Intermediate Microeconomics — Week 5

Professor Boyd **Sept. 20 & 22, 2022**

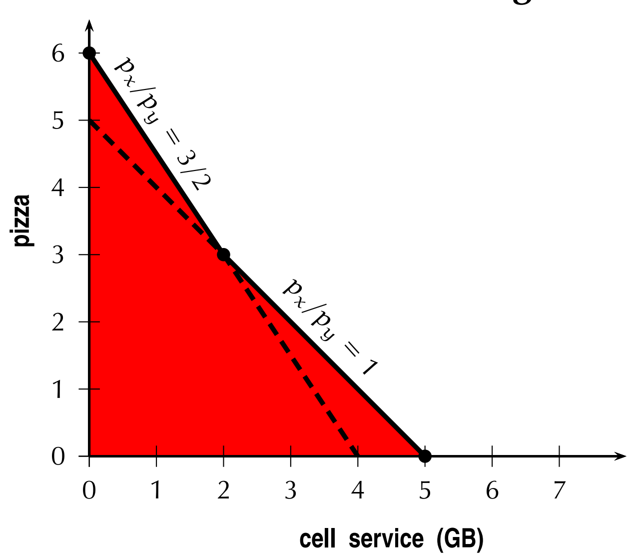
QUIZ: Thursday's quiz covers weeks 3 and 4, including sections 3.2 through section 4.3 (pp. 60–117).

4.4.11 A Non-Standard Budget Set: Quantity Discount **09/20/22**

Suppose we have two goods, pizza and cell service, measured in gigabytes (GB) per month. You have \$60 income. Each pizza costs \$10. Cell service is \$15 per GB for the first 2 GB, and \$10 per GB for each gigabyte past 2. The complication here is that the relative price changes, which puts a kink in the budget frontier.

If you buy only pizza, you can afford 6 of them. Cell service is more complicated. The first 2 GB cost a total of \$30, leaving \$30. At that point, the price changes to \$10 each. If you are buying only cell service, you can afford \$30 worth, which is 3 more GB, totalling 5 GB. If instead, you spend the rest on pizzas, you can buy 3 pizzas and 2 GB. We plot those three points, and then connect them.

Discounts and the Budget Set



The dashed lines show the budget frontier if the first 2 GB cost \$10 per GB, and additional GB cost \$15 per GB.

4.5 Consumer Choice

We've gathered the pieces of consumer theory, describing preferences and their properties, and examining budget constraints.

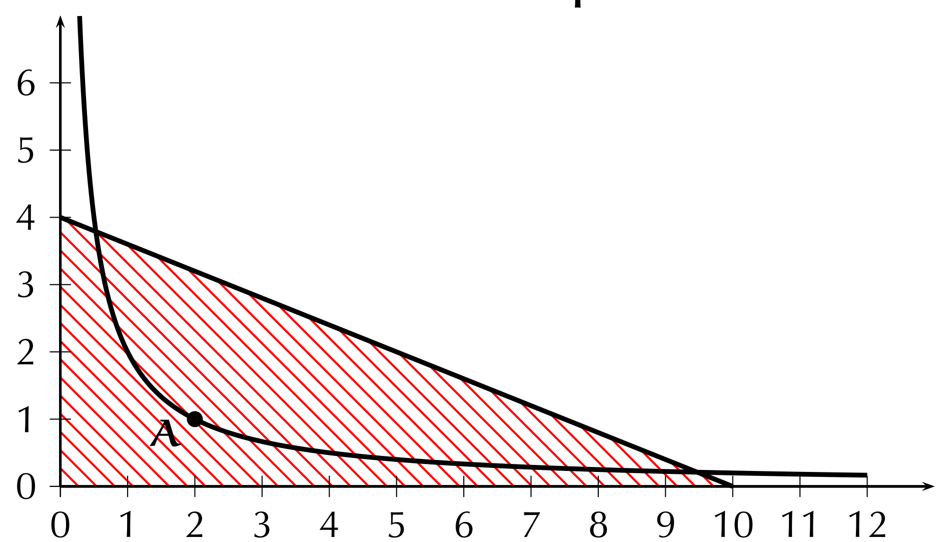
Now we're ready to put it all together and solve the consumer's choice problem: Maximizing utility subject to a budget constraint.

4.5.1 The Consumer's Problem

The consumer knows their utility function and has income m. They face prices p_x and p_y. How do they find the best point in the budget set? How do they maximize utility?

We start by asking if any points that are strictly inside the budget set (marked in red) can be best. We start with a point inside the budget set, marked A in the graph. We add an indifference curve through A. Completeness ensures there is such an indifference curve.

Is Bundle A Optimal?

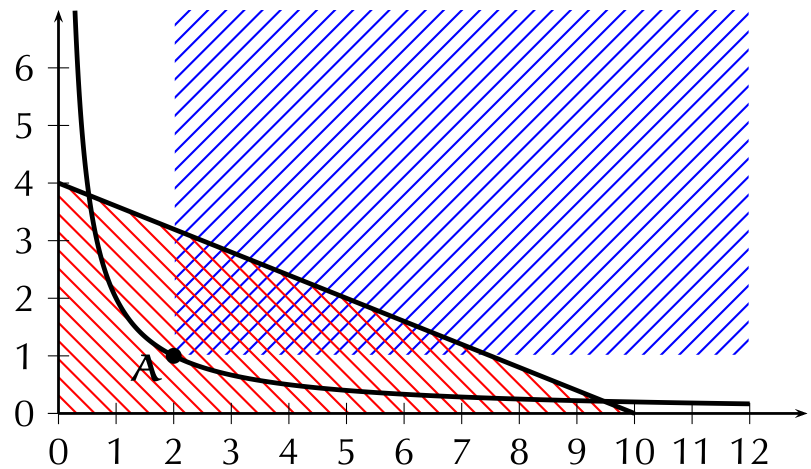


4.5.2 The Consumer's Choice Must be on the Budget Frontier

The red hatched area is the budget set. By monotonicity, our consumer prefers the consumption bundles in the area above and right of point A, the blue hatched area, to A itself. The consumer can afford to improve on A by choosing any point in the cross-hatched area.

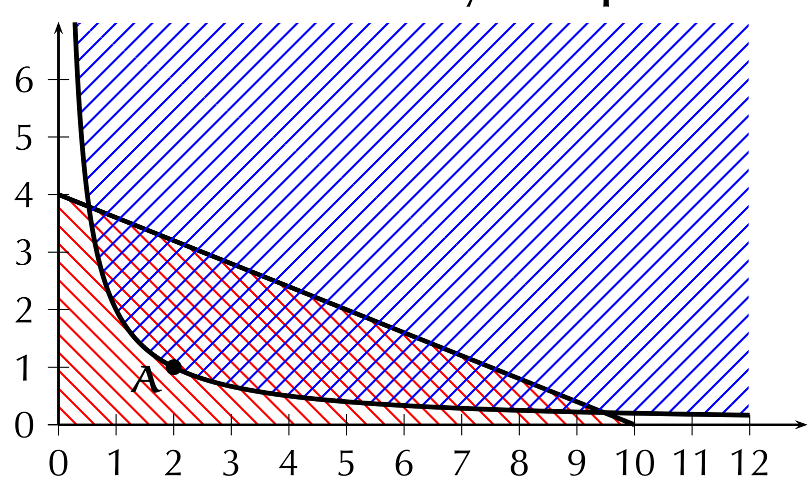
Indeed, as the lower figure shows, any point above the indifference through A and in the budget set (the cross-hatched part) is an improvement.

No, Bundle A Is Not Optimal!



Whenever a consumption bundle is not on the budget frontier, we can find such improvements. Conclusion: **The best affordable consumption bundle is on the budget frontier**.

Bundle A is Really Not Optimal!

