Intermediate Microeconomics — Week 2

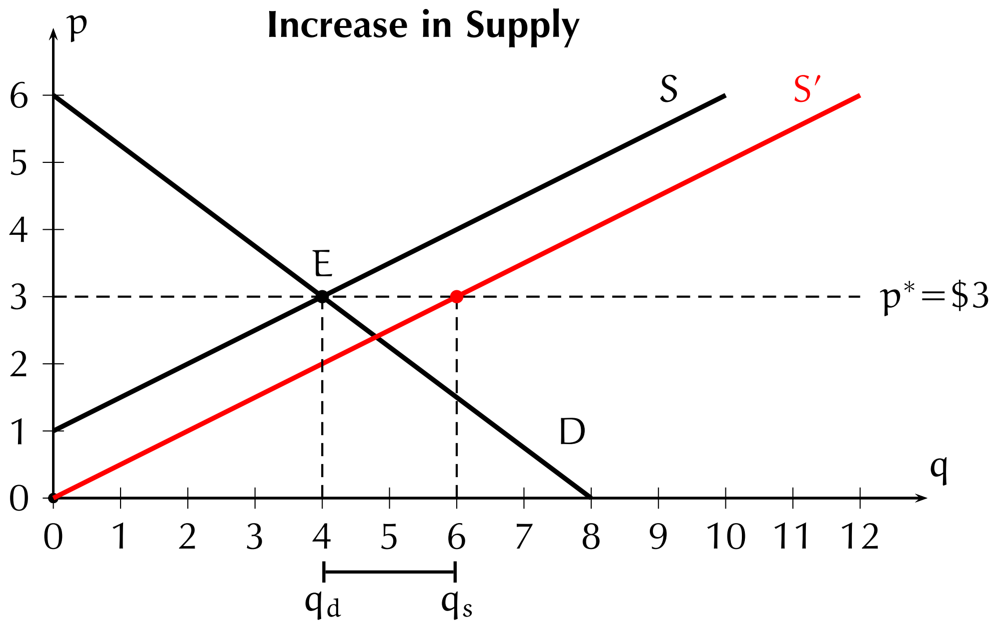
Professor Boyd **Aug. 30 & Sept. 1, 2022**

2.5 Disturbing the Equilibrium

Although we expect the market to stay at the equilibrium if it is not disturbed, we also expect things to change if supply or demand changes. We will take a quick look at the effects of changes in supply or demand.

2.5.1 Increase in Supply

Suppose that supply increases, shifting supply rightward from the old supply curve S to the new supply curve S' in the diagram below. The old equilibrium, marked at E, was $p^* = 3$, $q^* = 4$.

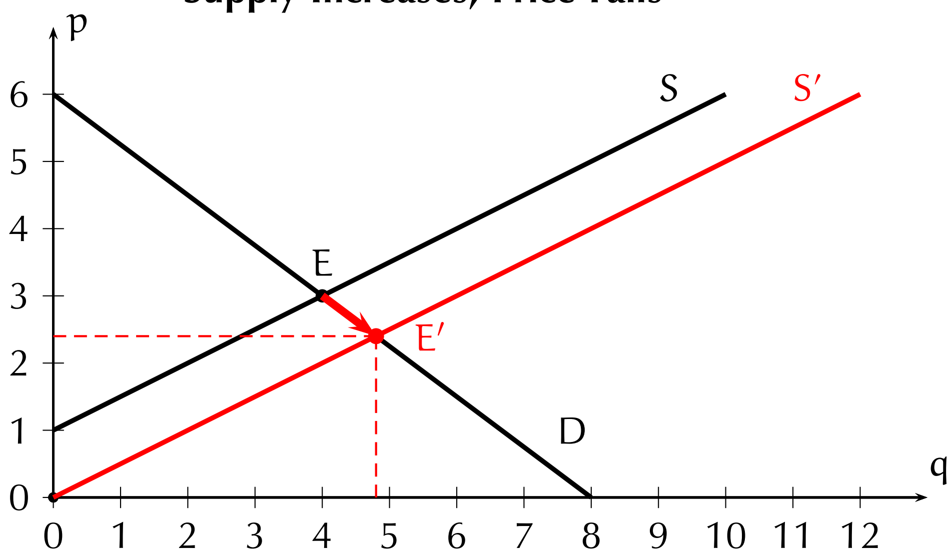


At the old price of \$3, we now have an excess supply of 2 because the quantity supplied q_s is now 6 and the quantity demanded q_d remains at 4. This means the price must fall.

2.5.2 Increase in Supply Decreases Price

The increase in supply, shifting supply to the new supply curve S' in the diagram below, caused an excess supply, forcing the market out of equilibrium. As a result, many willing sellers were unable to sell their product at the current market price. Many of them were willing to settle for less. This starts forcing the market price downward, a process that continues until we reach a new equilibrium at E′.

Supply Increases, Price Falls



At the old equilibrium, marked at E, price and quantity were $p^* = \$3$, $q^* = 4$. The new equilibrium, at E′, has price $p' = \$2.40$ and quantity $q' = 4.8$. The fall in market price is caused by a movement along the demand curve. This is illustrated by the red arrow on the diagram.

2.5.3 Changes in Supply

An increase in supply causes the equilibrium to move along the demand curve, decreasing price, and increasing quantity.

A decrease in supply causes the equilibrium to move along the demand curve, increasing price, and decreasing quantity.

In both cases, we move along the unchanged demand curve, which means that price and quantity must change in opposite directions. One goes up and the other goes down.

2.5.4 Disruption and Re-coordination

The activities of buying and selling lead to coordinated behavior in the market. At the old equilibrium, quantity supplied equaled quantity demanded. That meant that every would-be buyer at the market price was able to find a seller, and that every would-be seller was able to find a buyer. The plans of both buyers and sellers are in harmony at equilibrium.

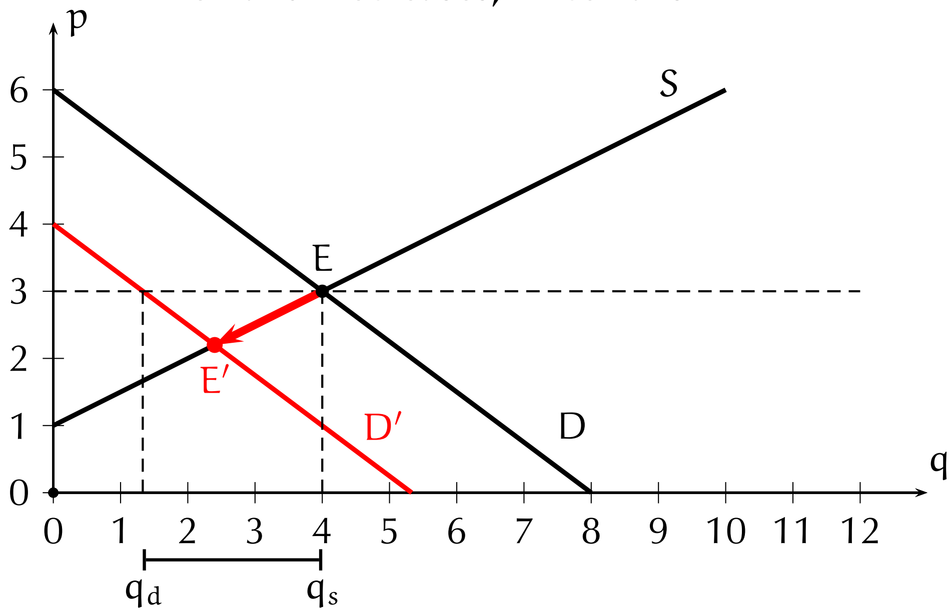
We disturbed this market by increasing supply. For a brief time, the buyers and sellers were not coordinating their actions. The increase in supply led to excess supply, too many sellers. However, competition, driven by the attempt of both buyers and sellers to gain, drove the price down, and established a new balance, a new equilibrium that re-coordinated the actions of buyers and sellers.

This happens throughout the economy, with buyers and sellers at all levels continuously re-coordinating in response to changing conditions.

2.5.5 Decrease in Demand

Suppose that demand decreases, shifting demand leftward from the old demand curve D to the new demand curve D′ in the diagram below. The old equilibrium, marked at E, was $p^* = \$3$, $q^* = 4$.

Demand Decreases, Price Falls



At the old price, we now have an excess supply of $2\frac{2}{3}$ because the quantity supplied remains at $q_s = 6$ and the quantity demanded q_d has been reduced to $1\frac{1}{3}$. This means the price must fall to the new equilibrium, which has price $p' = \$2.20$ and quantity $q' = 2.4$. This movement along the supply curve is illustrated by the red arrow on the diagram.

2.5.6 Changes in Demand

An increase in demand causes the equilibrium to move along the supply curve, increasing price, and increasing quantity.

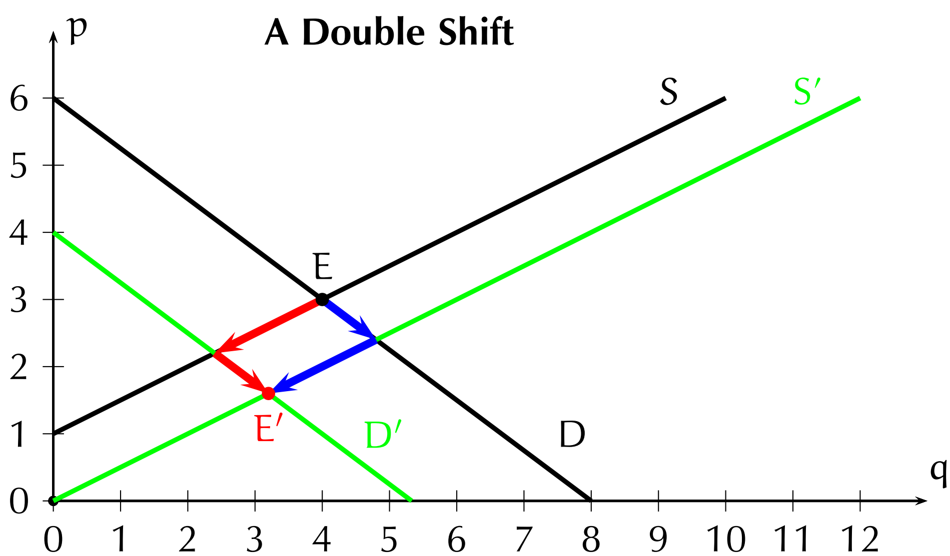
A decrease in demand causes the equilibrium to move along the supply curve, decreasing price, and decreasing quantity.

