Fall 01

Classical Physics

Name ______________________________

There are 9 problems in three sets (A, B, and C). You need to solve a total of 6 problems including at least 1 from each of the three sets. Circle your selection on this page.

Write your name on this page and on your solution pages. Staple your solutions together and use this page as the cover page.

A-1. XW

A-2. XW

B-1.

B-2.

B-3.

B-4.

C-1. XW

C-2. XW

C-3. XW

A-1. A sealed cylinder of volume $2V_0$ is separated into a right and a left chamber by an insulating, movable wall. Each chamber contains one mole of monoatomic ideal gas. Initially both chambers have the same temperature, T_0, and same volume, V_0. Heat is taken out slowly from the left chamber until the left chamber volume reduces to $\frac{1}{2}V_0$.

Calculate, in terms of P_0, V_0, T_0, and γ ($\gamma = C_p / C_v$), the following:

a) The final pressure of the right chamber
b) final temperature of the right chamber
c) final temperature of the left chamber
d) the amount of work done by the gas in the right chamber

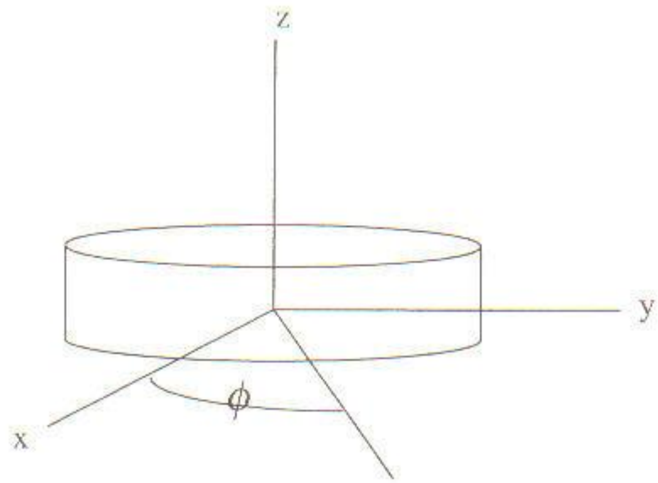
A-2. Given equation of state of a gas as $P(v-b) = RT$, show that the relationship between T and v during an adiabatic process is $T(v-b)^{R/c_v}$ =constant, where c_v is the constant volume specific heat.

B-1. An isolated spherical shell of radius a and negligible thickness has its surface maintained at a potential, which is a function of θ only, $\phi = \phi_0 \cos\theta$, where ϕ_0 is a constant. Determine the potential inside and outside the shell.
(Hint: The appropriate solution to Laplace's equation is
$\phi(r,\theta) = \sum_{l=1}^{\infty}(A_l r^l + \frac{B_l}{r^{l+1}})P_l(\cos\theta)$, where $P_0 = 1$, $P_1 = \cos\theta$, $P_2 = \frac{1}{2}(3\cos^2\theta - 1)$, etc)

B-2. Two coaxial conducting cylinders have a fixed electrostatic potential difference V. The radius r_1 of the inner cylinder can be adjusted relative to the fixed outer cylinder radius r_2 such that electric field at r_1 is minimal. Find r_1 in terms of V and r_2.

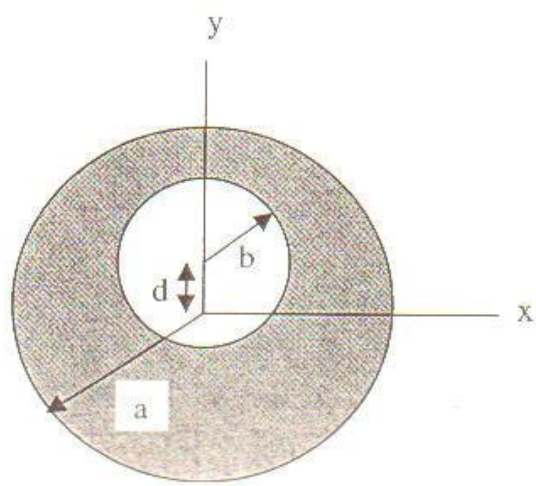
B-3. A dielectric disk of radius a and thickness from $z=-\frac{d}{2}$ to $z=\frac{d}{2}$ is uniformly polarized in the z direction ($\vec{P}=P\hat{z}$). It is rotating about the z axis with a constant angular velocity ω.

a) What is the surface current density, $\vec{K}$, on all 3 surfaces of the disk?
b) What is the total current through the plane $\phi=$constant?
c) What is the contribution to this current from each surface of the disk?
d) Find the magnetic field $\vec{B}$ on the z axis.