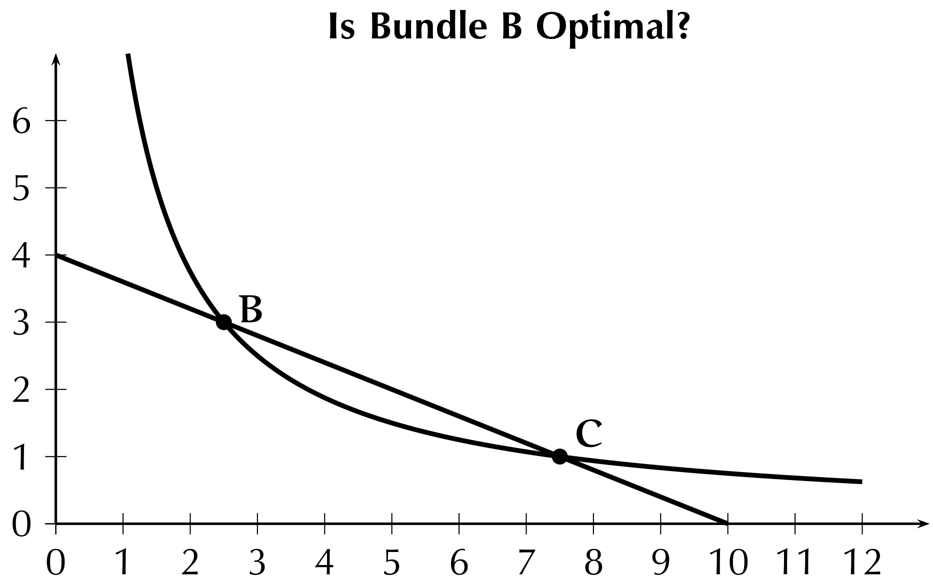


4.5.3 Finding the Best Bundle on the Frontier

Let's examine a point on the budget frontier. Call it B. Now take the indifference curve through B.

Can the consumer do better?

We start by marking the other point on the indifference curve through B that is on the budget frontier. Call it C.

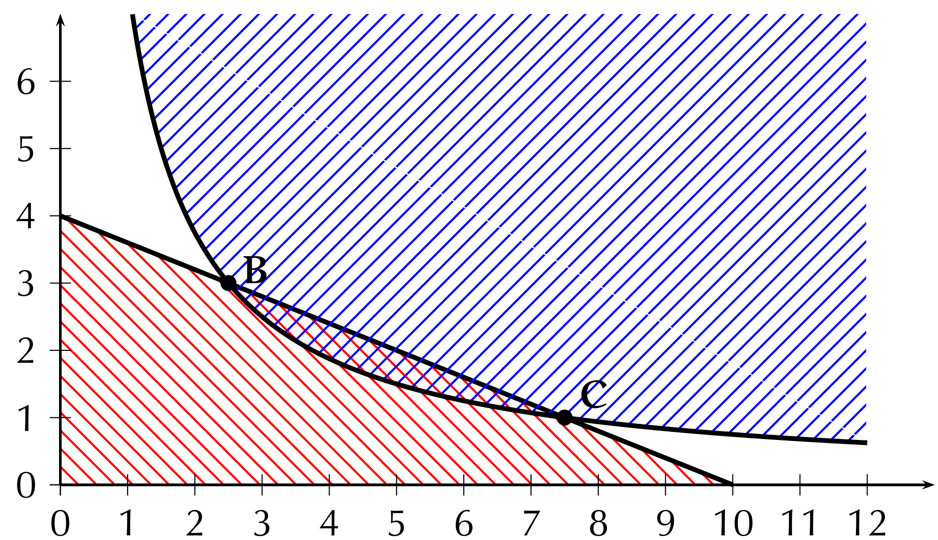


4.5.4 Finding the Best Bundle on the Frontier

We color in the budget set (red) and the bundles that are better than B (blue). This should make the situation clear.

The bundles on the budget frontier that lie between B and C are the affordable bundles that are better than B. We start moving down and right along the budget frontier to improve.

Bundle B is Still Not Optimal



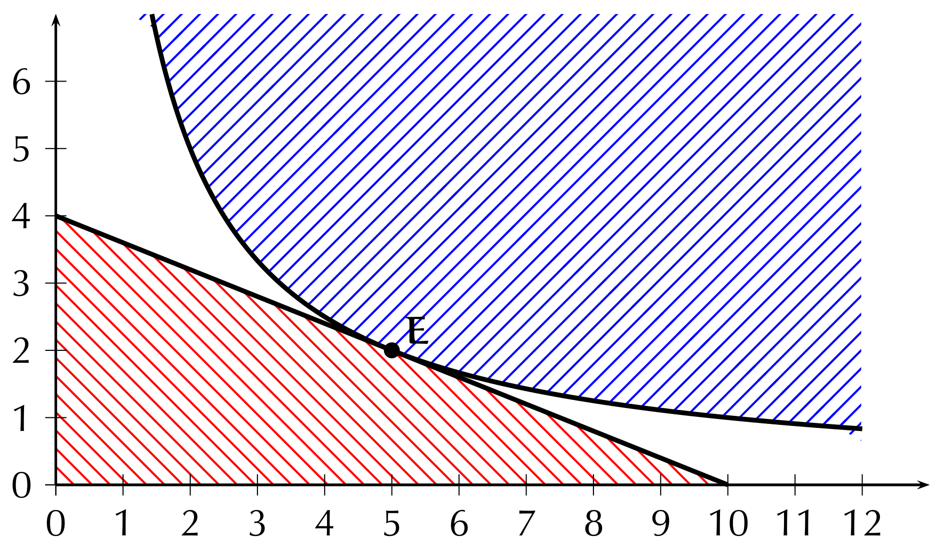
Another way of looking at this is that the slope of the indifference curve through B is higher than the slope of the budget line. This means that the marginal value of good X in terms of Y, the MRS, is higher than the its price in terms of Y, p_x/p_y. So the consumer can improve by buying more X and less Y.

4.5.5 The Consumer's Choice

The best consumption bundle the consumer can afford is at point E.

From point E, there are no more points that we can afford (red) that are also better than E (blue). That means that E is the best bundle that our consumer can afford—the consumer's optimum. This is the point in the budget set with the highest possible utility. It maximizes utility.

Bundle E is Optimal!!



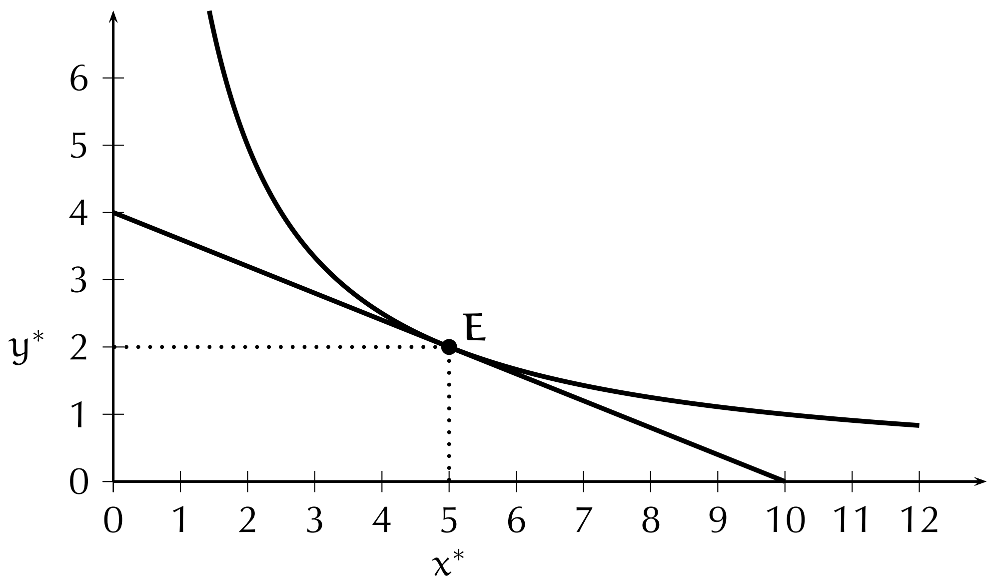
4.5.6 Characterizing the Utility Maximum

The utility maximizing bundle E has two properties:

1. The bundle E is on the budget frontier
2. The budget frontier is tangent to the indifference curve at E. As a result, the marginal rate of substitution at E is equal to the relative price.

The point $E = (x^*, y^*) = (5, 2)$ is our first point on the demand curve. Its location depends on the consumer's preferences (utility), income, and the prices of both goods.

