

Rules for Computing Probabilities

The **Union** of two events is the event that **A or B or both** occur on a single performance of an experiment. We use the notation: $A \cup B$.

The **Intersection** of two events **A and B** is the event that occurs if **both A and B** occur on a single performance of the experiment.

We use the notation: $A \cap B$

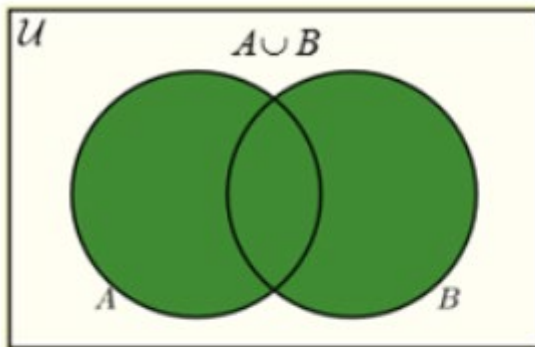
Example: A die is rolled once.

What is the probability of rolling **either a number larger than 4 or an even number**?

Event A - a number larger than 4 (sample points 5, 6)

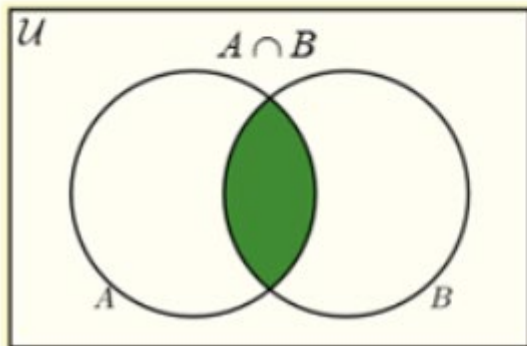
Event B – a number is an even number (sample points 2, 4, 6)

Venn Diagrams



A union B

Elements that belong to either A or B or both.



A intersect B

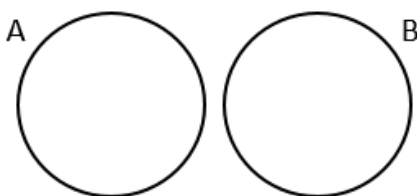
Elements that belong to both A and B .

Probability of cards

In a pack or deck of 52 playing cards, they are divided into 4 suits of 13

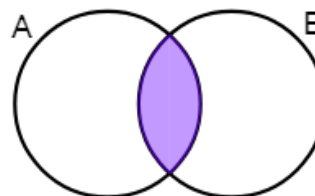


Mutually Exclusive Events



$$P(A \text{ or } B) = P(A) + P(B)$$

Non-Mutually Exclusive Events



$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

General Addition Rule: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

1. What is the probability that a card chosen at random from a standard deck of cards will be **either a King or a Heart**?
2. What is the probability that a card chosen at random from a standard deck is **either a Queen or a Red card**?
3. What is the probability that a randomly selected card is **either a face card (J, Q, K) or a spade**?
4. A fair six-sided die is rolled once. What is the probability of rolling **either a number larger than 2 or an even number**?
5. In a class of 100 students, **60 drink coffee, 45 drink tea, and 25 drink both**. What is the probability that a randomly selected student drinks **coffee or tea**?

For mutually exclusive events: $P(A \text{ or } B) = P(A) + P(B)$

6. What is the probability that a card chosen at random from a standard deck of cards will be **either a King or an Ace**?
7. A fair six-sided die is rolled once. What is the probability of rolling **either a 2 or a 5**?

General Multiplication Rule: $P(A \cdot B) = P(A) \cdot P(B|A)$

8. A card is chosen at random from a standard deck of 52 cards. What is the probability that the card is **a face card (J, Q, K) AND a heart**?
9. If 3 cards are selected **without replacement**, find the probability that all three are aces.
10. A die is rolled once. What is the probability of rolling **a number larger than 4 and an even number**?
11. A die is rolled. **Given that the result is greater than 3, what is the probability that it is even**?

Multiplication Rule for Independent events: $P(A \cdot B) = P(A) \cdot P(B)$

1. If 3 cards are selected **with replacement**, find the probability that all three are aces.
2. What is the probability of **flipping 3 coins** and observing **HHH**?
3. What is the probability of **rolling 3 dice** and observing the **number 6 on each die**?

Condition for Independence: $P(A) = P(A|B)$ and $P(B) = P(B|A)$

Check independence:

Example 1: Select a Card: A is a King card, and B is a Red card

Example 2: Select a Card: A is a Diamond card, and B is a Red card

Example 3: Roll a die. Event A: even N, event B: N larger than 3

A Complement Rule: $P(A) = 1 - P(A^c)$

$P(\text{at least 1}) = 1 - P(\text{none})$

4. What is the probability of **rolling 3 dice** and observing the **number 6 on at least 1 die**?
5. If 3 cards are selected **without replacement**, find the probability of choosing **at least 1 ace**.
6. A quiz consists of 4 multiple-choice questions. What is the probability of guessing **at least 1 correctly**?
7. A quiz consists of 4 multiple-choice questions. What is the probability of guessing **exactly 3 questions correctly**?
8. I set **three mechanical alarm clocks** before going to bed. Each alarm clock has a **90% probability of working properly**. I will wake up on time if **at least one** of the three alarm clocks works. **What is the probability that I will wake up on time the next morning**?

9. A system consists of two **independent components connected in series**. Each component has a **90% probability** of working properly. The system **works only if both components work**.

What is the probability that the system will work?

10. A system consists of **two independent components connected in parallel**. Each component has a **90% probability of working properly**. The system works if **at least one of the two components works**.

What is the probability that the system will work?

11. If 4 cards are selected **without replacement**, find the probability **that at least one of the cards selected is a diamond**.

12. A local country club has a membership of 600 and operates facilities that include an 18- hole championship golf course and 12 tennis courts.

A survey of the membership indicates **that 68% regularly use the golf course, 49% regularly use the tennis courts, and 5% use neither facility regularly**.

Find the probability that a randomly selected member **uses the golf or tennis facilities regularly**.