In both cases, we move along the unchanged supply curve, which means that price and quantity must change in the same way. Either both go up or both go down.

2.5.7 A Double Shift

We can also analyze markets where both supply and demand shift. In the diagram below demand has decreased and supply increased. If we shift demand and then supply, prices follow the red path. If we shift supply, then demand, prices follow the blue path. No matter which changes occur, in the end we end up at the equilibrium $E' = (3.2,, 1.6)$.

Compared to the original equilibrium of $E = (4, 3)$, the price and quantity have both fallen.



2.5.8 Analyzing the Double Shift

If we trace out the logic of the problem, the decrease in demand lowers both price and quantity; the increase in supply decreases price, but increases quantity. Putting them together, it's clear that price falls (it fell with each shift), but it's not clear what happens to quantity, with either rises before falling, or falls before rising. In this case, we can compute exactly where it ends up, with quantity falling.

Had demand shifted less, or supply more, the quantity might have risen, or even ended up being the same.

This is typical of double shifts. Without quantitative information, we can only be certain about what happens to price or to quantity, but not both.

2.6 Elasticity

Elasticity is a way of measuring how quantity demanded or supplied changes in response to changes in price and other economic variables (e.g., income, prices of other goods, taxes, minimum wage, etc.).

Although there are many kinds of elasticity, we will focus on four of them.

1. Price Elasticity of Demand
2. Income Elasticity of Demand
3. Cross-Price Elasticity of Demand
4. Price Elasticity of Supply

2.6.1 Measuring the Response of Demand to Price

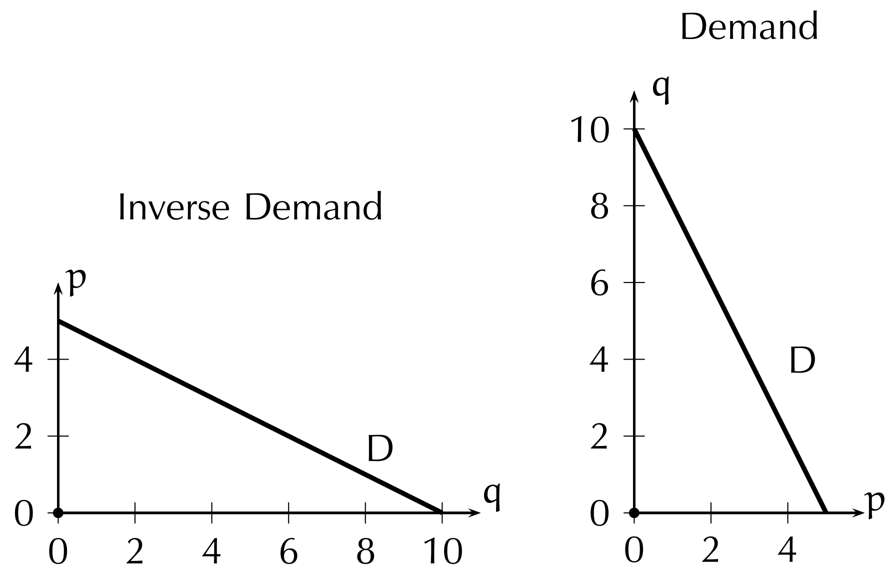
The first question is how to the responsiveness of quantity to changes in price. One obvious method is to use the slope of the demand curve $q(p)$.

This amounts to drawing the demand curve in non-standard fashion, with quantity on the vertical axis and price on the horizontal axis. Using our initial example, we would write $q(p) = 10 - 2p$ instead of $p(q) = 5 - \frac{1}{2}q$. The slope would be -2 instead of $-\frac{1}{2}$. The slope of the demand curve is always the inverse of the slope of the inverse demand curve.

2.6.2 Inverse Demand and Demand

The left diagram plots the demand curve $q(p) = 10 - 2p$ in conventional economic fashion, with price on the vertical axis. In other words, the equation of the line is better written using the inverse demand curve, $p(q) = 5 - \frac{1}{2}q$.

Demand and Inverse Demand



The right diagram is the same demand curve, drawn in mathematical fashion, with the independent variable (price) on the horizontal axis. In other words, the line has equation $q = 10 - 2p$. The slope -2 can be considered a measure of how quantity demanded responds to price.

2.6.3 Slope of the Mathematical Demand Curve

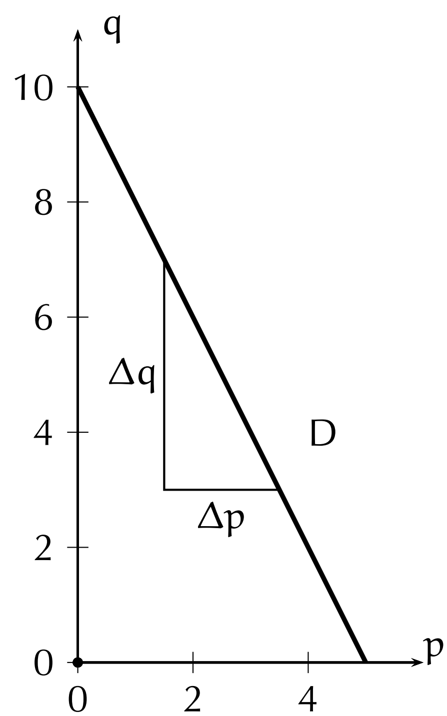
The slope of the mathematical demand curve is the rise over the run. The rise is the change in q, Δq, and the run is the change in p, Δp. Thus

$$\text{slope} = \frac{\Delta q}{\Delta p}.$$

Another way of stating it is the slope is the rate at which q changes relative to p. We measure it by making a marginal change in price (Δp), finding what change this makes in quantity (Δq), and dividing to find the rate of change.

This is illustrated on the diagram below.

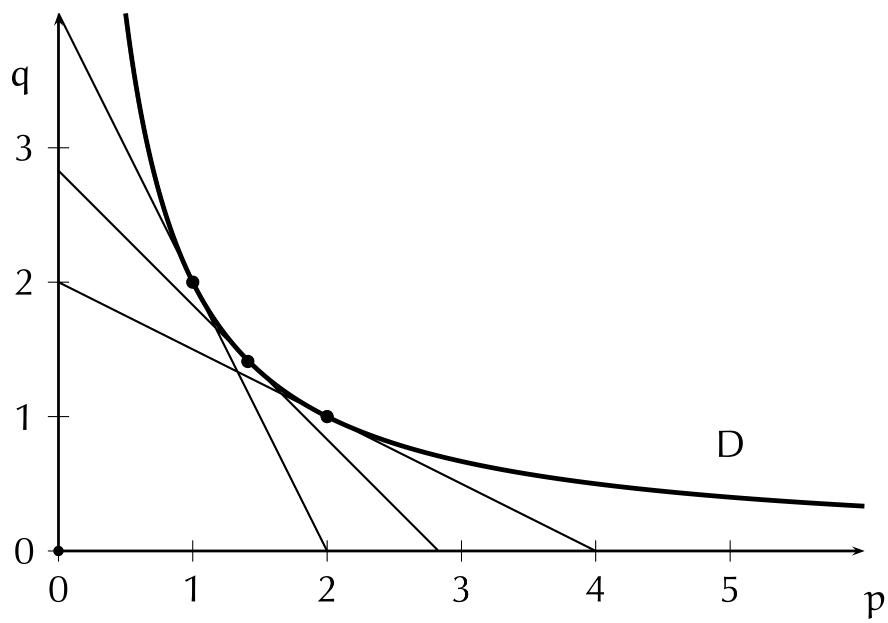
Slope is Rise over Run



2.6.4 Slope of the Tangent

For a curved demand curve, the rate of change of q relative to p is given by the slope of the tangent line. The **tangent line** at a point on a curve is the line that just touches the curve at that point. The slope of the tangent changes as you move along the curve.

Slopes of Tangent Lines



The tangents with slopes -2, -1, and $-\frac{1}{2}$ are shown on the diagram.

2.6.5 Calculating the Slope of the Tangent

To calculate the slope of the tangent, we use a little calculus. The slope of the tangent is the derivative dq/dp, which is the limit of slopes $\Delta q/\Delta p$ as the marginal change Δp goes to zero.

To calculate the derivative, we recall the power formula

$$\frac{d}{dp}p^n = np^{n-1},$$

and the fact that the derivative is linear.

In this case, the demand function is $q(p) = 2/p$ and $n = -1$, so the derivative is

$$\frac{dq}{dp} = -\frac{2}{p^2}.$$

One tangent line was drawn at $p = \sqrt{2}$, $q = \sqrt{2}$. The slope is -1 there. The formula shows us how the slope changes as we move along the curve.

2.6.6 What's Wrong with Using the Slope?

One problem with the slope can be seen if we ask exactly how it's measured. Consider the demand for milk.

Start with the price of milk. That price is stated in dollars per gallon. Currently, the price is about $3.50 per gallon. Changes in price are also measured in dollars per gallon.

Quantities are in gallons, as are changes in quantities.

The slope then, must be in gallons per (dollar per gallon). More simply, the slope is in gallons squared per dollar.

2.6.7 Changing the Units

What if we measure in quarts instead? We divide the \$3.50 per gallon by 4 to get the price per quart of \$0.875 per quart. Also, the quantity must be multiplied by 4 to translate the quantity in gallons to the quantity in quarts. This means that the slope gets multiplied by $4 \times 4 = 16$. The slope also depends on the monetary unit used, dollars, cents, peso, yuan, rupees, euros, pounds, shillings, etc.

This makes the size of the slope dependent on the units used for measurement. It makes numerical slopes without the units completely useless for comparison, and makes it impossible to compare sensitivity to price changes across commodities.

2.6.8 Price Elasticity of Demand

Fortunately, there is a universal measure of price sensitivity, a way to measure the sensitivity of the quantity demanded to price changes that is independent of how we measure things. That is the **elasticity**. More formally, we are computing the **price elasticity of demand**, E_d. It is defined as the percentage change in quantity demanded divided by the percentage change in price.

We write that as

$$E_d = \frac{\%\Delta q}{\%\Delta p} = \frac{\Delta q}{q} \bigg/ \frac{\Delta p}{p} = \frac{p}{q}\frac{\Delta q}{\Delta p}$$

This form is most useful for measuring sensitivity to changes over a price interval Δp. This version is referred to as the **arc elasticity of demand** or **interval elasticity of demand**.