Optimality Conditions



4.5.7 The Two Optimality Equations

We restate the two conditions for optimality as two equations:

$$\begin{aligned} m &= p_x x + p_y y && \text{Budget equation} \\ \mathrm{MRS} &= \frac{p_x}{p_y} && \text{Optimality equation.} \end{aligned}$$

Recall that

$$\mathrm{MRS} = \frac{\mathrm{MU}_x}{\mathrm{MU}_y} \quad \text{so} \quad \frac{p_x}{p_y} = \mathrm{MRS} = \frac{\mathrm{MU}_x}{\mathrm{MU}_y}.$$

4.5.8 An Alternative: Marginal Utility per Dollar

Let's take a closer look at the optimality equation.

$$\frac{MU_x}{MU_y} = \frac{p_x}{p_y} = MRS\,.$$

Ignoring the MRS term, divide by p_x and multiply by MU_y to obtain

$$\frac{MU_y}{p_y} = \frac{MU_x}{p_x} = \frac{[\text{utils}]/[x\text{-unit}]}{[\$]/[x\text{-unit}]} = \frac{[\text{utils}]}{[\$]}.$$

where the brackets indicate we are giving the units of each expression.

The MU/p terms are the marginal utility per dollar spent for each of the goods. To maximize utility, we must spend each dollar in a way that gives us the **biggest bang per buck**—the **biggest marginal utility per dollar**.

Later, in section 4.5.24, we will see that we can easily include the corner case.

4.5.9 Optimality by the Numbers I

The optimality graph uses the Cobb-Douglas utility function $u(x, y) = xy$. Prices are $p_x = 4$ and $p_y = 10$. Income is \$40. The relative price is then $p_x/p_y = 4/10 = 2/5$ and the marginal rate of substitution is y/x. That gives us two equations: the budget equation and the optimality equation $\text{MRS} = p_x/p_y$. In other words,

$$\begin{aligned} 4x + 10y &= 40 && \text{Budget} \\ \frac{y}{x} &= \frac{p_x}{p_y} = 2/5. && \text{Optimality} \end{aligned}$$

Eliminating the common factor of 2 from the first equation and clearing the denominators of the second, we find

$$\begin{aligned} 2x + 5y &= 20 \\ 5y &= 2x. \end{aligned} \tag{1}$$

4.5.10 Optimality by the Numbers II

We solve system (1) to find (x^*, y^*). First, let's repeat (1), emphasizing the $5y$ terms.

$$\begin{aligned} 2x + 5y &= 20 \\ 5y &= 2x. \end{aligned} \tag{1}$$

Substitute $2x$ for $5y$ in the first equation of (1). That gives us

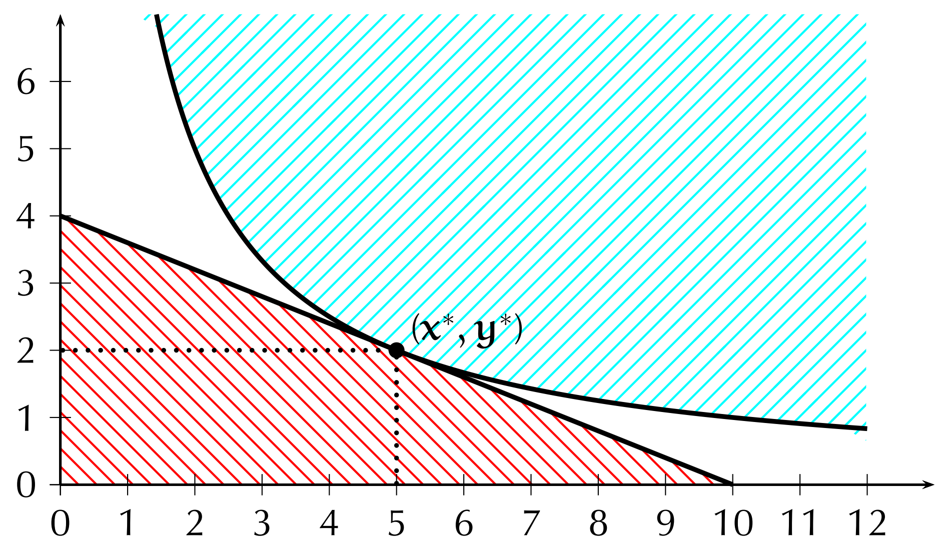
$$2x + 2x = 20 \quad \text{or} \quad 4x = 20.$$

Then $x^* = 5$. We now plug this in the budget equation to find y^*,

$$2(5) + 5y = 20 \quad \text{or} \quad 5y = 10.$$

It follows that $y^* = 2$. The optimal consumption point is $(x^*, y^*) = (5, 2)$. This tells us both the demand for X and for Y at current prices $(p_x, p_y) = (4, 10)$ and income $m = 40$.

The Optimal Consumption Point



4.5.11 Demand with Cobb-Douglas Utility I

To derive a demand function, we need to solve the consumer's problem for any price p_x. We consider the Cobb-Douglas Utility $u(x, y) = x^a y^b$ with $a, b > 0$. In this case, the marginal utilities are

$$\begin{aligned} MU_x &= a x^{a-1} y^b \\ MU_y &= b x^a y^{b-1}. \end{aligned}$$

The marginal rate of substitution is

$$MRS = \frac{MU_x}{MU_y} = \frac{a x^{a-1} y^b}{b x^a y^{b-1}} = \frac{a}{b}\frac{y}{x}.$$

4.5.12 Demand with Cobb-Douglas Utility II

Now

$$\begin{aligned} m &= p_x x + p_y y && \text{Budget equation} \\ \frac{a}{b}\frac{y}{x} &= \frac{p_x}{p_y} && \text{Optimality equation.} \end{aligned}$$

Multiplying the second equation by $(p_y x)$,

$$\begin{aligned} m &= p_x x + p_y y \\ \frac{a}{b} p_y y &= p_x x. \end{aligned}$$

Then we can write

$$\begin{aligned} m &= \frac{a}{b} p_y y + p_y y \\ &= \left(1 + \frac{a}{b}\right) p_y y \\ &= \left(\frac{a+b}{b}\right) p_y y. \end{aligned}$$

4.5.13 Demand and Cobb-Douglas Utility

Since

$$m = \left(\frac{a+b}{b}\right) p_y y,$$

we have

$$p_y y = \frac{b}{a+b} m,$$

showing that consumer spends a share $\frac{b}{a+b}$ of income on good y.

Demand for Y is then

$$y^* = \frac{b}{a+b}\left(\frac{m}{p_y}\right).$$

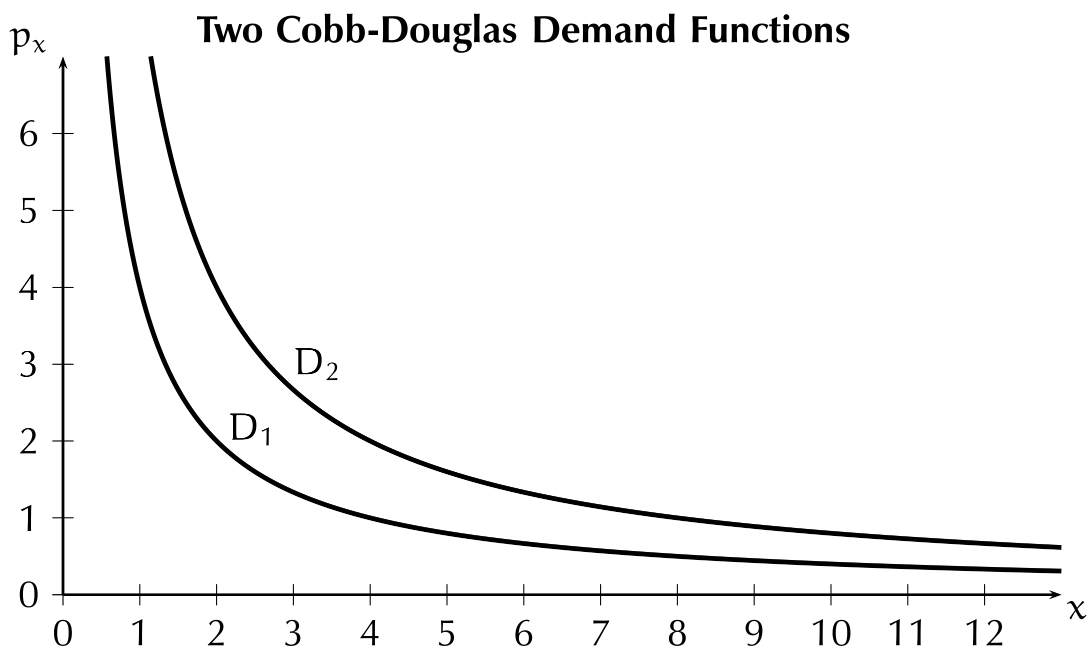
By the optimality equation

$$x^* = \frac{a}{a+b}\left(\frac{m}{p_x}\right).$$

The equations tell us not only demand for X and Y as a function of their own prices, but also tell us how demand **varies with the preference parameters** a and b, **with prices of both goods** p_x and p_y, and with **income**. In this case, p_y does not affect demand for X.

