1. a) Let $G$ be the solid defined by the inequalities: $\sqrt{x^2 + y^2} \leq z \leq 20 - x^2 - y^2$. Find the coordinates of the centroid of $G$. b) Find the mass and center of gravity of the solid $G$ enclosed by the portion of the sphere $x^2 + y^2 + z^2 = 2$ on or above the plane $z = 1$ if the density is $\delta = \sqrt{x^2 + y^2 + z^2}$.

2. a) State the fundamental theorem of line integral. b) Let $F(x, y) = (2xy + x) \mathbf{i} + (x^2 + 2y) \mathbf{j}$. Show that $F$ is conservative. b1) Find a potential function $\varphi$ for $F$. b2) Evaluate the line integral $\int_{\sigma} (2xy + x) \, dx + (x^2 + 2y) \, dy$ along the curve $C$ parametrized by $\mathbf{r}(t) = (t + 1) \mathbf{i} + \sin^{-1} t \mathbf{j}$, $0 \leq t \leq 1$.

3. a) Set up, but do not evaluate, two iterated integrals equal to the surface integral $\iint_{\sigma} F \cdot d\mathbf{S}$, where $(u,v)$ are the cylindrical coordinates of a point on the surface. b) Consider the parametric surface given by $x = u \cos v, y = u \sin v, z = u$. b1) Find the area $\iint_{\sigma} \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F} = (x^2 + y^2) \mathbf{i} + (x^2 - y^2) \mathbf{j} + (2xy + z^2) \mathbf{k}$. b2) Find a potential function $\varphi$ for $\mathbf{F}$. b3) Evaluate the line integral $\int_{\gamma} (2xy + x) \, dx + (x^2 + 2y) \, dy$ along the curve $C$ the line integral along $C$ of the del operator $\nabla$ acting on $F$.

4. Let $F(x, y) = (x^3 + 4e^{-2x}) \mathbf{i} + (x^4 + y^2) \mathbf{j}$. a) Show that $F$ is conservative. b) Find a potential function $\varphi$ for $F$. c) Evaluate the line integral $\int_{C} (x^3y + 4e^{-2x}) \, dx + (\frac{x^4}{4} + y^2) \, dy$ along the curve $C$ parametrized by $\mathbf{r}(t) = \cos^3 t \mathbf{i} + \sin^3 t \, \mathbf{j}$, $0 \leq t \leq \pi$.

5. a) Let $F(x, y, z) = (x^2 - 2xy) \mathbf{i} + (3y^2 - 2yz) \mathbf{j} + (5z^2 - 2xz) \mathbf{k}$. Find $\operatorname{div} \mathbf{F}$ and $\operatorname{curl} \mathbf{F}$. Evaluate the line integral $\int_{C} \mathbf{F} \cdot d\mathbf{r}$, where $C$ is the triangle with vertices $(0,0)$, $(0,2)$ and $(2,0)$. b) Evaluate $\int_{C} x^2y^2z^2 \, ds$. c) Evaluate $\int_{C} y \, dx + z \, dy - x \, dz$ along the helix $x = \cos(\pi t)$, $y = \sin(\pi t)$, $z = t$ from the point $(1,0,0)$ to $(-1,0,1)$. d) Find the mass of a thin wire shaped in the form of the curve $x = e^t \cos t$, $y = e^t \sin t$, $0 \leq t \leq 1$ if the density function $\delta$ is proportional to the distance to the origin.

6. a) Find parametric equations for the paraboloid $z = x^2 + y^2$ in terms of the parameters $\theta$ and $\phi$, where $(\rho, \theta, \phi)$ are spherical coordinates of a point on the surface. b) Find a parametric representation of the portion of the sphere $x^2 + y^2 + z^2 = 9$ on or above the plane $z = 2$ in terms of the parameters $\rho$ and $\theta$, where $(\rho, \theta, \lambda)$ are the cylindrical coordinates of a point on the surface.

7. a) Let $F(x, y, z) = \sqrt{x^2 + y^2} \mathbf{k}$. Find the flux of $\mathbf{F}$ across $\sigma$, where $\sigma$ is the portion of the cone $\sigma(u, v) = u \cos v \mathbf{i} + u \sin v \mathbf{j} + 2u \mathbf{k}$, $0 \leq u \leq \sin v \leq 0 \leq v \leq \pi$. b) Let $F(x, y, z) = x \mathbf{i} + y \mathbf{j} + 2z \mathbf{k}$. Find the flux of $\mathbf{F}$ across $\sigma$ for the portion of the paraboloid below the plane $z = y$, oriented by downward unit normals.

8. a) Let $F(x, y, z) = \sqrt{x^2 + y^2} \mathbf{k}$. Use the Divergence Theorem to find the flux of $\mathbf{F}$ across $\sigma$, where $\sigma$ is the boundary of the solid $G$, bounded above by the sphere $z = \sqrt{4 - x^2 - y^2}$ and below by the $xy$-plane, with outward orientation.

9. a) $\sigma$ is the boundary of the solid $G$, bounded above by the sphere $z = \sqrt{4 - x^2 - y^2}$ and below by the $xy$-plane, with outward orientation. b) $\sigma$ is the boundary of the cylindrical solid enclosed by $x^2 + y^2 = 4$, $z = 0$ and $z = 1$ with outward orientation.

10. a) Use Green’s theorem to evaluate the line integral $\int_{C} (4y + \cos(1 + e^{-x})) \, dx + (2x - \sec^2 y) \, dy$, where $C$ is the circle $x^2 + y^2 = 9$ going from $(0,3)$ to $(0,3)$ counterclockwise.

11. Review the Fundamental Theorem of Line Integral, Green’s Theorem and the Divergence Theorem.