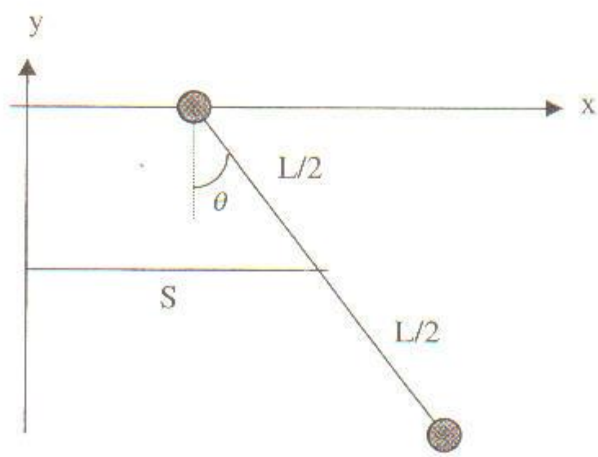
B-4. The diagram shows a cross section through an infinite cylinder of charge of radius a, with an infinite cylindrical cavity of radius b. The axis of the cylinder is the z axis and the axis of the cavity is at $y=d$ ($d<a-b$). The charge density, ρ, is constant.

Show that the electric field inside the cavity is given by $\frac{\rho d}{2\varepsilon_0}\hat{y}$.

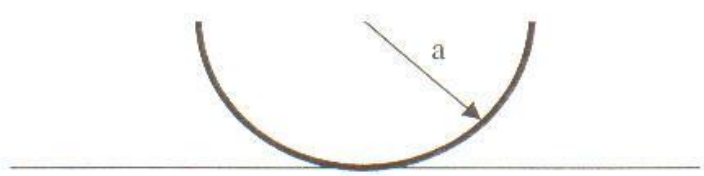


C-1. A simple pendulum, length l, mass m, is constrained to move in a plane. The point of support is attached to another mass m with can move on a horizontal rail in the same plane. Work with generalized coordinates S and θ as is shown in the figure.

a) Write down the Lagrangian L, simplify as much as possible.
b) Derive the canonical momenta p_S and p_θ.
c) Derive the equations of motion.
d) Solve the equations of motion for S.
e) What is the frequency of small oscillations of θ?



C-2. A thin, uniform cylindrical shell of mass M and semi-circular radius a, performs small oscillations with its curved surface in contact with a rough horizontal surface. What is the period of the small oscillations?



C-3. A uniform distribution of dust in the solar system adds to the gravitational attraction of the sun on a planet an additional force $\vec{F} = -mC\hat{r}$. Here m is the mass of the planet, C is a constant proportional to the gravitational constant and the density of the dust, and $\hat{r}$ is the radius unit vector from the sun to the planet (both considered as points). This additional force is very small compared to the direct sun-planet gravitational force.

a) Calculate the period for a circular orbit of radius r_0 of the planet in this combined field.

b) Calculate the period of radial oscillations for slight disturbances from this circular orbit.

Fall 02

Classical Physics

Name ____________________________

There are 9 problems in three sets (A, B, and C). You need to solve a total of 6 problems including at least 1 from each of the three sets. Circle your selection on this page.

Write your name on this page and on your solution pages. Staple your solutions together and use this page as the cover page.

A-1. YZ

A-2. YZ

A-3. YZ

A-4. YZ

B-1. OVm

B-2. OVm

B-3. OVm

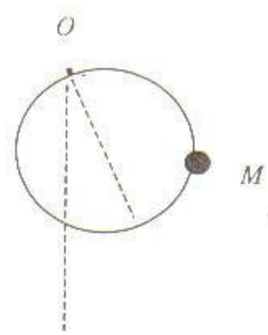
C-1. XW

C-2. XW

A-1. A square board of side 2a and mass m is lying on a table when struck on a corner by a bullet of mass m with velocity $\bar{V}$ parallel to one of the edges of the board. The bullet is lodged in the corner of the board. Find the velocity of the center of mass and the angular velocity of the board plus the bullet.

A-2. Assume that the Earth's orbit to be circular and that the sun's mass suddenly doubles, find the subsequent motion and the orbit of the Earth.

A-3. A thin hoop of radius R and mass M is allowed to swing with point O on the loop fixed. Attached to the hoop is a small mass M, which is constrained to move without friction along the hoop. Consider only small oscillations and show that the eigen-frequencies are $\omega_1 = \sqrt{\frac{2g}{R}}$ and $\omega_2 = \sqrt{\frac{g}{2R}}$

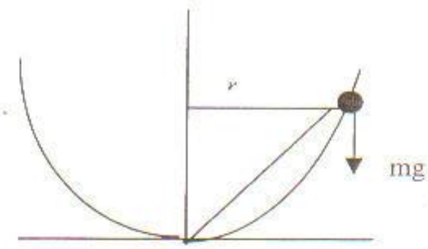


A-4. A particle of mass m slides without friction under the influence of gravity inside a paraboloid of revolution with a vertical axis. Using the coordinates r and θ as shown,

(a) find the angular momentum for which the motion is in a horizontal circle,

(b) show that if this circular motion is perturbed slightly, the particle will oscillate about the circular path, and

(c) show that r sweeps out equal areas in equal time.