4.5.14 Graphing Cobb-Douglas Demand

To graph the demand for X, we must hold everything but p_x constant. For this example, we set $a = 1$, $b = 2$, and $m = 12$ (p_y does not appear in the demand equation, so it is irrelevant). Then demand D_1 is $x = 4/p_x$, which is solely a function of p_x. Inverse demand is $p_x = 4/x$. If we change income to $m = 24$, we get the demand curve D_2, $x = 8/p_x$.



Changing preferences (a, b) or income (m) will shift demand. If we change b to 4, holding $m = 12$, we get $x = 2.4/p_x$, a decrease in the demand for X (not illustrated).

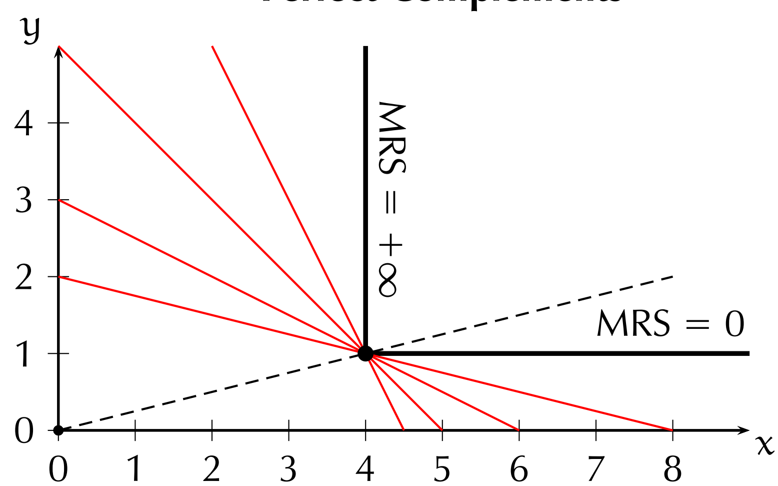
Notice that p_y does not affect the demand for X.

4.5.15 Demand with Perfect Complements I

We start with the utility function $u(x, y) = \min\{ax, by\}$ where $a, b > 0$. This means that goods X and Y are perfect complements in a ratio of a/b units of Y for every unit of X.

As we saw in section 4.3.16, there is an issue about how to interpret the marginal rate of substitution for such utility functions. When $ax > by$, we are on the horizontal part of some indifference curve and MRS $= 0$. When $ax < by$, we are on the vertical portion of some indifference curve, and MRS $= +\infty$. It is only when $ax = by$ that the tangent lines can have finite, non-zero slopes. As the figure below illustrates, tangents with any slope are possible there.

Perfect Complements



4.5.16 Demand with Perfect Complements II

This slope problem means that the only place that the optimality condition can be satisfied is along the line $ax = by$. We again have two equations:

$$\begin{aligned} m &= p_x x + p_y y && \text{Budget equation} \\ ax &= by && \text{Optimality equation.} \end{aligned}$$

Then $y = (a/b)x$, and the budget equation becomes

$$m = p_x x + \frac{a}{b} p_y x = \left(\frac{bp_x + ap_y}{b} \right) x$$

so

$$x^* = \frac{bm}{bp_x + ap_y}.$$

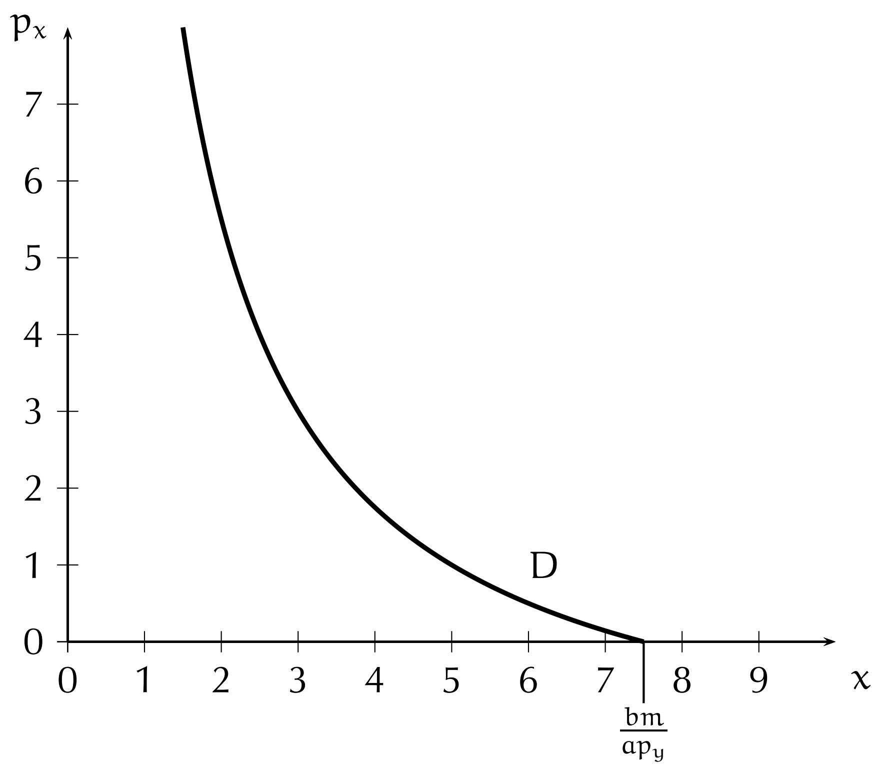
Using $y = (a/b)x$ yields

$$y^* = \frac{am}{bp_x + ap_y}.$$

4.5.17 Graphing Demand with Perfect Complements

To graph the demand for X, we must hold everything but p_x constant. We do that by setting $a = b = 1$, $p_y = 2$, and $m = 15$. Then demand is $x = 15/(2 + p_x)$. Inverse demand is $p_x = 15/x - 2$.

A Leontief Demand Function



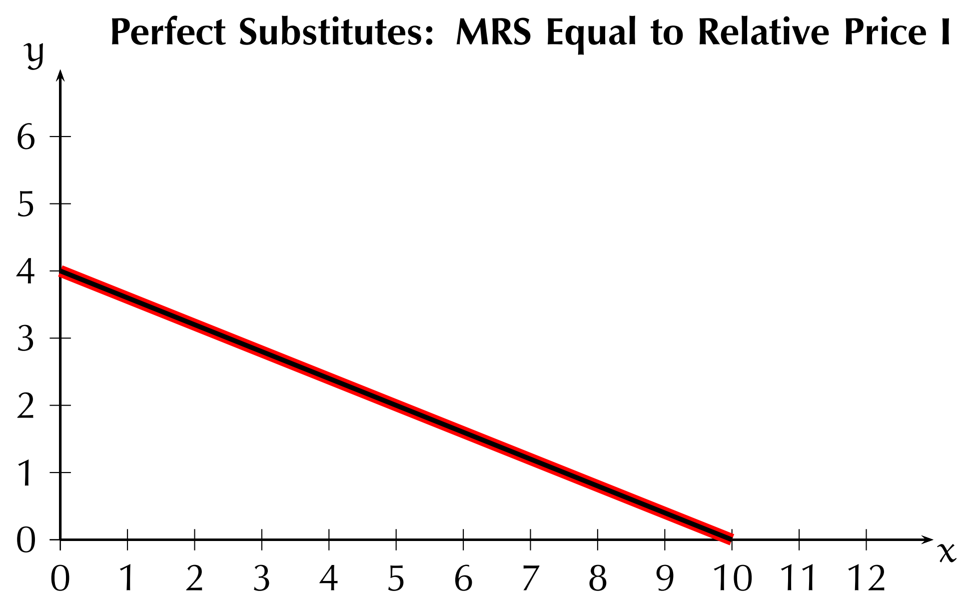
Changing preferences (a, b), income (m), or the price of the complementary good (p_y) will shift the demand curve.

