarrow B, B \rightarrow aC, C \rightarrow bD, C \rightarrow cA, A \rightarrow bA, A \rightarrow cB, D \rightarrow cD, D \rightarrow aE \)

\( E \rightarrow aA, E \rightarrow bC, E \rightarrow \lambda \).

3. (a) Corresponding GFA

(b) Eliminate \( \lambda \)
(a) \( L(H) = (abc)^*, ab\cdot c, (bbc + aba/c + bcb^c)(abc)^*ab^c) \).

(b) \( A = ((A \cup F) - F) \cup (AnF) \).

Since \( F \) is finite and \( AnF \subseteq F \), 
\( AnF \) is also finite. Now any 
finite set is regular. So \( F \) and \( AnF \) are regular. Also 
\( AUA \) is given as regular. Hence \( (A \cup F) - F \) is regular
and so \( A = ((A \cup F) - F) \cup (AnF) \) will be regular by the 
closure theorem.

4 (a) The initial functions are: (i) the constant 0, (ii) the zero func.
of 1 var, \( z(x) = 0 \), (iii) the successor function, \( s(x) = x+1 \); and
(iv) the projective functions \( I^0_k(x_1, \ldots, x_n) = x_k \).
The primitive recursive functions are the functions that can
be obtained from the initial functions by using a finite number of
applications of compositions and primitive recursions.

(b) \( f(x,0) = 3x + 4(0) + 2 = 3x + 2 \), so \( 9(x) = 3x + 2 \)
f(x,y+1) = 3x + 4(y+1) + 2 \quad \rightarrow \quad g(0) = 3(0) + 2 = 2 = g_1 \)
\( = (3x + 4y + 2) + 4 \quad \rightarrow \quad g(y+1) = 3(y+1) + 2 = 3y + 2 + 3 \)
\( = f(x, y) + 4 \quad \rightarrow \quad g(y) + 3 = h(y, g(y)) \)
\( = h(x, y, f(x, y)) \quad \rightarrow \quad h = sosososos I^3 \)
\( \therefore f = \text{prim. rec. } (g, h) = \text{prim. rec. } (\text{prim. rec. } (sosos, sosos I^3, sosos I^3)) \)
5(a) Halting Problem: Is there a TM $H$ such that for an arbitrary TM $M$ and an arb. input $w$ for $M$:
$H$ halts on $c(M)\#\langle w \rangle$ in an ac. st. if $M$ halts on $w$, and $H$ halts on $c(M)\#\langle w \rangle$ in a non-ac. st. if $M$ does not halt on $w$?
(Here $c(M)\#\langle w \rangle$ is $\langle M, w \rangle$ coded into the alphabet of $H$)

(b) $\langle A, 10101 \rangle$ $\vdash$ $\langle D, 00101 \rangle$ $\vdash$ $\langle D, 01101 \rangle$ $\vdash$ $\langle C, 01101 \rangle$ $\vdash$ $\langle B, 01111 \rangle$
$\vdash$ $\langle C, 01110 \rangle$ $\vdash$ $\langle F, 01110 \rangle$.

(c) $L(M) = 10^k + 01(01)^* + 10^k(01)^*$.

6(a) $L_1 = \{a^k b^n : n \leq 2n^2 \text{ mod } 3 \} = \{a^k b^n : k = 2n^2 - 1 \text{ mod } 3 \}$

$N = 0 \text{ (mod 3)} \Rightarrow k = 2(0)^2 - 1 = -1 \equiv 2 \text{ (mod 3)}$

$N = 1 \text{ (mod 3)} \Rightarrow k = 2(1)^2 - 1 = 1 \text{ (mod 3)}$

$N = 2 \text{ (mod 3)} \Rightarrow k = 2(2)^2 - 1 = 7 \equiv 1 \text{ (mod 3)}$.

So a reg. expr. for $L_1$ is $aa(aaa)^* bbb^* = a(aaa)^* b(bbb)^* + a(aaa)^* b(bbb)^*$. Hence $L_1$ is a regular language.

(b) $L_2 = \{b^k c^n : k + 1 > n^2 \}$. Suppose $L_2$ is regular. Then we can find an NFA $N$ such that $L(N) = L_2$. Let $N$ be the number of states in $M$. Then $b^{n^2}c^n \in L(M)$ because $N+1 > (N)^2$.

(Here $k = N^2$ and $n = N$) So $b^{n^2}c^n$ will be accepted by $N$. Since $M$ has only $N$ states and it takes $N+1$ states to process $c_N$, the acceptance track of $b^{n^2}c^n$ must have a loop as shown below with $j \geq 1$.

Now if we ride this loop twice we will see that $M$ accepts $b^{n^2}c^j c^k c^{N-j} = b^{n^2}c^{N+1}$. But $N+1 \neq (N+j)^2$, so this contradicts the fact that $L(M) = L_2$. Hence $L_2$ is not regular.