B-1. Two very long concentric conducting cylindrical shells have radii a and b ($b > a$). The outside cylinder is maintained at potential V relative to the inside cylinder. The volume between the cylinders is filled with material with a **non-uniform** conductivity, $\sigma_c = k / r$, where r is radial distance measured from the common axis of the cylinders. A steady state current flows radially from the inside cylinder to the outside cylinder.

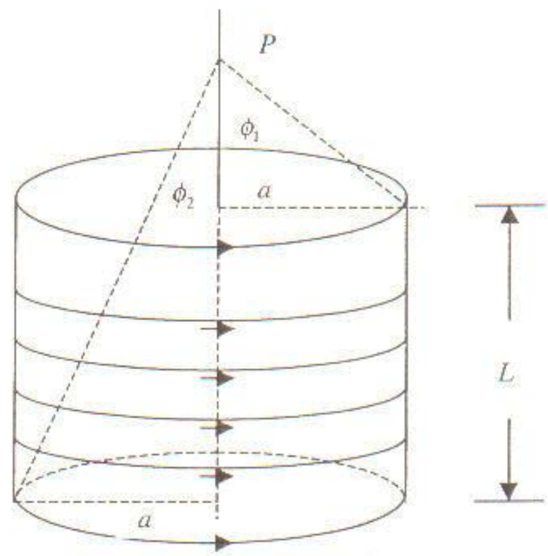
(a) Show that the electrical field in the region between the cylinders is constant and determine its strength.

(b) Calculate the resistance of the arrangement over a length L.

B-2. A solenoid of radius a, length L and with a total of N windings is oriented with its axis along the z-axis. The solenoid carries a current I through each of its windings in the direction indicated. We wish to find an expression for the magnetic field on the axis of the solenoid at point P.

(a) Using the Law of Biot-Savart, first determine the strength and direction of the magnetic field on the axis of a single loop of current.

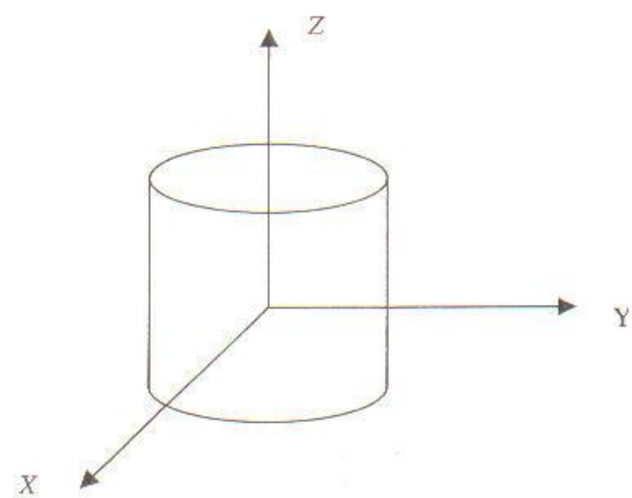
(b) Use the result of part (a), to derive an expression for the solenoid magnetic field at point P in terms of the two angles ϕ_1 and ϕ_2 as defined in the figure.



B-3. A very long ferroelectric cylinder of radius a is oriented with its axis along the z-axis. The cylinder has uniform permanent polarization vector $\vec{P}$ that is perpendicular to the cylinder axis (choose $\vec{P}$ along the x-axis).

(a) Show that **inside** the cylinder, the electric field is constant and is directed opposite to $\vec{P}$. Find the strength of this electric field.

(b) Derive an expression for the electric field **outside** the cylinder. Express your result in terms of the cylindrical coordinates r and ϕ, which are the radial distance measured from the cylinder axis and the azimuthal angle around the cylinder axis.

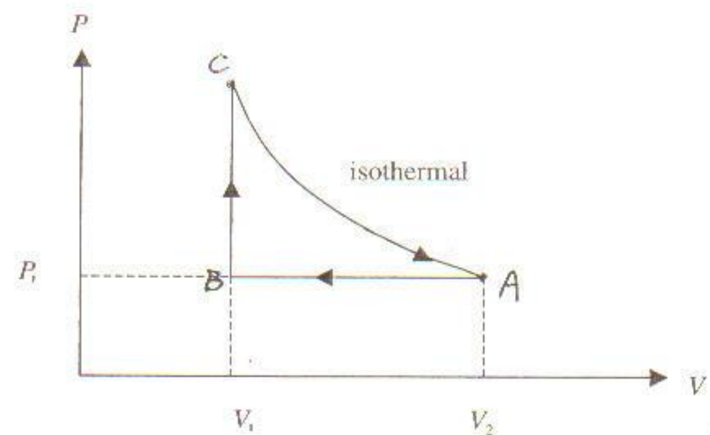


C-1. The equation of state of a certain gas is $(P+b)v = RT$ and its specific internal energy is given by $u = aT + bv + u_0$

