MAC 2313 (Calculus III)  
Test 2 Review - Spring 2015

1. Describe the domain of the function $f$ in words. a) $f(x, y, z) = \ln(z^2 - x^2 - y^2)$, b) $f(x, y, z) = \cos^{-1}(x^2 + y^2 + z^2)$.

2. Sketch the largest region where $f$ is continuous. a) $f(x, y) = \sqrt{x^2 + y^2 - 4}$, b) $f(x, y) = \sin^{-1}(y - x)$.

3. a) Find an equation for the level curve of the function $f$ that passes through the point $P$. i) $f(x, y) = \int_{x}^{y} \frac{dt}{t^2 + 1}$, $P(-\sqrt{3}, \sqrt{3})$. ii) $f(x, y) = \sum_{n=0}^{\infty} (x/y)^n$, $P(1, 2)$. b) Find an equation for the level surface of the function $f$ that passes through the point $P$. i) $f(x, y, z) = \sum_{n=0}^{\infty} (-1)^n (xyz)^n$, $P(\sqrt{2}, 1, 1/\sqrt{2})$. ii) $f(x, y, z) = f_x \frac{\partial f}{\partial x} + f_y \frac{\partial f}{\partial y} + f_z \frac{\partial f}{\partial z}$, $P(0, 1/2, 2)$. c) Identify the level surfaces of $f(x, y, z) = \ln(x^2 + y^2 + z^2)$ for $k = -1, 0, 1$.

4. a) Let $f(x, y, z) = x^2y^3\sin(x^2z^2)$. i) Find $f(y, z, x)$ and $f(z, x, y)$. ii) Find $f_x(x, y, z)$, $f_y(x, y, z)$ and $f_z(x, y, z)$. b) Use implicit partial differentiation to find $\partial x/\partial y$ and $\partial x/\partial z$ if $x^2 + y^2 - z^4 + 4 = 0$ defines $x$ as a function of $y$ and $z$. c) If $x = v \ln u$, $y = u \ln v$, use implicit partial differentiation to find $u_x$, $u_y$, $v_x$ and $v_y$. If we set $z = \tan(2u - 3v)$, use the chain rule to find $z_x$ and $z_y$. d) Answer the same questions as c) if $x = u^2 - v^2$, $y = u^2 - v$, and $z = u^2 + v^2$.

5. Evaluate each limit.

a) $\lim_{(x,y,z)\to(-1,2,1)} \frac{x^2}{\sqrt{x^2 + 2y^2 + 3z^2}}$, b) $\lim_{(x,y)\to(1,1)} \frac{x^2 - 2xy + y^2}{x - y^2}$, c) $\lim_{(x,y)\to(-1,1)} \frac{2x^3 + 3xy^2 - 2y^2 - 3y^2}{2x^2 + xy - y^2}$, d) $\lim_{(x,y,z)\to(0,0,0)} \frac{xyz}{x^2 + y^2 + z^2}$, e) $\lim_{(x,y,z)\to(2,2,1)} \frac{\sin(2x - 5y + 6z)}{(2x - 5y + 6z)(y + z)}$, f) $\lim_{(x,y)\to(0,0)} \frac{1 - \cos(x^2 + y^2)}{(x^2 + y^2)^2}$.

6. Find an equation for the tangent plane and parametric equations for the normal line to the given surface at the given point. a) $3x^2 - 2y^2 + 4z^2 = 5$, $P(-1,1,1)$. b) $\frac{x + 2y}{2y + z} = 1$, $P(1,2,1)$.

7. Find parametric equations for the tangent line to the curve of intersection of the given surfaces at the given point. a) $z = x^2 + y^2$ and $z = 8 - x^2 - y^2$, $P(\sqrt{2}, -\sqrt{2}, 4)$. b) $x^2 + y^2 = 2$ and $x + 2y + 3z = 6$, $P(1,1,1)$.

8. Let $f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2 - 2x + 3y}, & (x, y) \neq (0,0), \\ 0, & (x, y) = (0,0). \end{cases}$ a) Find $f_x(0,0)$, and $f_y(0,0)$. b) Show that $f$ is not continuous at $(0,0)$. c) Is $f$ differentiable at $(0,0)$?

9. a) Write down the definition of “$f$ is differentiable at $(x_0, y_0)$”. Use the definition in a) to show that the function $f$ given by $f(x, y) = 2x - 3xy$ is differentiable at the point $(1, -2)$.

10. a) Let $f(x, y, z) = x^3e^{yz}$. i) Find the differential $df$. ii) Find the local linear approximation for $f$ about $P(1, -1, -1)$, and use it to approximate $f(Q)$ with $Q(0.99, -1.01, -0.98)$. b) Answer the same questions for $f(x, y, z) = yz \ln(xy)$, $P(e, 1, 1)$ and $Q(2.72, 0.99, 1.01)$. i) $f(x, y, z) = \tan^{-1}(xyz)$, $P(1,1,1)$ and $Q(0.98,1.01,0.99)$

11. a) Find the gradient of $g$ at $P(1, 1, 1)$. b) Find a unit vector in the direction in which $g$ decreases most rapidly at the point $Q(-1,1,1)$, and find the rate of change of $g$ at $Q$ in that direction. c) Find the directional derivative of $g$ in the direction of the vector $\bar{v} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ at the point $A(1, -1, 1)$.

12. Find all the critical points of $f$ as points of local minimum, local maximum, or saddle points. a) $f(x, y) = xy + 2x - \ln(x^2y)$, b) $f(x, y) = x^3 - y^3 - 2xy + 6$, c) $f(x, y) = 4xy + x^4 + y^4$, d) $f(x, y) = x^4y^4$, e) $f(x, y) = 2y^2x - x^2y + 4xy$. 13. Find all the first partial derivatives of $f$ if a) $f(x, y) = \log_x(y)$, b) $f(x, y) = \int_x^y g(t)\ dt$, c) $f(x, y, z) = yz \ln(xy)$, d) $f(x, y, z) = yz \sec(xz)$.

14. a) Find the point on the paraboloid $z = x^2 + y^2 + 10$ that is closest to the plane $x + 2y - z = 0$. b) Find three positive numbers whose sum is 48 and their product is as large as possible. c) Problems 42 and 44 in 13.8, p. 987.

15. Review the true/false problems in the text.