When that interval is small, we can use the formula for elasticity at a point

$$E_d = \frac{p}{q}\frac{dq}{dp}.$$

This gives us the **point elasticity of demand**. When we use calculus to calculate the elasticity, we obtain the point elasticity.

2.6.9 How to Use the Elasticity of Demand: Questions

(1) Suppose the elasticity of demand for gasoline is $E_d = -0.26$ and the price of gasoline rises by 20%. How much does the quantity demanded fall?

(2) Suppose the elasticity of demand for potato chips is -3 and that a reduction in supply has caused the quantity demanded to fall by 12%. What was the change in price?

2.6.10 How to Use the Elasticity of Demand: Answer #1

(1) Suppose the elasticity of demand for gasoline is $E_d = -0.26$ and the price of gasoline rises by 20%. How much does the quantity demanded fall?

We set

$$\begin{aligned} E_d &= \frac{\%\Delta q}{\%\Delta p} \\ -0.26 &= \frac{\%\Delta q}{20} \\ -5.2 &= \%\Delta q, \end{aligned}$$

so the quantity demanded fell by 5.2%.

2.6.11 How to Use the Elasticity of Demand: Answer #2

(2) Suppose the elasticity of demand for potato chips is -3 and that a reduction in supply has caused the quantity demanded to fall by 12%. What was the change in price?

We set

$$\begin{aligned} E_d &= \frac{\%\Delta q}{\%\Delta p} \\ -3 &= \frac{-12}{\%\Delta p} \\ \frac{1}{4} &= \frac{1}{\%\Delta p} \\ \%\Delta p &= 4. \end{aligned}$$

This means the price of potato chips rose by 4%. E.g., if it was \$4 per bag, it is now $1.04 \times 4 = 4.16$ per bag.

2.6.12 Calculating the (Point) Elasticity of Demand

Consider the demand curve $q = 2/p$. We already found that its slope is $dq/dp = -2/p^2$. Let's find the formula for elasticity of demand.

$$
\begin{aligned}
E_d &= \frac{p}{q}\frac{dq}{dp} = \frac{p}{q}\left(\frac{-2}{p^2}\right) \\
&= \frac{1}{q}\left(\frac{-2}{p}\right) = \frac{p}{2}\left(\frac{-2}{p}\right) \\
&= -1
\end{aligned}
$$

Such a curve, where the elasticity of demand is always -1, is called **unit elastic**. Unit elastic demand curves all have the form

$$q = \frac{a}{p}$$

for some positive number a.

2.6.13 Unit Elastic Demand

Unit elastic demand curves have a very special property. No matter what the price is, consumers always spend the same amount of money on them.

Consumer spending on a good is the price times the quantity demanded, $p \times q$. When demand is $q = a/p$, spending is

$$p \times q = p \times \frac{a}{p} = a.$$

Spending is always a, and doesn't depend on the price p one bit.

2.6.14 Elasticity of Linear Demand Curves I

Suppose we start with the inverse demand curve $p = 5 - \frac{1}{2}q$. The slope of the inverse demand is $-\frac{1}{2}$, so the slope of demand itself is the inverse of that, -2. This is also the derivative dq/dp.

Then the elasticity of demand is

$$\begin{aligned} E_d &= \frac{p}{q}\frac{dq}{dp} = \frac{p}{q} \times (-2) \\ &= \frac{-2(5 - \frac{1}{2}q)}{q} = \frac{-10 + q}{q}. \end{aligned}$$

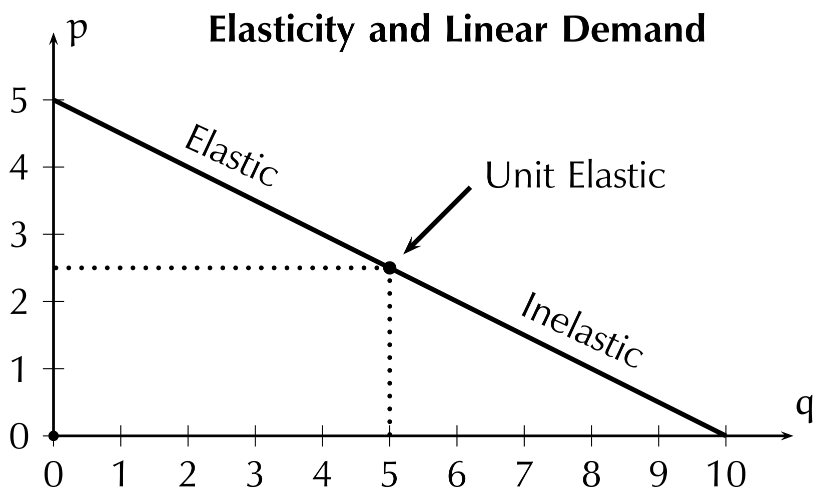
This elasticity changes as we move along the demand curve. If $q = 0$, we interpret the elasticity as $-\infty$, while if $q = 10$, the elasticity is 0. It runs over the whole range of negative numbers.

- Demand is **inelastic** is $0 \leq |E_d| < 1$.
- Demand is **unit elastic** is $|E_d| = 1$.
- Demand is **elastic** is $1 < |E_d|$.

We compare elasticities by comparing the absolute values. That means demand with an elasticity of -0.8 is more elastic than demand with elasticity -0.2, and the latter is more inelastic than the former.

2.6.15 Elasticity of Linear Demand Curves II

Here's how the elastic and inelastic regions compare on a linear demand curve.



For some numbers: $E_d(1) = -9$, $E_d(2) = -4$, $E_d(4) = -3/2$, $E_d(5) = -1$, $E_d(7) = -0.43$, $E_d(8) = -0.25$, and $E_d(9.5) = -0.53$.

Linear demand curves are always unit elastic at the midpoint, elastic to the left, and inelastic to the right.

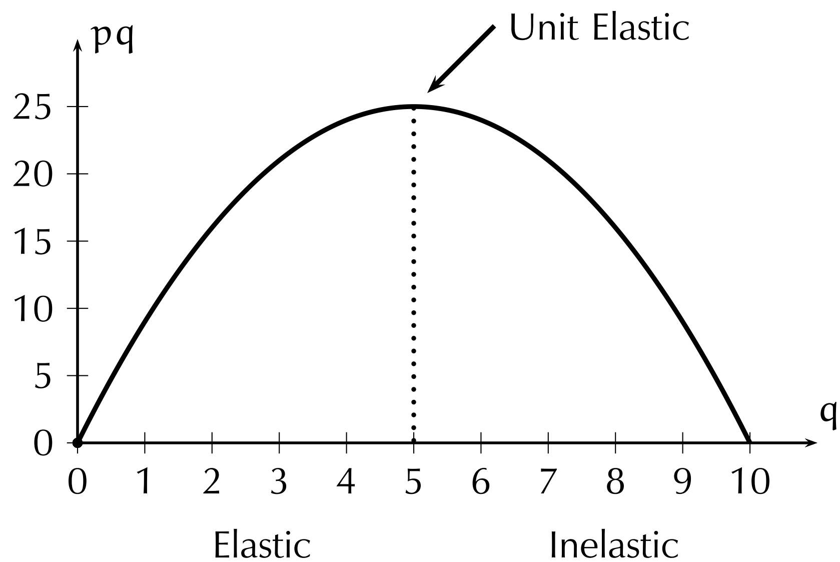
2.6.16 Spending and Elasticity

We can also graph consumer spending as a function of quantity. Then spending is

$$p(q) \times q = (5 - \frac{1}{2}q)q = 5q - \frac{1}{2}q^2$$

giving us the spending hill.

The Spending Hill and Elasticity



2.6.17 Some Demand Elasticities

Here are some estimated price elasticities of demand:

Natural Gas (short-run): -0.1

Coffee: -0.25

Legal Services: -0.4

Movies: -0.9

Owner-occupied Housing (long-run): -1.2

Automobiles (includes SUV's): -1.3

Restaurant Meals: -2.3

Chevrolet automobiles: -4.0

Fresh Tomatoes: -4.6

2.6.18 Factors Affecting Elasticity of Demand

There are several things that have a predictable affect on the elasticity of demand. More **close substitutes** make demand more elastic. E.g., automobiles in general (-1.3) have fewer substitutes than Chevys (-4.0).

The **share of income** spent on a product affects demand elasticity. Demand is more elastic when goods are a small part of consumer spending than a large part.

2.6.19 Elasticity and Time to Adjust

Generally speaking, demand is more elastic when consumers have more time to adjust their behavior. A 1996 study of the elasticity of demand for gasoline found that in the short-run (1 year or less), that elasticity was about -0.26, which is pretty inelastic. In the long-run (over 1 year), demand was more elastic, -0.58. In other words, the long-run change in quantity demanded for same price change was double that in the short run.[1]

There are usually more opportunities to make adjustments in the long-run. In the case of gasoline, consumers will eventually buy new cars. If there's a long-run increase in price, those cars are more fuel-efficient than they would otherwise be.

[1] Molly Espey, (1996), Explaining the Variation in Elasticity Estimates of Gasoline Demand in the United States: A Meta-Analysis, *Energy Journal* 17, 49–60.

2.6.20 Elasticity of Supply

Another elasticity is the elasticity of supply. Definitions of the interval and point elasticities are

$$E_s = \frac{\%\Delta q}{\%\Delta p} \quad \text{and} \quad E_s = \frac{p}{q}\frac{dq}{dp},$$

respectively.