4.5.18 Demand with Perfect Substitutes I

Perfect substitutes can be described by linear utility functions of the form $u(x, y) = ax + by$, where $a, b > 0$. The marginal rate of substitution is $\text{MRS} = a/b$. The optimality condition requires that

$$\frac{a}{b} = \text{MRS} = \frac{p_x}{p_y}.$$

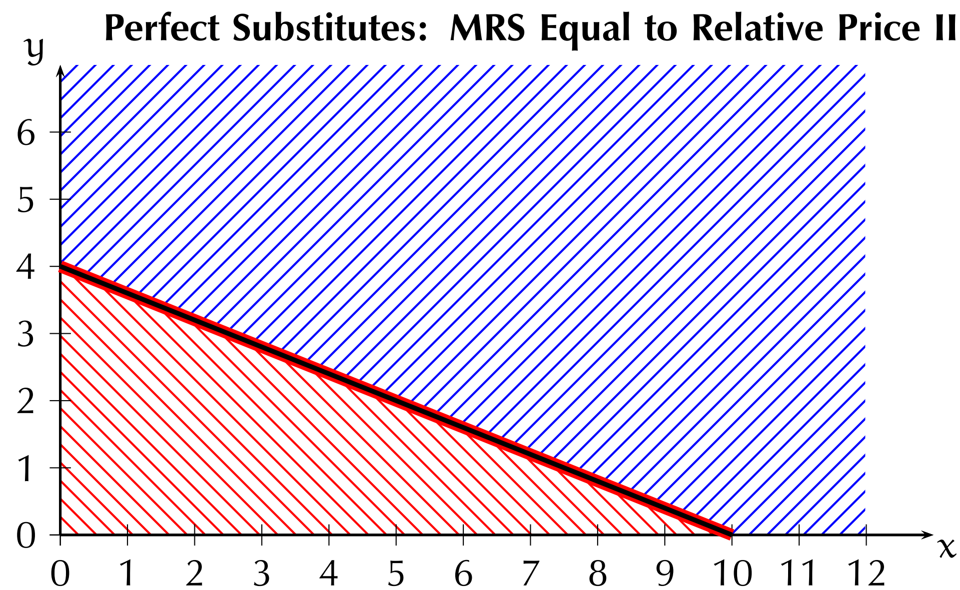
Since a and b are constants that describe preferences, there is only one relative price where the optimality condition holds. At any other relative prices, the optimality condition can never be satisfied. We'll look more at that in a bit. When optimality holds, the budget line coincides with the highest indifference curve that touches the budget set. Every point on the budget frontier is equally good, and maximizes utility.



On the diagram, the budget line is the thick red line and the maximizing indifference curve is shown as black, superimposed on the budget line. Any point on the budget line is an optimal point.

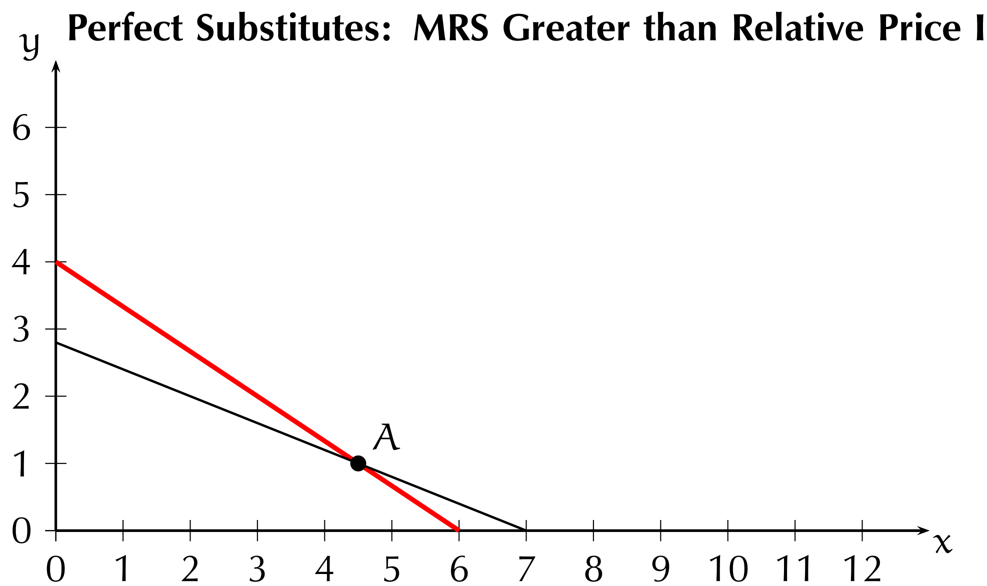
4.5.19 Demand with Perfect Substitutes II

We repeat the previous diagram, with the entire budget set hatched red and the consumption bundles with higher utility than the budget frontier hatched blue, allowing you to see the division into better bundles (blue), which the consumer cannot afford, and worse bundles (red hatched), which the consumer can afford. These regions are separated by the budget frontier/indifference curve.



4.5.20 Demand with Perfect Substitutes III

What happens if the budget frontier and indifference curves don't align? This happens in the diagram below, where the budget line is again shown in red.



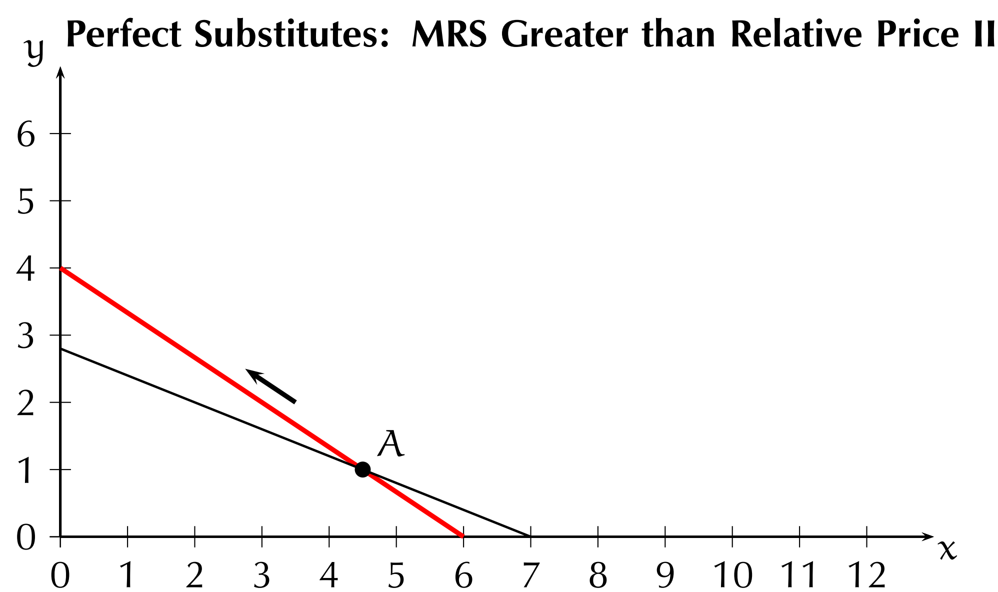
Although the point A is on the budget frontier, it fails optimality. We know the optimal point is on the budget frontier. Where is it? Which way should we move along the budget frontier to increase utility.

4.5.21 Demand with Perfect Substitutes IV

Here the relative price of X, p_x/p_y, is greater than the marginal rate of substitution. The relative price is the absolute slope of the red line, the budget frontier. This is clearly higher than marginal rate of substitution, the absolute slope of the black line, the budget frontier.