(a) Find c_v

(b) Show that $c_p - c_v = R$

C-2. One kilomole of a monatomic ideal gas is carried around the reversible closed cycle shown in the figure below. Here $P_1 = 10\,\text{atm}$, $V_1 = 2\,\text{m}^3$, and $V_2 = 4\ \text{m}^3$. Calculate the change in entropy for each leg of the cycle and hence show that the entropy change for the complete cycle is zero.



Fall 03

Classical Physics

Name ____________________

There are 9 problems in three sets (A, B, and C). You need to solve a total of 6 problems including at least 1 from each of the three sets. Circle your selection on this page.

Write your name on this page and on your solution pages. Staple your solutions together and use this page as the cover page.

yz A-1. 6

yz A-2. 10

yz A-3. 5

ovm B-1. 3

ovm B-2. 8

ovm B-3. 5

ovm B-4. 2

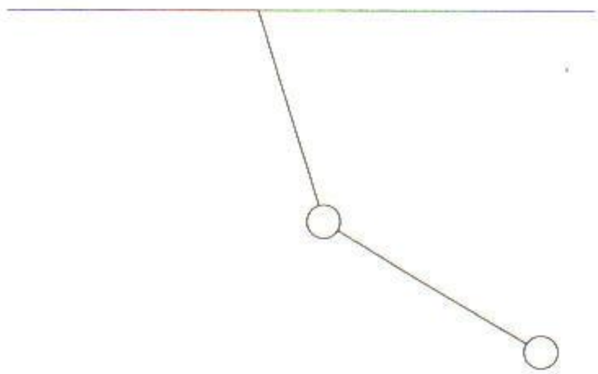
xw C-1. 6

xw C-2. 6

45

/90

A-1. Two identical simple pendulums (length l and mass m) are coupled together as shown in the figure. Consider small oscillations, calculate the characteristic frequencies and derive the normal modes.

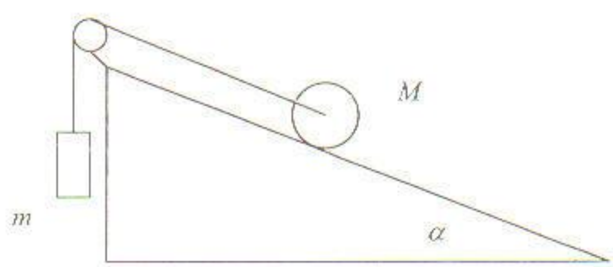


A-2. A thin rod of mass M and length l rests on a smooth table. An impulse I is suddenly applied perpendicularly to one end of the rod.

(a) Find the translational kinetic energy and rotational kinetic energy of the rod.

(b) After the rod makes one complete revolution, how far has the rod moved?

A-3. A solid disk of mass M and radius R is rolling without slipping on a slope with an inclination angle α. The disk has a short, weightless axle of negligible radius. A string is fixed to the axle, goes over a frictionless pulley, and is attached to an object of mass m hanging vertically down. Derive the acceleration of the object m from the Lagrange equations of motion.



B-1. Under a Lorentz transformation from one frame of reference to another, the electric and magnetic field components parallel to the transformation velocity do not change, but the perpendicular components transform according to the equations:

$$\vec{E}_\perp' = \gamma(\vec{E}_\perp + \vec{v} \times \vec{B}), \qquad \vec{B}_\perp' = \gamma(\vec{B}_\perp - \vec{v}/c^2 \times \vec{E})$$

where $\vec{v}$ is the velocity of the primed system relative to the unprimed system and

$\gamma = (1 - v^2/c^2)^{-\frac{1}{2}}$ is the usual Lorentz factor.

(a) Show that the combination $E^2 - c^2 B^2$ is a Lorentz invariant.

(b) Show that $\vec{E}'$ is perpendicular to $\vec{B}'$ if and only if $\vec{E}$ is perpendicular to $\vec{B}$.

(c) Show that there exists a frame of reference with transformation velocity given by $\vec{v}/(1 + v^2/c^2) = (\vec{E} \times \vec{B})/(B^2 + E^2/c^2)$ in which $\vec{E}'$ and $\vec{B}'$ are parallel.

B-2. According to classical electromagnetic theory, the power radiated by an accelerated charged particle moving at non-relativistic velocities is given by $P = \mu_0 q^2 a^2/(6\pi c)$ where q is the charge and a is the acceleration.