A supply curve is unit elastic if it is a straight line through the origin. Such curves have the form $q = ap$ where a is a positive number. Then $dq/dp = a$ (the slope is a) and $p/q = 1/a$, so

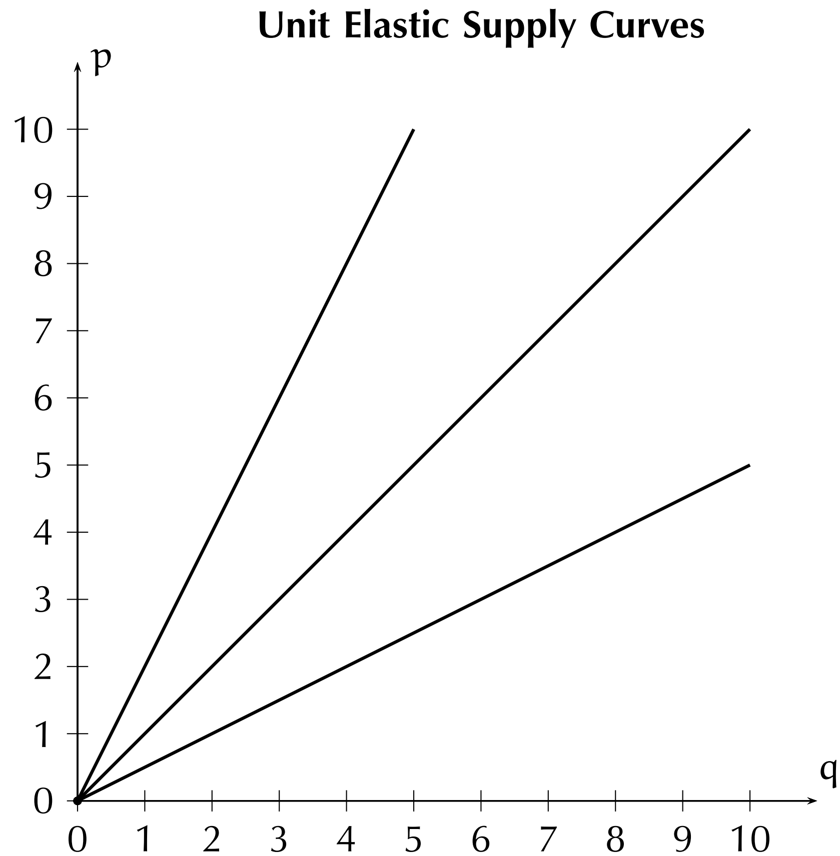
$$E_s = \frac{p}{q}\frac{dp}{dq} = \frac{1}{a} \times a = 1.$$

The inverse supply curve is $p = q/a$ and has slope $1/a$.

These are the only supply curves that are always unit elastic.

2.6.21 Unit Elastic Supply

Here are some unit elastic supply curves.



2.6.22 Income Elasticity of Demand

Another type of elasticity is the income elasticity of demand, E_I. Let m be income. Definitions of the interval and point elasticities are

$$E_I = \frac{\%\Delta q}{\%\Delta m} \quad \text{and} \quad E_I = \frac{m}{q}\frac{dq}{dm},$$

respectively.

2.6.23 Inferior, Normal, and Superior Goods

(1) A good is **inferior** if its income elasticity of demand is negative.

(2) A good is **normal** if its income elasticity of demand is positive.

(3) A good is **superior** or a **luxury good** if its income elasticity of demand is larger than 1.

2.6.24 Spending on Luxury Goods Increases with Income

When a good is luxury (superior) good, consumers spend a larger share of income on it as incomes increase.

Suppose Chris's income elasticity of demand for education is 2.5 and Chris spends 25% of income on education. What happens to Chris's spending on education if Chris's income increases by 10%?

Call Chris's income before the increase m. Since spending was 25% of income, Chris's original spending on education was $.25m$.

Income increased by 10%, to $1.1m$. The 10% increase in income leads to a $10 \times 2.5 = 25$ per cent increase in spending on education. That is, education spending increases to $1.25 \times .25m = .3125m$.

As a fraction of Chris's new income ($1.1m$), spending on education is

$$\frac{.3215m}{1.1m} = .284.$$

so Chris now spends 28.4% of income on education.

2.6.25 Cross-Price Elasticity of Demand

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Let p_x and q_x be the price and quantity demanded of good x and p_y and q_y the price and quantity demanded of good y. Then the **cross-price elasticity of demand for x with respect to the price of y** is

$$E_{xy} = \frac{\%\Delta q_x}{\%\Delta p_y} \quad \text{and} \quad E_{xy} = \frac{p_y}{q_x}\frac{dq_x}{dp_y}$$

2.6.26 Substitutes and Complements

- Two goods are **substitutes** if their cross-price elasticity of demand is positive.
- Two goods are **complements** if their cross-price elasticity of demand is negative.

3. Using Supply and Demand

3.1 Gains From Trade

When a consumer buys a product, it indicates they expect what they are buying to be worth at least as much as the money they are paying for it. If they didn't think so, they wouldn't buy it. This is a form of what economists call **revealed preference** where the *actions* of a buyer or seller *reveal* what they prefer.

Trade requires mutual agreement of both buyer and seller. This mutual consent tells us that both sides expect to gain from trade. Neither side would agree to the trade if they expected to lose from it. Trade is **positive-sum**, not zero-sum.

Involuntary exchange often involves losses for one side. If you are the victim of a robber who takes $100 from you, you have lost $100, and the robber has gained $100.

A much more common involuntary exchange is taxation. We will count the taxes paid as loses for the taxpayer, and gains for the government.

3.1.1 Measuring Gains From Trade

What we need as economists, is a way to measure how big or small those gains are. And when I say small, we should remember than in highly competitive industries such as retail trade, those gains are quite small compared to total sales. In the case of supermarkets, net profits are about 1-2% of total sales.

While 1-2% profit is small compared to market prices, major supermarket chains sell a lot of goods. In absolute terms, profits for the largest chains can be $10-30 billion.

As economists, we will be interested in measuring those gains and including them on supply and demand diagrams.

3.2 Consumer's Surplus

Consumer's surplus measures the gains to consumers from buying and consuming goods and services. Remember, the value to the consumer usually comes from consumption, and only occasionally from possession (e.g., artwork).

To compute consumer's surplus, we must account for both benefits and costs of the consumer. Both are measured in dollar terms (or whatever currency is appropriate).

We start by considering the benefits, the gains to the consumers. Consumer's surplus (CS) is a way of putting a dollar value on that gain. Reducing everything to dollars facilitates comparisons across different goods and different consumers.

Consumer's surplus is the difference between the amount a product is worth to a consumer—meaning the maximum they are willing to spend for it—and the cost, the amount they paid for it.

Expressed another way,

$$\text{Consumer's Surplus} = \text{Value} - \text{Cost}$$

3.2.1 Calculating Consumer's Surplus

Surplus from a single transaction is easy to calculate, provided we know the numbers. Just use the formula.

$$\text{Consumer's Surplus} = \text{Value} - \text{Cost}$$

One case of beer has a price of $20, and Joe is willing to pay a maximium of $30 for it, his consumer's surplus for that case is $30 − $20 = $10. It measures Joe's gain from this transaction.

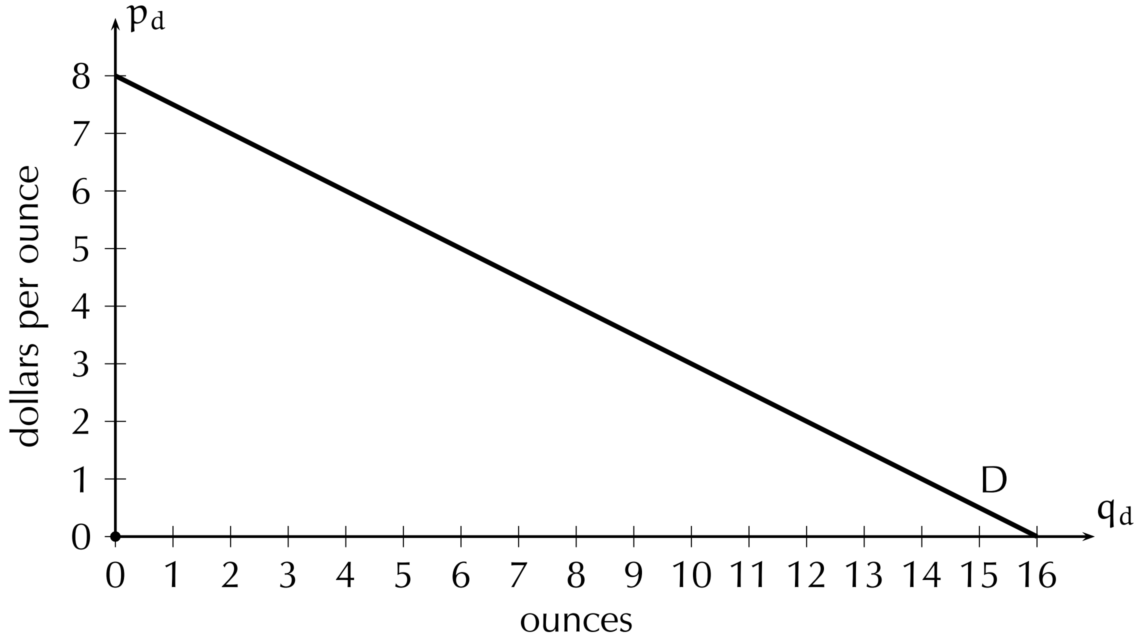
If Joe thinks he will have a use for another case soon, that second case would likely have a lower value, perhaps $25, yielding an additional $5 of surplus.

By buying two cases, Joe would have a surplus of $10 + $5 = $15.

3.2.2 Demand and Consumer's Surplus

We can also calculate surplus from consumer demand curves. Suppose inverse demand is $p_d(q_d) = 8 - \frac{1}{2}q_d$, as illustrated below. We use p_d for the demand price as we will be working with both inverse supply and inverse demand. The supply price will be p_s.

A Consumer Demand Curve



3.2.3 The Value of Each Unit Bought

The demand curve shows us that at a price of $8 per ounce, no consumer is willing to buy anything. No one values this product at more than $8 per ounce.

At $7.50 per ounce, some consumer is willing to buy a single ounce of our product, but no more.

We can infer that the first ounce has a value between $8.00 and $7.50 to one of the potential consumers.

At a price of $7.00 per ounce, a second ounce can be sold. There is some consumer who values that ounce at more than $7.00 and less that $7.50. We don't know if it's the same consumer buying a second ounce or another buying their first.

At a price of $6.50 per ounce, a third unit can be sold. There is some consumer who values that ounce at more than $6.50 and less that $7.00.

We can continues this process all the way down the demand curve, but for the present, we stop at $q = 4$. The 4^{th} unit is bought at a price of $6 and has a value between $6.50 and $6.00.

3.2.4 The Value of Each Unit Bought, with Chart

The demand curve shows us that at a price of $8 per ounce, no consumer is willing to buy anything. No one values this product at more than $8 per ounce.

At $7.50 per ounce, some consumer is willing to buy a single ounce of our product, but no more.

We can infer that the first ounce has a value between $8.00 and $7.50 to one of the potential consumers.