This means that the marginal value of X (MRS) is less than its cost (relative price), so the consumer benefits from cutting back on consumption of X

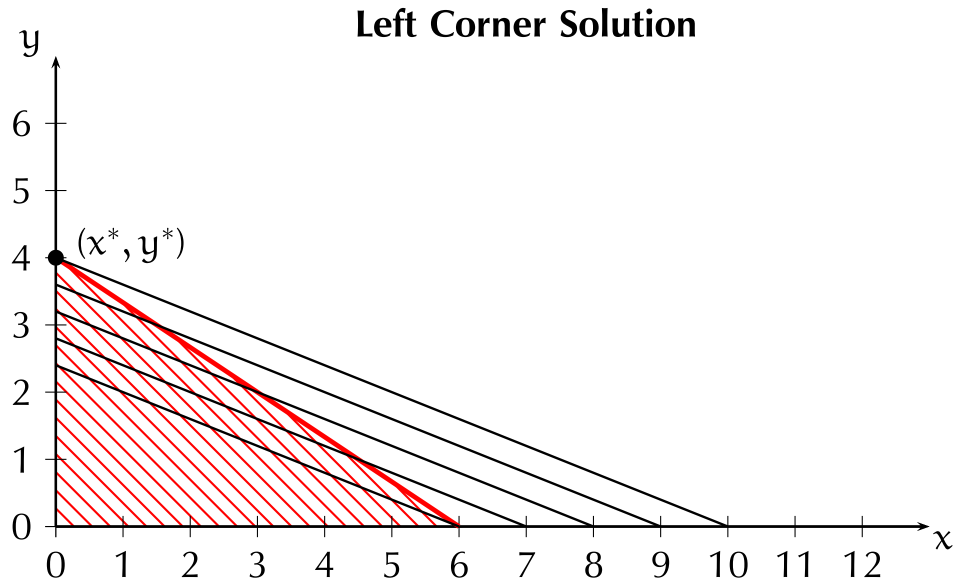
Our consumer needs to reduce purchases of X and increase purchases of Y. This will move them up to the left along the budget frontier.



How far left do we have to move?

4.5.22 Left Corner Solution

We move left as far as we can, until we can't reduce our consumption of X any more. That doesn't happen until we hit the Y-axis, where $x = 0$.



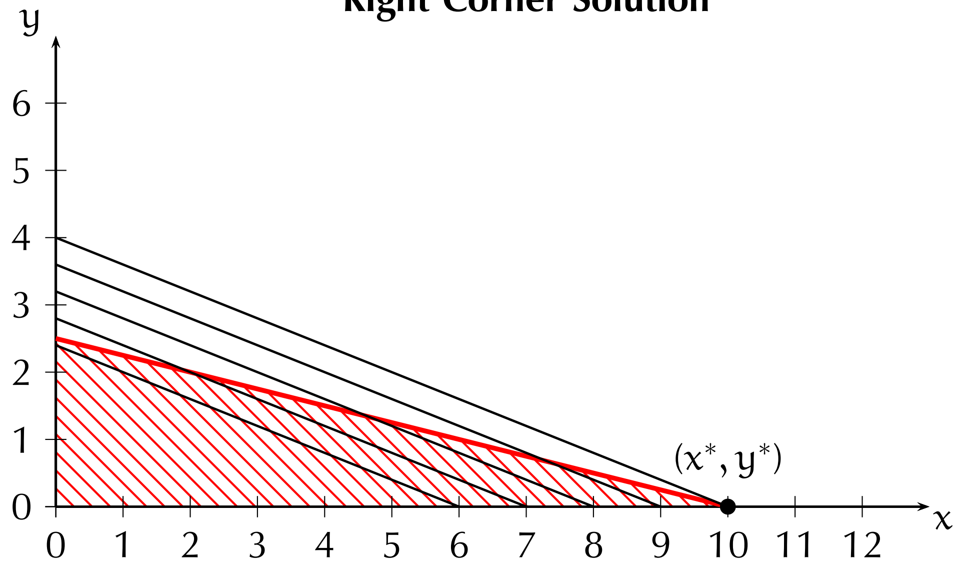
As you can see on the diagram, the highest indifference curve that touches the budget line is the one through $(x^*, y^*) = (0, 4)$, making it the optimal choice.

When the relative price is **always greater than** the MRS, the optimal consumption bundle will be the **vertical intercept** of the budget frontier. This solution at the corner of the budget set is called a **corner solution**.

4.5.23 Right Corner Solution

When the relative price is less than the (constant) marginal rate of substitution, the situation reverses. We move **right** as far as we can, not left. We stop when we can't reduce our consumption of Y any more. That doesn't happen until we hit the X-axis, where $y = 0$.

Right Corner Solution



As you can see on the diagram, the highest indifference curve that touches the budget line is the one through $(x^*, y^*) = (10, 0)$, making it the optimal choice.

When the relative price is **always less than** the MRS, the optimal consumption bundle will be the **horizontal intercept** of the budget frontier.

4.5.24 Corner Solution Summary

The **right corner** of the budget set, $(m/p_x, 0)$ **maximizes utility** whenever

$$\text{MRS}(m/p_x, 0) \geq \frac{p_x}{p_y}$$

at the corner. When MRS diminishes, it must be higher everywhere to the left of the corner, meaning that everywhere else on the budget frontier

$$\text{MRS} > \frac{p_x}{p_y}.$$

This means that the usual optimality condition that $\text{MRS} = p_x/p_y$ cannot be satisfied.

Similarly, if the **left corner** obeys

$$\text{MRS}(0, m/p_y) \leq \frac{p_x}{p_y},$$

a diminishing marginal rate of substitution means that everywhere else on the budget frontier

$$\text{MRS} < \frac{p_x}{p_y}.$$

Again, the usual optimality condition fails away from the corner. In this case, the **left corner maximizes utility**.

4.5.25 Consumer Choice with a Plethora of Goods

The bang for the buck formulation has the advantage that it applies no matter how many goods the consumer considers for purchase. The **goods bought** all **have the same marginal utility per dollar**. Any good where the marginal utility per dollar at zero consumption is lower than that common value will not be bought.

This is extremely helpful when dealing with fact that in modern economies, most consumers do not purchase any the vast majority of goods that are available. If you do not believe this, take a trip to your favorite supermarket, and starting counting the number of goods available to estimate the total (count each brand and variety separately, don't just say "bread"). Then ask yourself what fraction of those goods you, or even your family, have ever bought.

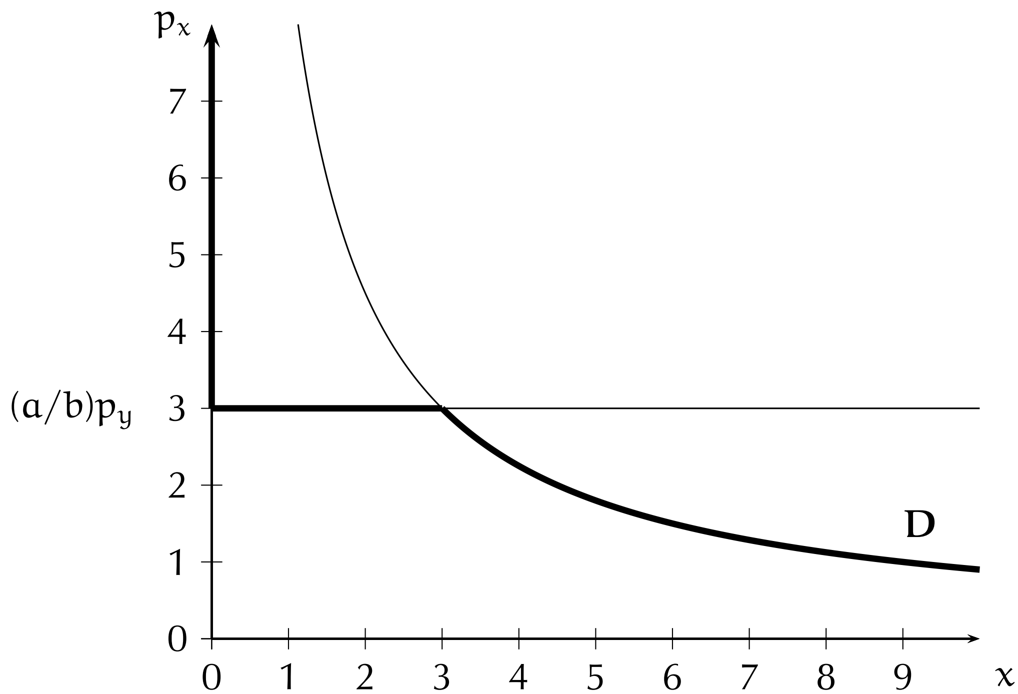
4.5.26 Graphing Demand with Perfect Substitutes I

To graph the demand for X, we again hold everything but p_x constant. To do this, we set $a = 3$, $b = 1$ (MRS = 3), $p_y = 1$, and $m = 9$. The key price p_x, where we switch from one corner to the other, is when $MRS = p_x/p_y$, meaning $p_x = (a/b)p_y$. For us, $(a/b)p_y = 3$.