(a) In the Bohr model of the hydrogen atom, the electron moves in a circular orbit around the proton. Determine the total energy of the electron in this model as a function of r, which is the radius of its orbit. Note that the total energy includes both the kinetic energy and the electrical potential energy.

(b) Derive an expression for the classically radiated power as a function of r.

(c) Using energy conservation and your results for part (a) and (b), derive an expression for the time that would be required classically for the electron to spiral into the nucleus starting from $r = r_0$ at time $t = 0$, taking $r_0 = 0.529$ angstroms. (data: $e = 1.6 \times 10^{-19}$ C, $m_e = 9.11 \times 10^{-31}$ kg, $\frac{\mu_0}{4\pi} = 10^{-7}$ N/A^2, $c = 3.0 \times 10^8$ m/s, $\varepsilon_0 \mu_0 = 1/c^2$).

B-3. Two long conducting cylinders of equal length L and radii a and b $(b > a)$ are oriented vertically in a large container of oil so that they are concentric, i.e., have the same axis. The inner cylinder $(r = a)$ is maintained at potential V, while the outer cylinder is grounded $(V = 0)$. It is found that the oil, which has mass density ρ and dielectric constant κ, rises up to a height h, $h < L$, between the two cylinders.

(a) Determine the capacitance of the two cylinder system assuming that L and h are both large compared with a or b. Note that your result should depend on all four distances.

(b) Derive an expression for the equilibrium value of h in terms of the quantities given. Hint: the electrical force on the oil is given by the relation $\vec{F} = \vec{\nabla} U$ where U is the energy stored by the capacitor.

B-4. A conducting sphere of radius a is surrounded concentrically by a thin, non-conducting shell of radius b, $b > a$. The non-conducting shell carries a non-uniform surface charge density given by $\sigma(\theta) = \sigma_0 \cos\theta$, where σ_0 is a constant and θ is the usual polar angle in spherical coordinates. The potential on the inner conducting sphere is V_0.

(a) Write down two boundary conditions that must be satisfied by the potential on the surface at $r = b$.

(b) Assuming that the potential is finite everywhere, derive expressions for the potential at arbitrary points within the two regions of $a \le r \le b$ and $r \ge b$, where r is the radial coordinate. Hint: the general solution to Laplace's equation in spherical coordinates with azimuthal symmetry is given by: $V(r,\theta) = \sum_l (A_l r^l + B_l / r^{l+1}) P_l(\cos\theta)$, where $P_l(\cos\theta)$ is the Legendre polynomial of order l.

(c) Determine the surface charge density induced on the surface at $r = a$ and then determine the total charge of the whole system.

C-1. One mole of an ideal monatomic gas is expanded adiabatically and reversibly from temperature T_0 and volume V_0 to a volume twice as large. It is then compressed reversibly and isothermally back to its original volume.

(a) Calculate the change in internal energy of the gas in terms of T_0, V_0 and the gas constant R.

(b) Calculate the change in entropy of the gas.

C-2. One mole of H_2O is cooled from $25°C$ to $0°C$ and frozen. All the heat is delivered to a second mole of H_2O initially at $25°C$, heating it to $100°C$. The refrigerating machine is operating at maximum theoretical efficiency (no entropy created).

(a) How much work must be done by the refrigerator?

(b) What fraction of the second mole is converted to steam at $100°C$? (the heat capacity of water is 18 cal/mole/degree; the heat of vaporization at $100°C$ is 9730 cal/mole; the heat of fusion at $0°C$ is 1438 cal/mole.)

Fall 04

Classical Physics

Name ____________________________

There are 9 problems in three sets (A, B, and C). You need to solve a total of 6 problems including at least 2 from A and B set and at least 1 from the C set. Circle your selection on this page.

Write your name on this page and on your solution pages. Staple your solutions together and use this page as the cover page.

A-1.

A-2.

A-3.

B-1.

B-2.

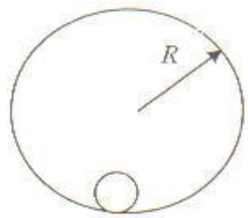
B-3.

B-4.

C-1.

C-2.

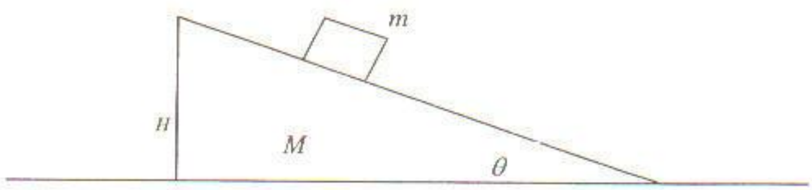
A-1. A **sphere** of radius r is constrained to roll without slipping on the inner surface of a hollow, fixed **cylinder** of inner radius R . Derive the Lagrange's equations of motion and find the frequency of small oscillations near the bottom of the cylinder.