At a price of $7.00 per ounce, a second ounce can be sold. There is some consumer who values that ounce at more than $7.00 and less that $7.50. We don't know if it's the same consumer buying a second ounce or another buying their first.

At a price of $6.50 per ounce, a third unit can be sold. There is some consumer who values that ounce at more than $6.50 and less that $7.00.

We can continues this process all the way down the demand curve, but for the present, we stop at $q = 4$. The 4^{th} unit is bought at a price of $6 and has a value between $6.50 and $6.00.

Units	Upper Value	Lower Value
1	$8.00	$7.50
2	$7.50	$7.00
3	$7.00	$6.50
4	$6.50	$6.00

3.2.5 Four Ounces are Worth at Least $27.00

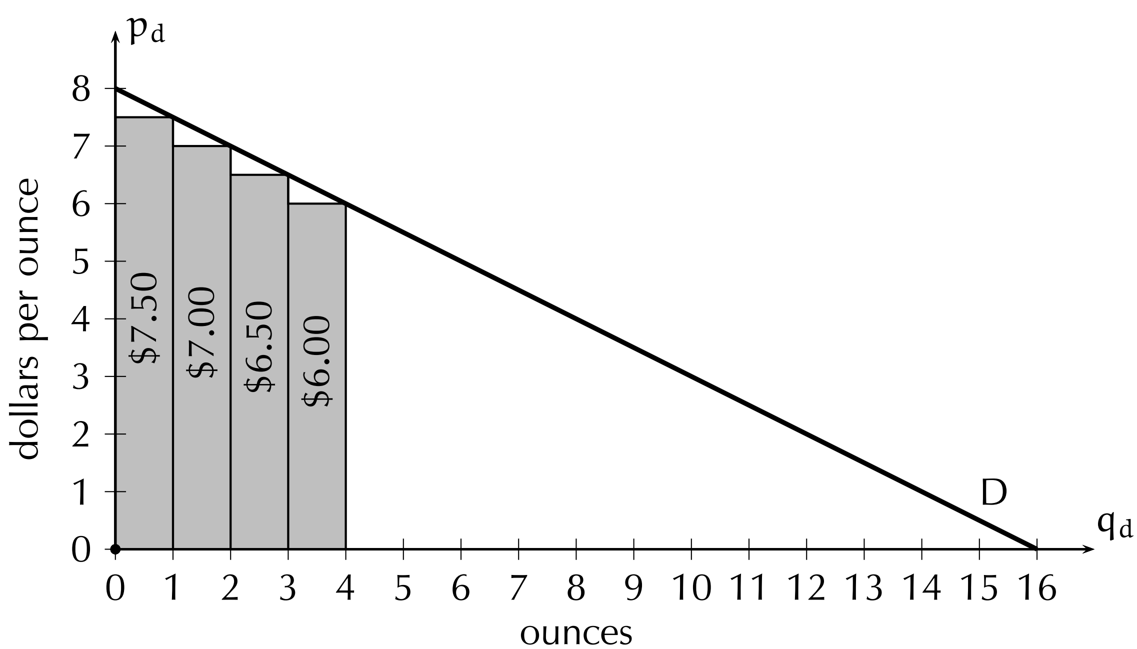
The shaded area has been constructed from the lower values, $7.50 for the first unit, $7.00 for the second, $6.50 for the third, and $6.00 for the fourth, a total of $27.00 (7.5 + 7 + 6.5 + 6 = 27).

This is also the shaded area. The first vertical box has a base of 1 ounce and a height of $7.50/oz., so its area is the product,

$$1 \text{ oz.} \times \frac{\$7.50}{\text{oz.}} = \$7.50.$$

Similarly, the others have areas of $7.00, $6.50, and $6.00, as marked on the diagram. This adds to $27.00.

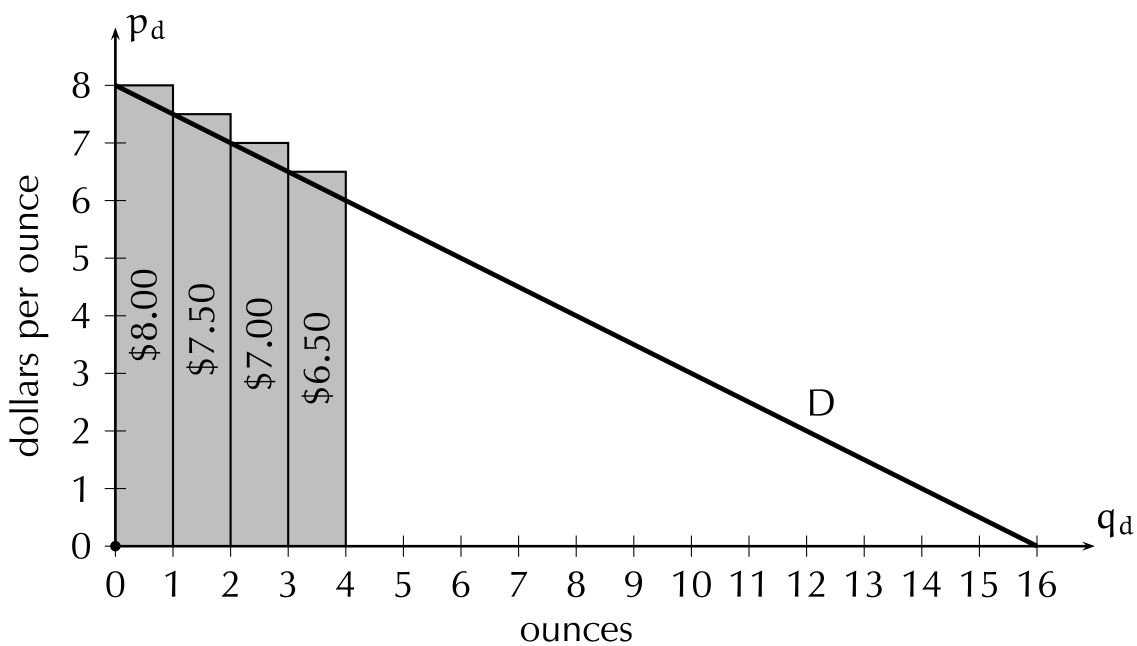
Value of Four Ounces: Lower Bound



3.2.6 Four Ounces are Worth at Most $29.00

We now put an upper bound on the value of 4 ounces by adding up the upper values: $8.00, $7.50, $7.00, and $6.50 to obtain $29.00. Once again these can be represented by vertical boxes based on the demand curve.

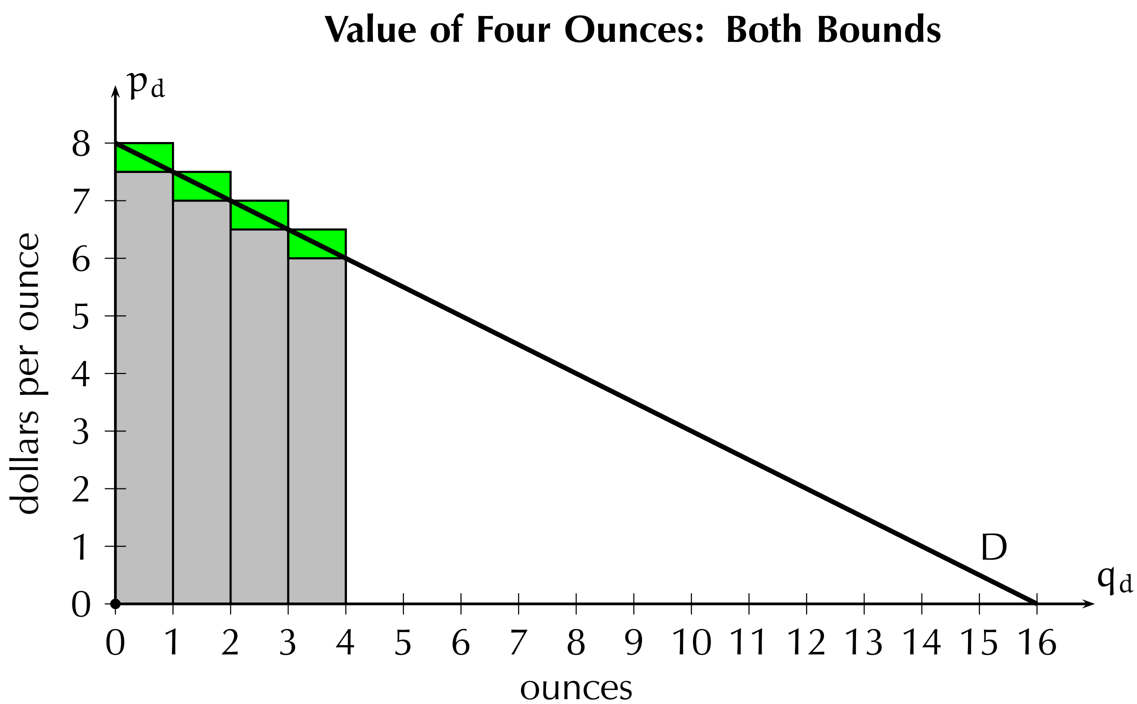
Value of Four Ounces: Upper Bound



We now know that 4 ounces are worth between $27.00 and $29.00 to our consumers. But we can do better.