When $p_x > (a/b)p_y = 2$, we consume no X. When $p_x < (a/b)p_y = 2$, we spend everything on X, so $x = m/p_x = 6/p_x$.

When $p_x = (a/b)p_y = 2$, we can consume any amount of X we can afford, anything from 0 to $m/2 = 3$.

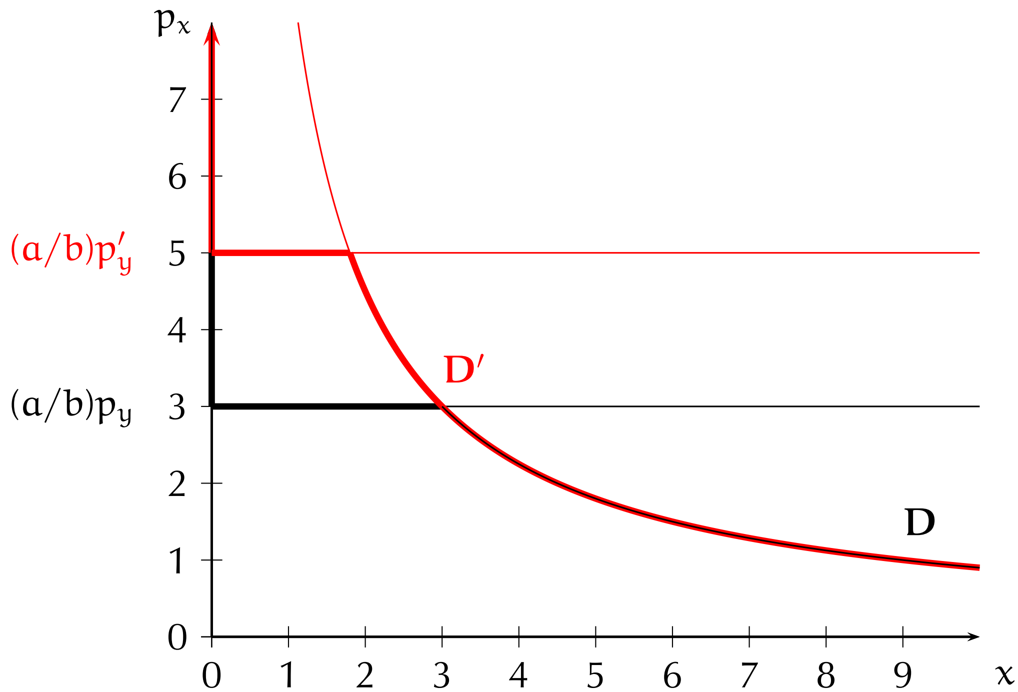
Demand under Perfect Substitutes



4.5.27 Graphing Demand with Perfect Substitutes II

Here we increase the price of Y from $p_y = 1$ to $p'_y = 5/3$. Notice the change in demand.

Demand Increase under Perfect Substitutes



See how demand switches from the one curve to the other when the price of X reaches the critical level $(a/b)p_y$.

Demand again depends on prices, the preference parameters, and income.

4.5.28 Summary of Calculated Demand Functions

Cobb-Douglas utility, $u(x, y) = x^a y^b$:

$$x^* = \frac{a}{a+b}\left(\frac{m}{p_x}\right).$$

Perfect Complements, $u(x, y) = \min\{ax, by\}$:

$$x^* = \frac{bm}{bp_x + ap_y}.$$

Perfect Substitutes (linear), $u(x, y) = ax + by$:

$$x^* = \begin{cases} 0 & \text{when } p_x > (a/b)p_y \\ [0, \frac{m}{p_x}] & \text{when } p_x = (a/b)p_y \\ \frac{m}{p_x} & \text{when } p_x < (a/b)p_y \end{cases}$$

4.5.29 Price Elasticities for Cobb-Douglas Demand

We can take a quick look at elasticities for Cobb-Douglas demand

$$x^* = \frac{a}{a+b}\left(\frac{m}{p_x}\right).$$

Spending on X is

$$p_x x^* = \left(\frac{a}{a+b}\right) m.$$

Since this is independent of p_x, demand is unit elastic. The price elasticity of demand is -1.

Demand is independent of p_y, so the cross-price elasticity of demand is 0.

4.5.30 Income Elasticity for Cobb-Douglas Demand

Finally, we consider income elasticity. To simplify the calculation, let $z = a/p_x(a+b)$, so that $x^* = zm$. Since z doesn't depend on income, $dx^*/dm = z$. Then

$$E_I = \frac{m}{x^*}\frac{dx^*}{dm} = \frac{m}{zm} \times z = +1.$$

This indicates that X is a normal good, and is right at the borderline of being a superior good (recall that superior means $E_I > 1$). The share of income spent on X is

$$\frac{p_x x^*}{m} = \frac{a}{a+b}.$$

4.5.31 Price Elasticity for Perfect Complements

09/22/22

Demand is

$$x^* = \frac{bm}{bp_x + ap_y}.$$

To find the price elasticity of demand, we first take the derivative with respect to p_x:

$$\begin{aligned}\frac{\partial x^*}{\partial p_x} &= -b\frac{bm}{(bp_x + ap_y)^2} \\ &= \left(\frac{-b}{bp_x + ap_y}\right)\left(\frac{bm}{bp_x + ap_y}\right) \\ &= \frac{-b}{bp_x + ap_y}x^*.\end{aligned}$$

Then

$$|E_d| = \left|\frac{p_x}{x^*}\frac{\partial x^*}{\partial p_x}\right| = \frac{bp_x}{bp_x + ap_y} < 1$$

Demand is always inelastic.

4.5.32 Cross-Price and Income Elasticity for Perfect Complements

Demand is

$$x^* = \frac{bm}{bp_x + ap_y}.$$

To find the cross-price elasticity of demand, we first take the derivative with respect to p_y:

$$\begin{aligned}
\frac{\partial x^*}{\partial p_y} &= -a\frac{bm}{(bp_x + ap_y)^2} \\
&= \left(\frac{-a}{bp_x + ap_y}\right)\left(\frac{bm}{bp_x + ap_y}\right) \\
&= \frac{-a}{bp_x + ap_y}x^*.
\end{aligned}$$

Then

$$E_{xy} = \frac{p_y}{x^*}\frac{\partial x^*}{\partial p_y} = \frac{-ap_y}{bp_x + ap_y} < 0.$$

This is always negative, indicating that X and Y are complements.

Since x^* is again proportional to m, its income elasticity is again $+1$.

5.1 Chapter 5: Individual and Market Demand

In this chapter we take a closer look at demand, focusing on general characteristics rather than specific demand curves.