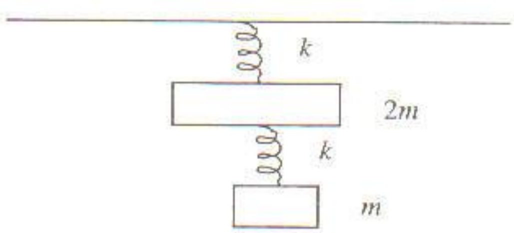
A-2. A particle of mass m slides down a frictionless inclined plane of a triangular wedge of mass M . The bottom of the wedge can slide without friction on a horizontal table.

a) Find the acceleration of the wedge.

b) If the particle m starts from rest on the top of the inclined plane, find the velocity of the wedge after the particle has reached the bottom of the wedge.



A-3. Two masses m and $2m$ are suspended by two springs of the same spring constant k as shown in the figure. Calculate the amplitude ratio of the two masses and the oscillation frequencies of the normal modes.



B-1. The space between two concentric, conducting spherical shells of radii R_1 and R_2 ($R_1 < R_2$) is filled with a conducting material with conductivity σ_c. The inner shell is maintained at constant electric potential V_o, while the outer shell is grounded (V=0). An electric current flows radially from the inside shell to the outside shell.

a) Using Gauss' law, determine the electrical field at an arbitrary point between the two shells. Express your result in terms of V_o.

b) Calculate the electrical resistance of the system.

B-2. A solid sphere of radius R has a uniform volume charge density ρ. The sphere rotates with angular velocity ω about an axis that passes through its center.

a) Determine the volume current density at an arbitrary point within the sphere. Express your result in spherical coordinates and give both the magnitude and the direction of the current density.

b) Calculate the magnetic moment of the sphere.

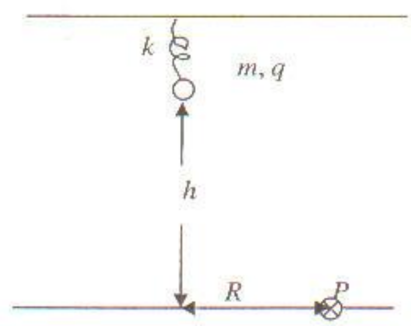
B-3. An inductor with inductance L and a resistor with resistance R are connected together. At time t=0, the current through the resistor has the initial value I_o.

a) Derive a differential equation that must be satisfied by the current as a function of time. Hint: recall that the total electrical potential difference around any closed loop in a circuit must vanish.

b) Find the time required for the current through the resistor to decrease to one third of its initial value.

c) Prove that energy is conserved in this system; i.e., prove that at any time t (t>0), the total electrical energy dissipated by the resistor is exactly equal to the difference between the initial energy stored by the inductor and the energy stored at time t.

B-4. A particle of mass m and charge q oscillates at the end of a spring with spring constant k. The spring is located at height h above the ground, and the particle's oscillations have amplitude z_o, where $z_o \ll h$.



a) In the radiation field approximation, the Poynting vector associated with the radiation from an oscillating electric dipole is given by the expression:

$$\vec{S} = \mu_0 (d^2 p / dt^2)^2 \sin^2\theta / (16\pi^2 r^2)\hat{r}$$

where $\mu_0 = 4\pi \times 10^{-7}$ N/A^2, $\vec{p}$ is the time dependent dipole moment of the charged particle, $\vec{r}$ is the vector from the dipole to the point of observation, $\hat{r} = \vec{r}/r$, and θ is the angle between $\vec{r}$ and $\vec{p}$. Using this expression, find the time averaged radiation intensity from the oscillating charge at point P shown in the figure. Note that only the vertical component of the Poynting vector contributes to the intensity at any point on the ground.

b) Assuming that the ground is infinite in extent, use your result for part (b) to find the total power radiated onto the ground by the oscillating charge

C-1. The latent heat of fusion for ice at a pressure of 1 atm (1.013×10^5 Pa) and $0^\circ C$ is 3.348×10^5 J kg^{-1}. The density of ice under these conditions is 917 kg m^{-3} and the density of water is 999.8 kg m^{-3}. If 1 kilomole of ice is melted, what will be:

a) the amount of work done?

b) the change in internal energy?

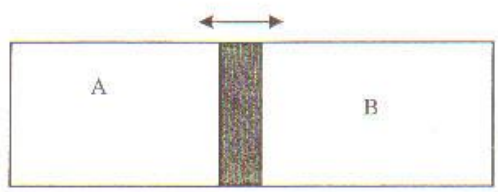
c) the change in entropy?