3.2.7 Upper and Lower Bounds Together

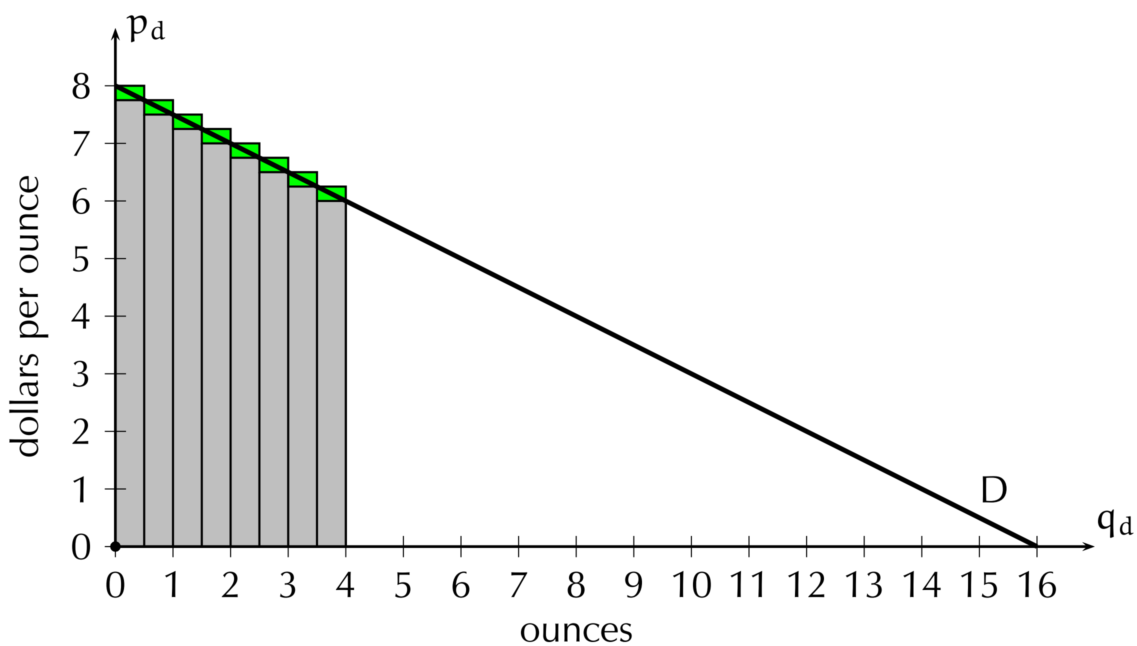
We now put the upper and lower bounds on the same diagram. The green area is included in the upper bound but not the lower.



3.2.8 Refining the Estimate

Suppose we thought in terms of selling a half-ounce at a time. At a price of \$7.75 per ounce, some consumer is willing to buy the first half-ounce. This costs them $\frac{1}{2} \times 7.75 = 3.875$ dollars. We again construct the lower and upper bounds for each half ounce, yielding a new diagram.

Value of Four Ounces: Squeezing the Bounds

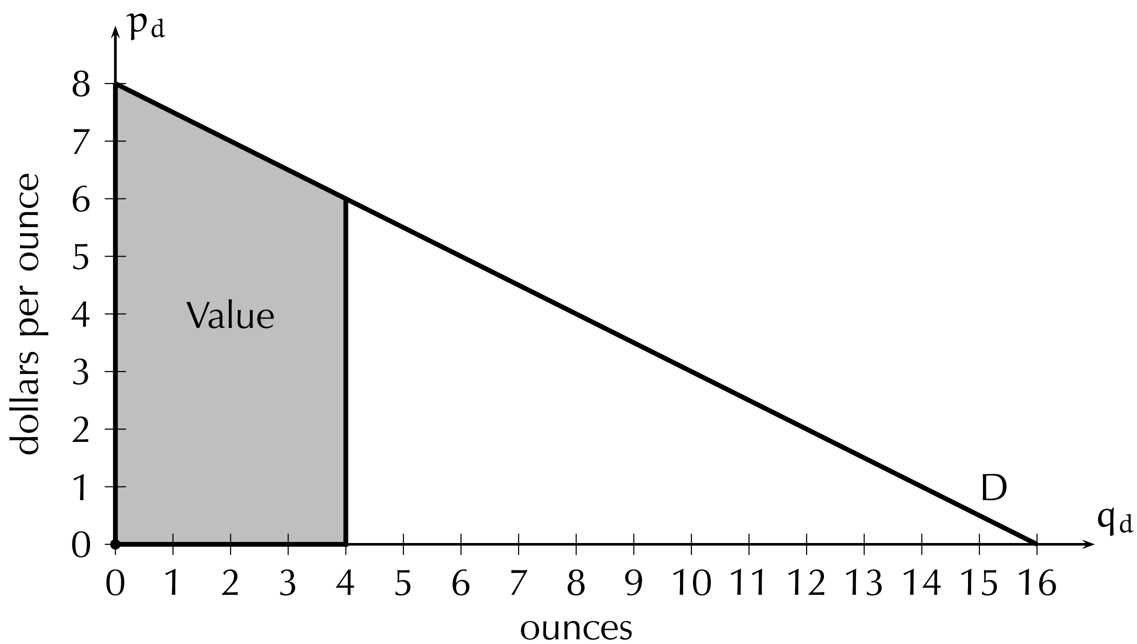


3.2.9 The Value is the Area Under the Demand Curve

Taking smaller and smaller steps, we approach the area under the demand curve, up to the quantity consumed. This may remind you of integral calculus. That's because it is!

The shaded area is the value to consumers of consuming 4 ounces. It is measured in dollars.

Value of Four Ounces: Limit Value



3.2.10 The Calculating the Consumer's Value

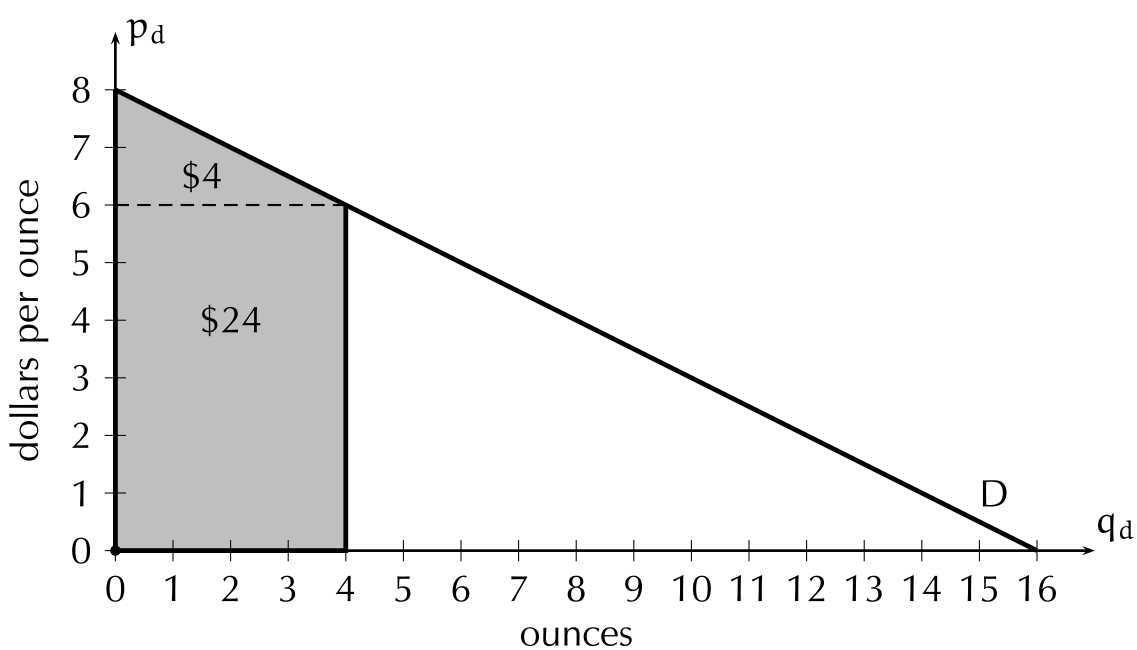
Now divide the consumer's value into a triangle and rectangle as shown. This will make it easy to calculate the area.

The rectangle has base $b = 4 - 0 = 4$ and height $h = 6 - 0 = 6$. For rectangles, the area is $b \times h = 4 \times 6 = 24$. (More pedantically, \$24.)

The triangle has base $b = 4$ and height $h = 2$. For triangles, the area is a bit different, $\frac{1}{2}b \times h$. Substituting, we find it $\frac{1}{2}b \times h = \frac{1}{2} \times 2 \times 4 = 4$. (Actually, \$4.)

We add these areas together to find the value of four ounces of our product to consumers. It is **\$28**.

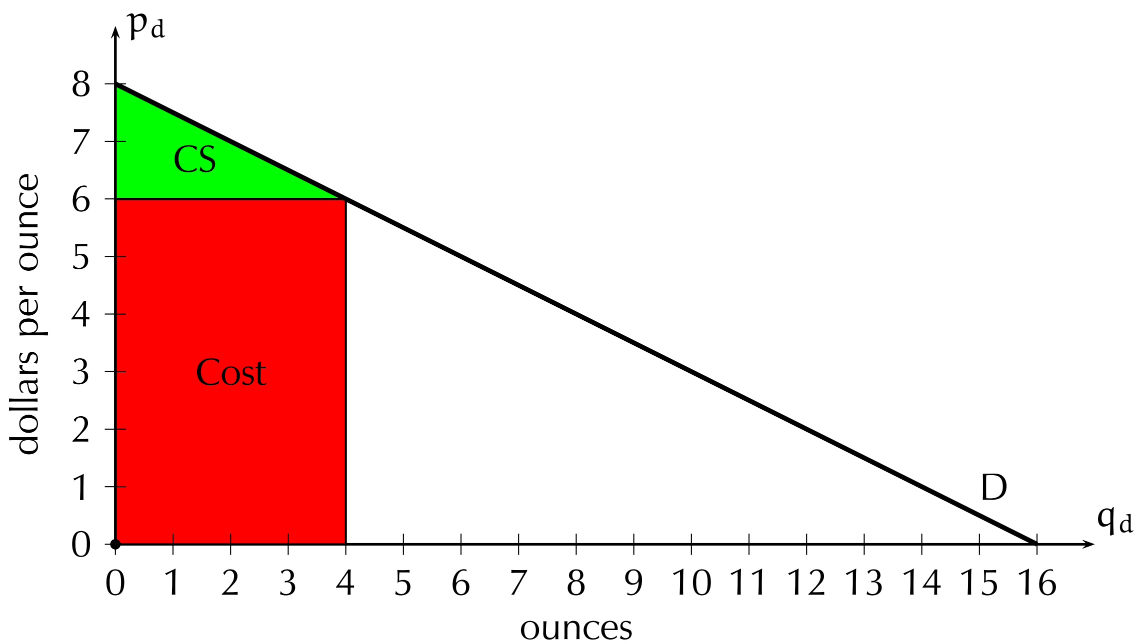
Calculating the Consumer's Value of Four Ounces



3.2.11 Consumer's Surplus Again

To compute the consumer's surplus, we subtract the cost from the value. The value of 4 ounces is \$28. Four ounces are demanded at a price of \$6, so the cost is $\$6 \times 4 = \$24$, leaving a consumer's surplus of $CS = \$28 - \$24 = \$4$.

Calculating the Consumer's Surplus from Four Ounces



The consumer's surplus is the area under the demand curve and above the price, up to the quantity consumed.

