

Sampling Distribution for a Sample Mean ($\bar{x}$)

1. The mean of a Sampling Distribution is denoted by $\mu_{\bar{x}} = \mu$.
2. The SD of a Sampling Distribution is denoted by $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$.
3. When the population is normal or when $n \geq 30$, $\bar{X}$ follows a normal distribution.

1. The heights of women are normally distributed with a mean of 63.6 inches and a standard deviation of 2.5 inches. If 16 women are selected at random, find the mean and standard deviation of the population of sample means.

$$\mu_{\bar{x}} = \mu = 63.6 \qquad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2.5}{\sqrt{16}} = 0.625$$

2. The weight of corn chips dispensed into a 10-ounce bag by the dispensing machine has been identified as possessing a normal distribution with a mean of 10.5 ounces and a standard deviation of .2 ounces. Suppose 100 bags of chips were randomly selected from this dispensing machine. Find the probability that the sample mean weight of these 100 bags is less than 10.45 ounces.

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{10.45 - 10.5}{\frac{0.2}{\sqrt{100}}} = -2.50 \qquad P = 0.0062$$

3. As part of a quality control program at a factory, random samples of 36 6-ounce cans of juice are taken, and the contents are carefully measured. When the manufacturing process is working properly, the cans contain an average of 6.04 oz of juice. The standard deviation is 0.02 oz. When the process is working properly, what is the probability that a sample of 36 cans of juice would contain an average of 6.03 oz or less?

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{6.03 - 6.04}{\frac{0.02}{\sqrt{36}}} = -3.00 \qquad P = 0.0013$$

Sampling Distribution of Sample Proportion ($\hat{p}$)

1. The shape of the sampling distribution of $\hat{p}$ is normal if $np(1 - p) \geq 10$.

2. The mean of the sampling distribution of $\hat{p}$: $\mu_{\hat{p}} = p$

3. The standard deviation of the sampling distribution of $\hat{p}$: $\sigma_{\hat{p}} = \sqrt{\frac{p(1 - p)}{n}}$

1. Historically, 40% of college students reported using marijuana in the past six months. In a recent random sample of 200 college students, 64 (32%) reported such use. Does this provide convincing evidence that the proportion has decreased?

2. A company reports that 43% of its customers are women and 57% are men. To attract more female customers, the company launches an aggressive marketing campaign. After the campaign, a random sample of 200 customers reveals that 48% are women. Does this result provide convincing evidence that the campaign was successful?

3. A company has historically produced 3% defective items. To reduce this percentage, the company implemented a quality improvement program. After the program was introduced, a random sample of 1,000 items was inspected, and 18 were found to be defective. Is there sufficient evidence to conclude that the program was effective?

Normal and Sampling Distribution Examples

1. Problem: Tire Lifetimes

A tire manufacturer states that a certain type of tire has a **mean lifetime of 60,000 miles**. Suppose **tire lifetimes are normally distributed** with SD $\sigma = 3,500$ miles.

Part (a)

Find the probability that if you **buy one such tire**, it will last only 57,000 miles or fewer.

If you had this experience, is it strong evidence that the tire is not as good as claimed?

Solution

Let X be the lifetime of a single tire. $X \sim N(60,000, 3,500)$.

$$Z = \frac{X - \mu}{\sigma} = \frac{57,000 - 60,000}{3,500} = -0.86$$

$$P(X \leq 57,000) = P(Z \leq -0.86) \approx 0.1957$$

Interpretation: This is not an unusually rare outcome, so one such tire does not provide particularly strong evidence against the claim.

Part (b)

A consumer group **buys 16 such tires** and tests them. Find the probability that **the average lifetime of the 16 tires** will be 57,000 miles or less. Does the sample provide strong statistical evidence that the tire is not as good as claimed?

Solution

Let $\bar{X}$ be the sample mean lifetime for $n = 16$ tires. **Because tire lifetimes are normally distributed, the sample mean is also normally distributed.**

$$\mu_{\bar{X}} = 60,000 \text{ miles} \quad \sigma_{\bar{X}} = \sigma / \sqrt{n} = 3,500 / \sqrt{16} \approx 875 \text{ miles.}$$

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{57,000 - 60,000}{\frac{3,500}{\sqrt{16}}} = -3.43$$

$$P(\bar{X} \leq 57,000) = P(Z \leq -3.43) \approx 0.0003.$$

Interpretation: This is an extremely unlikely event. Therefore, the sample provides very strong statistical evidence that the true mean tire lifetime is lower than the manufacturer's claimed 60,000 miles.

2. Problem: Battery Life

An automobile battery manufacturer claims that its battery has a **mean life of 50 months** with a **standard deviation of 6 months**. Suppose the distribution of battery lives is approximately normal.

- a. On the assumption that the manufacturer's claims are true, find the probability that a randomly selected battery of this type **will last less than 47 months**. If you had this experience, is it strong evidence that the battery is not as good as claimed?
- b. On the same assumption, find the probability that the mean of a random sample of **36 batteries** will be **less than 47 months**. Does the sample provide strong statistical evidence that the battery is not as good as claimed?

3. Problem: Proportion of passenger vehicles in operation

An economist wants to investigate whether people are keeping their cars longer than they did in the past. Five years ago, **38%** of all passenger vehicles in operation were at least ten years old. In a recent random sample of **325** passenger vehicles currently in operation, **132** were at least ten years old.

- a. Calculate the sample proportion of vehicles that are at least ten years old.
- b. Assuming that the population proportion is still **0.38**, find the probability of obtaining a sample proportion at least as large as the one observed in a random sample of 325 vehicles.
- c. Does the sample provide convincing evidence that the proportion of passenger vehicles in operation that are at least ten years old has increased?

4. Problem: Proportion of orders shipped within 12 hours.

An online retailer claims that **90%** of all orders are shipped within 12 hours of being received. To evaluate this claim, a consumer group placed **121** orders. Of these orders, **102** were shipped within 12 hours.

- a) Calculate the sample proportion of orders shipped within 12 hours.
- b) Assuming the retailer's claim is true, find the probability of obtaining a sample proportion at least as low as the one observed in a sample of 121 orders.
- c) Does the sample provide convincing evidence that less than 90% of all orders are shipped within 12 hours?