C-2. A gas-tight, frictionless piston of small thermal conductivity slides in a thermally insulated cylinder. Both compartments A and B contain one mole of an ideal monatomic gas. Assume that initially the temperature of the gas is T_0 in A, and $3T_0$ in B. The system is to be considered as in mechanical equilibrium at all times, and eventually it will be in thermal equilibrium as well.

a) What will be the ratio of the volume of A to that of B initially, and at $t=\infty$?

b) What will be the total change in the entropy of the entire system when it reaches thermal equilibrium?

c) How much useful work could have been done by the system if the transfer of heat from one compartment to the other had been accomplished reversibly?



Spring 05

Classical Physics

Name ____________________________

There are 9 problems in three sets (A, B, and C). You need to solve a total of 6 problems including at least 2 from the A set, at least 2 from the B set and at least 1 from the C set. Circle your selection on this page.

Write your name on this page and on your solution pages. Staple your solutions together and use this page as the cover page.

A-1. RB

A-2. XW

A-3. RN

A-4. XW

B-1. RN

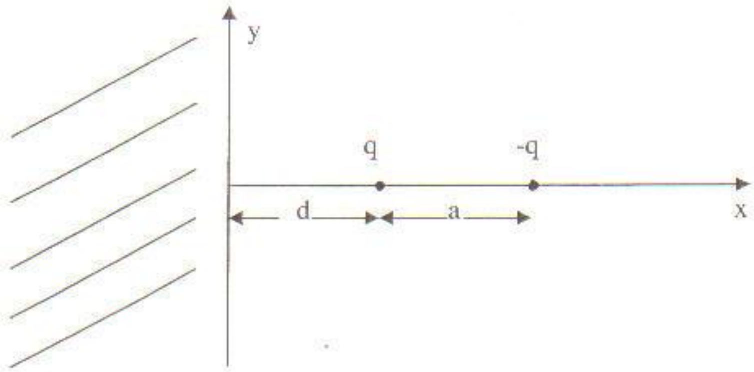
B-2. YZ

B-3. YZ

C-1. ~~XW~~ RN

C-2. XW

A-1. A dipole is located as shown in front of a semi-infinite, grounded conductor, which occupies the space to the left of the y-z plane. Determine the electric potential <u>everywhere</u>. What is the electric field in the vacuum region at the conductor's surface? What is the induced surface charge density at the origin?

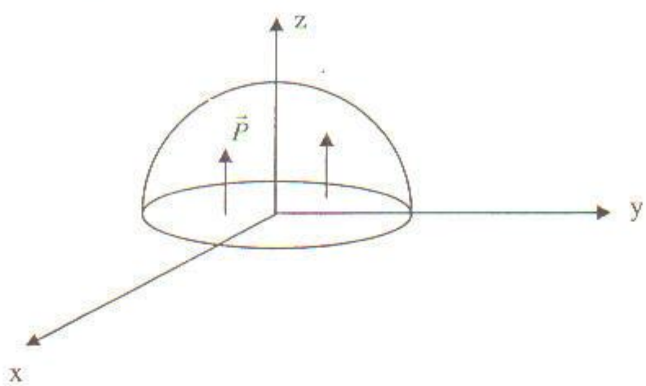


A-2. A hemisphere of dielectric has its flat surface in the x-y plane. It is uniformly polarized in the z-direction, $\vec{P} = P\hat{z}$.

a) Find the bound volume charge density and the bound surface charge density on the flat as well as on the hemispherical surfaces.

b) Find by integration the net bound charge on the hemisphere. Explain why your answer is the expected one.

c) Find by integration the dipole moment ($\vec{p}_{tot}$) of the bound charge relative to an origin at the center of the flat surface. Does this answer agree with the value obtained directly from the definition of $\vec{p}_{tot}$? ($\vec{p}_{tot} = \int \vec{P} d^3 r$).



A-3. Find the electric field everywhere due to a uniform spherical charge distribution. The charge density is $\rho\ (\rho \geq 0)$ and the radius of the sphere is R. A very small negative point charge, $-e$, and mass m is placed on the surface of this charge distribution. Determine the motion of this small negative point charge assuming that it is free to move through the charged sphere without affecting the uniform charge distribution.

A-4. Within the region of space defined by the restriction $0 \leq x \leq a$, the electric and magnetic field are given by the expressions:

$$\vec{E} = E_0 \sin(\pi x / a)\cos(kz - \omega t)\hat{y}$$

$$\vec{B} = (E_0 / \omega)[-k\sin(\pi x / a)\cos(kz - \omega t)\hat{x} + (\pi / a)\cos(\pi x / a)\sin(kz - \omega t)\hat{z}]$$

Where a, E_0, ω and k are constants. At $x = 0$ and $x = a$, there are two infinite conducting surfaces perpendicular to the x-axis. Assume that $\vec{E}$ and $\vec{B}$ are both zero on the other side of the conducting surfaces (i.e., for $x \leq 0$ and $x \geq a$).

a) Find the relationship between ω, k, a that is required to make $\vec{E}$ and $\vec{B}$ satisfy Maxwell's equations in the absence of free charges and currents in the volume defined by $0 \leq x \leq a$, and then show explicitly that Maxwell's equations are satisfied by these fields.