3.2.12 Value and Consumer's Surplus via Calculus

For those who know calculus, we can write the value and consumer's surplus in terms of integrals. Suppose the quantity is q_0 and the price is $p_0 = p(q_0)$.

The value of quantity q_0 to consumers is

$$\text{Value} = \int_0^{q_0} p_d(q)\, dq$$

and the consumer's surplus at price p_0 and quantity q_0 is

$$\begin{aligned} \text{CS} &= \int_0^{q_0} \left[p_d(q) - p_0\right] dq \\ &= \int_0^{q_0} p_d(q), dq - \int_0^{q_0} p_0\, dq \\ &= \int_0^{q_0} p_d(q), dq - p_0 q_0 \end{aligned}$$

The last form writes the surplus as the difference between value and cost.

3.2.13 A Calculus Example

The inverse demand function is $p_d(q) = 8 - \frac{1}{2}q$. Use calculus to find the value to consumer's of q_0.

The value V is

$$\begin{aligned} V(q_0) &= \int_0^{q_0} 8 - \frac{1}{2}q \, dq \\ &= 8q - \frac{1}{4}q^2 \Big|_0^{q_0} \\ &= 8q_0 - \frac{1}{4}q_0^2. \end{aligned}$$

3.3 Producer's Surplus

Surplus works a bit differently for sellers. The price is no longer a cost, but rather a benefit. The supply curve does not represent value, as demand does for buyers. Rather, it represents cost. This can be financial cost as is typical with firms (we will see in chapters 7 and 8 that the supply curve is the marginal cost curve). Supply can also represent opportunity cost.

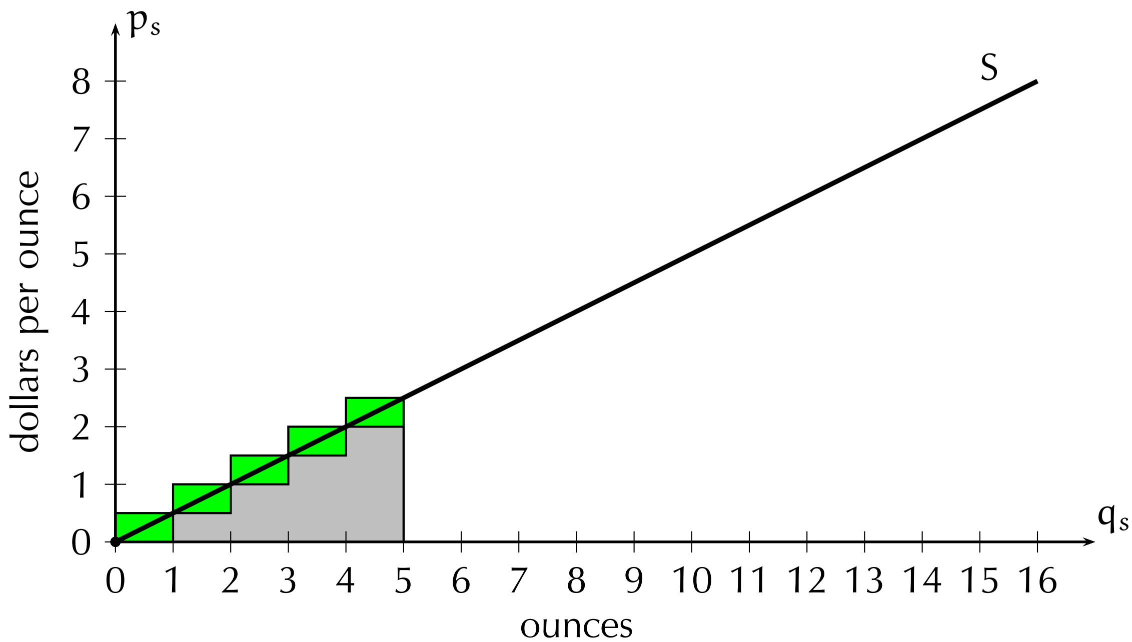
The benefit the seller receives from a sale of each unit of a good is the price paid for it.

The supply curve tells us how much sellers are willing to sell at each particular price. As with the demand curve, we can ask how much it takes to get the seller to part with each unit of the good in question. This brackets the supply curve as we did the demand curve, with the supply curve telling us how much the product is worth to the sellers—what they are giving up. In other words, the supply curve gives us the cost of selling.

3.3.1 Supply and Producer's Value

We take the supply curve with inverse supply function $p_s(q_s) = \frac{1}{2}q_s$, and apply the same stairstep method, but this time to the supply curve .

Calculating Producer Cost: Both Bounds



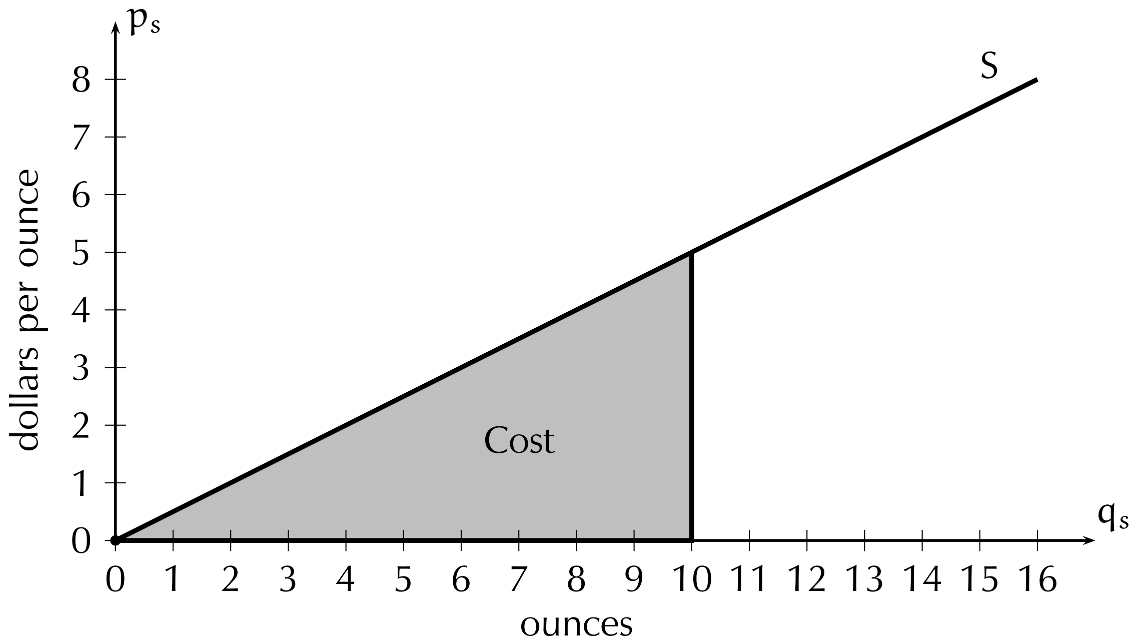
3.3.2 The Cost is the Area Under the Supply Curve

As with demand and value, we use the supply curve to approximate cost above and below, obtaining the area under the supply curve as the limit.

We've illustrated the cost of $q_0 = 10$ ounces using the supply price $p_s(10) = \$5$. Since the cost is a triangle with base 10 and height 5, it has area

$$\frac{1}{2}b \times h = \frac{1}{2} \times 10 \text{ oz.} \times \frac{\$5}{\text{oz.}} = \$25.$$

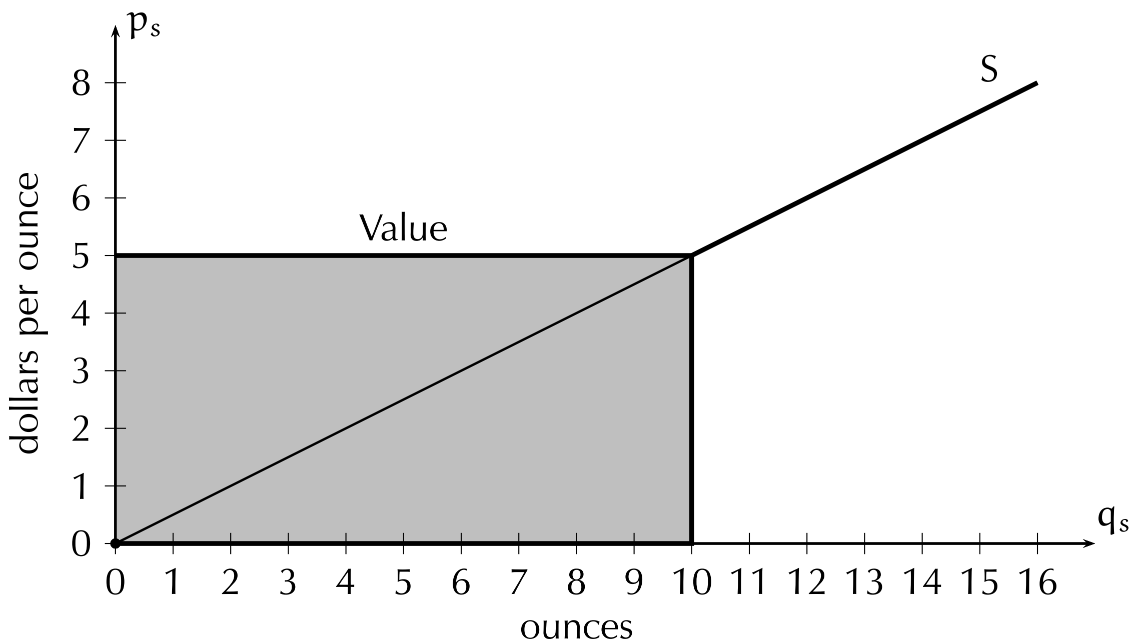
Calculating the Producer Cost: Limit Value



3.3.3 Producer's Value and the Supply Curve

The value of $q_0 = 10$ at $p_s(q_0) = \$5$ is $\$5 \times 10 = \$50$.