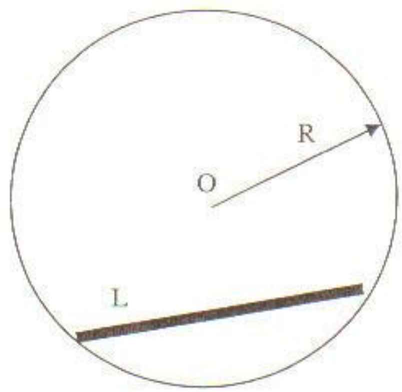
b) By making use of the appropriate boundary conditions on the conducting surface at $x = 0$, determine the surface charge and current densities for arbitrary points on that conducting surface. Note that the surface current density is a vector. The direction of this vector must be indicated.

B-1. Show that the equation of motion for a rocket projected vertically upward in a uniform gravitational field, g, neglecting atmospheric resistance, is

$$m\frac{dv}{dt} = -v'\frac{dm}{dt} - mg\ .$$

Where m is the mass of the rocket, v is the velocity of the rocket, v' is the velocity of the ejected gases relative to the rocket. Integrate this equation to obtain v as a function of m, assuming a constant rate of loss of mass.

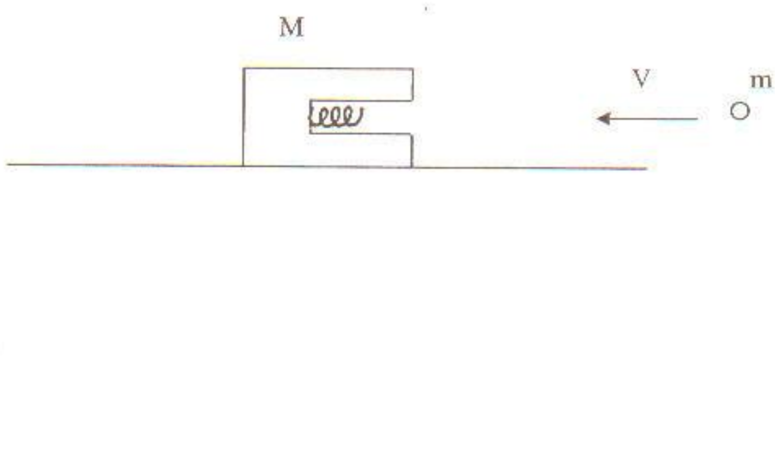
B-2. Consider the motion of a uniform rod of length L and mass M whose ends are constrained to move on a vertical circle of radius R. Assume that there is no friction between the rod and the circular track. Find the frequency of small oscillations about the equilibrium position.



B-3. A projectile of mass m is shot (with a velocity V) at the target of mass M. The target has a hole containing a spring with a force constant k. The target is initially at rest but can slide without friction on a horizontal surface.

a) Find the maximum compression of the spring.

b) Find the maximum speed mass M acquires.



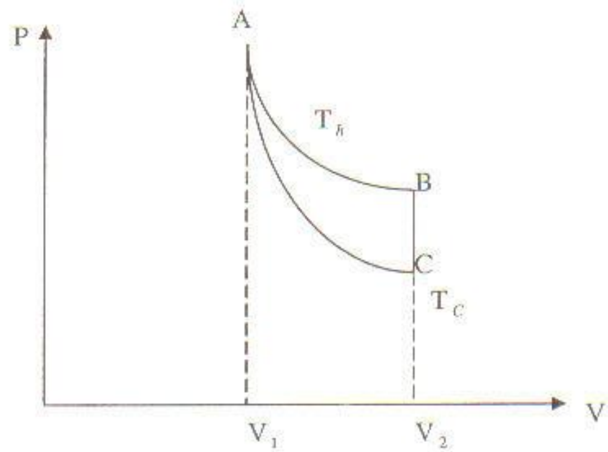
C-1. Compute the efficiency of a heat engine with the following cycles, assuming that the gas is ideal:

(1) Isothermal expansion from state A to state B at temperature T_h;

(2) Cooling at constant volume from state B at T_h to state C at T_c;

(3) Adiabatic compression from state C back to state A.

Compare this efficiency to that of a heat engine operating on a Carnot cycle between T_h and T_C.



C-2. The equation of state of a certain gas is given by $(P+b)V = RT$ and its internal energy is $U = aT + bV + U_0$, where a, b, and U_0 are all constants.

a) Find the constant volume heat capacity C_V.

b) Calculate the constant pressure heat capacity C_P and show that $C_P - C_V = R$.