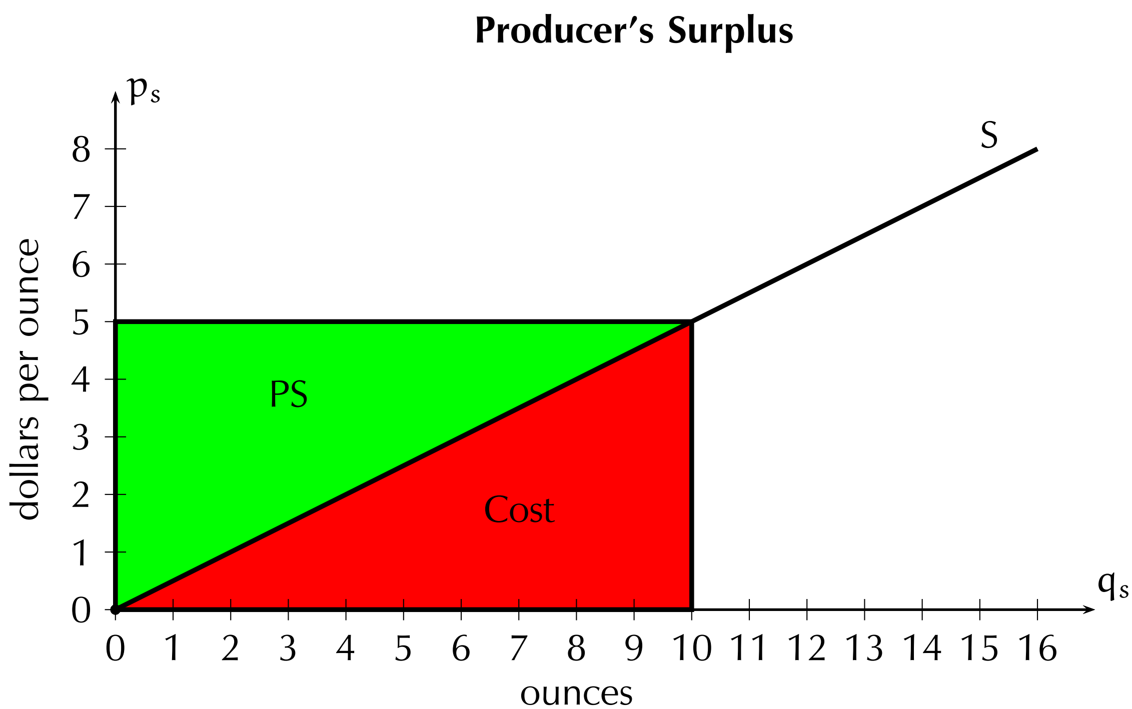
Calculating Producer's Value from Supply



3.3.4 Calculating Producer's Surplus

Producer's surplus (PS) is given by PS = Value − Cost, just like consumer's surplus. In our case it is \$50 − \$25 = \$25.

We can also calculate as the area of the upside down triangle below the price and above the supply curve. That triangle has height of 5 and base of 10 (on top), so the area is $\frac{1}{2} \times 5 \times 10 = \$25$.



3.3.5 Cost and Producer's Surplus via Calculus

As with consumer's surplus, there is are calculus formulas for both cost and producer's surplus when supplying quantity q_0

The cost of supplying quantity q_0 is

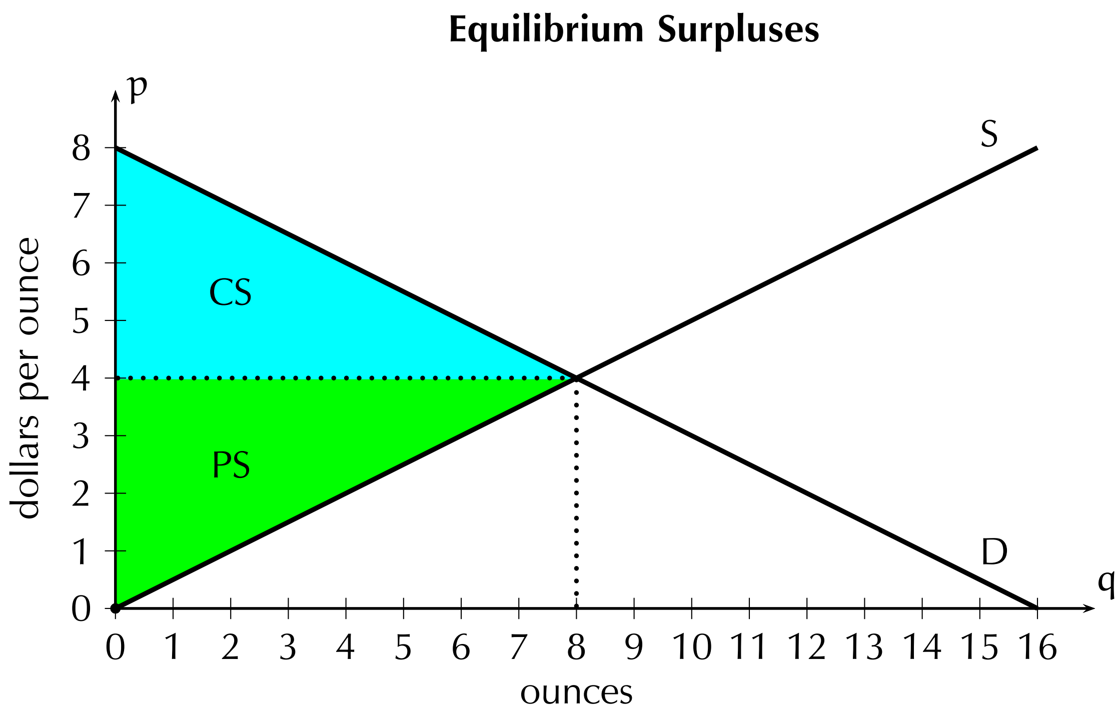
$$\text{Cost} = \int_0^{q_0} p_s(q)\, dq$$

and the producer's surplus at price p_0 is

$$\text{PS} = \int_0^{q_0} \left[p_0 - p_s(q)\right]\, dq.$$

3.4 Equilibrium Market Surpluses

Putting together our supply ($p_s = \frac{1}{2}q$) and demand ($p_d = 8 - \frac{1}{2}q$) curves, we find the market equilibrium is at $p^* = 4$, $q^* = 8$. The equilibrium consumer's and producer's surpluses are shown below.



3.4.1 Total Surplus

Let $p_d^c = 8$ and $p_s^c = 0$ be the demand and supply choke prices, respectively. Then

$$CS = \frac{1}{2}(p_d^c - p^*) \times q^* = \frac{1}{2}(8 - 4) \times 8 = \$16$$

and

$$PS = \frac{1}{2}(p^* - p_s^c) \times q^* = \frac{1}{2}(4 - 0) \times 8 = \$16.$$

The **total surplus** TS is the sum of the consumer's and producer's surpluses. Here

$$TS = CS + PS = \$16 + \$16 = \$32.$$

The total surplus measures all of the gains from trade. We call a market **efficient** if the total surplus is as large as possible.

3.4.2 Total Surplus and Price

One interesting thing about the total surplus, is that if we exchange the same amount q but change the price, the surplus remains unchanged. The price comes into the calculation of total surplus in two places.

We subtract the total cost of consumption pq from the value of consumption (which depend only on q) to get consumer's surplus.

We take the revenue pq and subtract the cost of supply (which depends only on q) to get producer's surplus.

The total surplus is the sum of these, and the $-pq$ from consumer's surplus and $+pq$ from producer's surplus cancel out. The total surplus is the value to the consumers minus the cost to the producers, both of which depend only on q.

We have

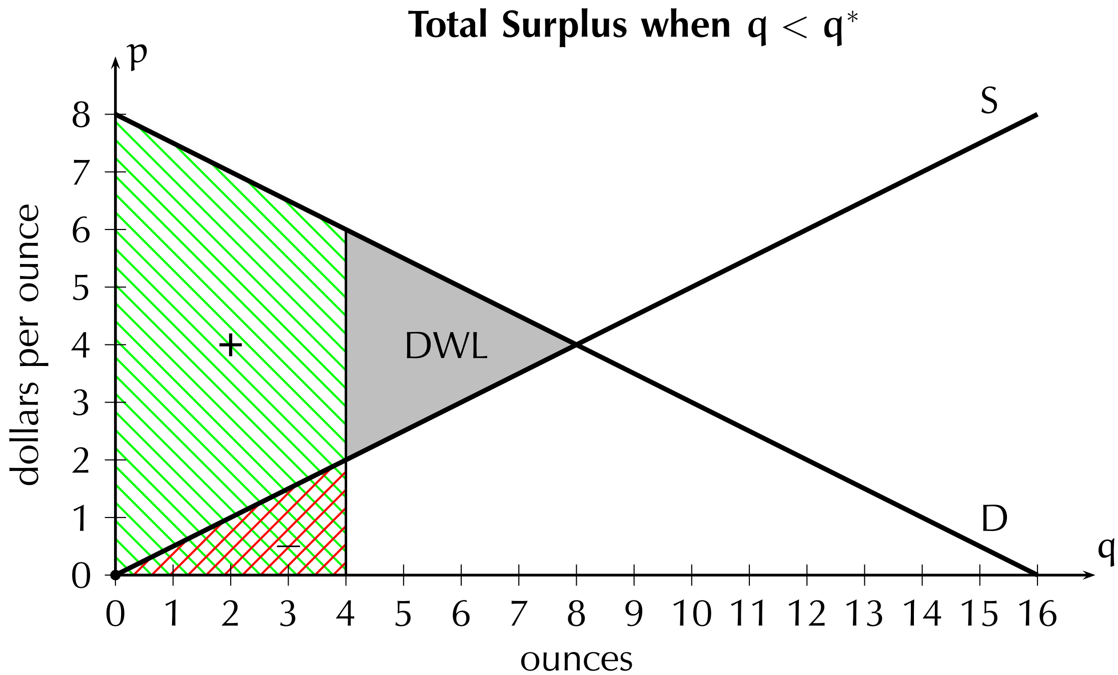
$$\text{CS} = \text{Consumer's Value} - pq$$
$$\text{PS} = pq - \text{Producer's Cost}$$

Adding them together gives us total surplus:

$$\text{TS} = \text{Consumer's Value} - \text{Producer's Cost}$$

3.4.3 Maximizing Total Surplus I

What if we trade a quantity less then q^*. What happens to total surplus? Since $q^* = 8$, let's try $q = 4$.



The green hatching represents benefits to consumers, and the red hatching represents costs to producers. The costs and benefits cancel out in the area marked in both green and red. That means that are hatched only in green is the total surplus.

Compared to the equilibrium quantity, where the surplus was \$32, we have lost the gray area. This lost surplus is referred to as the **deadweight loss**, **DWL**. Since it is a triangle, its area is easily computed as \$8. The total surplus is \$8 less than at equilibrium.

September 6, 2022