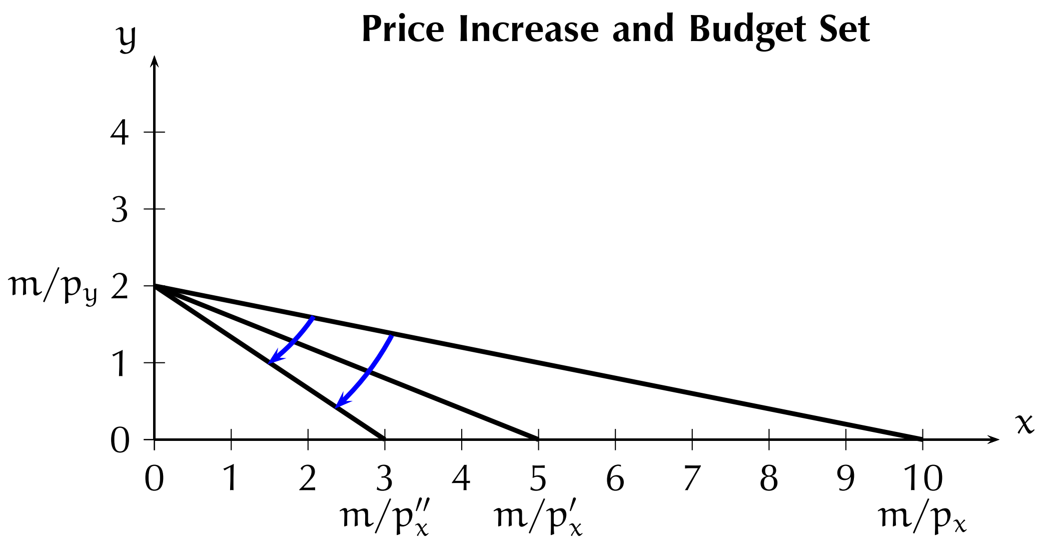
We know that demand describes the relation between the price of a product and the quantity demanded—the quantity buyers wish to purchase—all while holding other relevant variables constant. At the end of the last chapter, we examined the effects of some of these other variables—income, prices of other goods, preference parameters.

Quantity demanded depends on many variables, not just price. One of these variables, income, is tied up with demand itself. This is because changes in the price of X lead to changes in real income.

5.1.1 Price and Real Income

We earlier considered the budget set with income \$10, $p_y = \$5$ and three different prices for X: $p_x = \$1$, $p'_x = \$2$, and $p''_x = \$3.33$.

As the price of X increases, the budget line rotates clockwise about the Y anchor point at (0, 2).



The budget set shrinks as the price of X increases, and expands as the price decreases. Since there are more options available at lower prices, and fewer at higher prices, we can say that real income (the consumer's consumption possibilities) increased as prices fell, and shrank as prices rose.

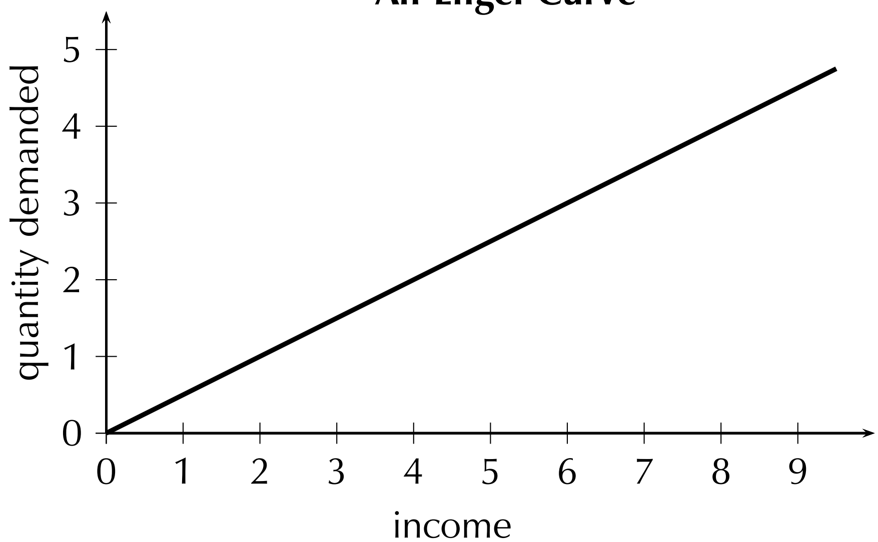
When we change the price of X, it doesn't make sense to think about income being unchanged. Even when the dollar income is unchanged, real income still changes. The effect due to changing income is called the income effect. We will need to take the income effect into account when analyzing demand.

5.1.2 The Engel Curve

We use an **Engel curve** to focus on the effect of income. The Engel curve graphs quantity demanded as a function of income rather than price.[1] All other variables, including price, are held constant.

For the demand functions we studied in detail, quantity demanded is proportional to income, as illustrated below.

An Engel Curve



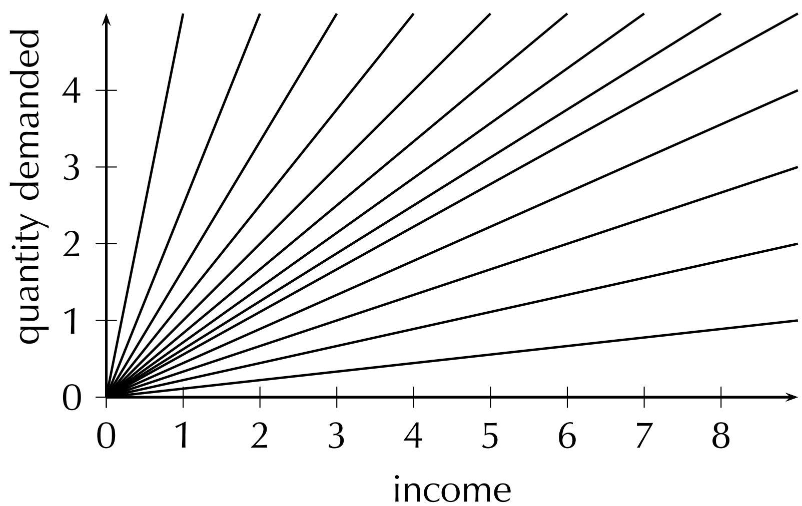
A positive slope corresponds to a normal good ($E_I > 0$), while a negative slopes indicates the good is inferior ($E_I < 0$). Straight lines through the origin indicate the income elasticity $E_I = 1$.

[1] Ernst Engel (1821–1896) was a German statistician who was the first to systematically study the relationship between expenditures and income. He discovered that consumer spend a smaller share of their on food as income increases. This is known as Engel's Law.

5.1.3 Unit Income Elastic Engel Curves

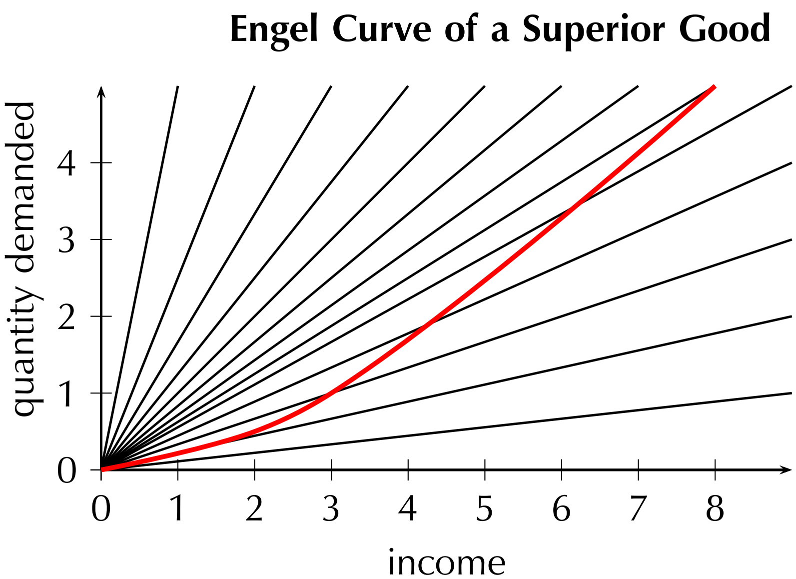
The diagram illustrates some unit income elastic Engel curves. Engel curves for luxury goods will cross the unit elastic lines that lie to the left of them, while goods with income elasticity less than one cross unit elastic lines lying to the right.

A Gaggle of Engel Curves



5.1.4 Engel Curve of a Superior Good

For example, the red Engel curve below keeps crossing unit elastic Engel curves to the left of it. It must be a superior good.

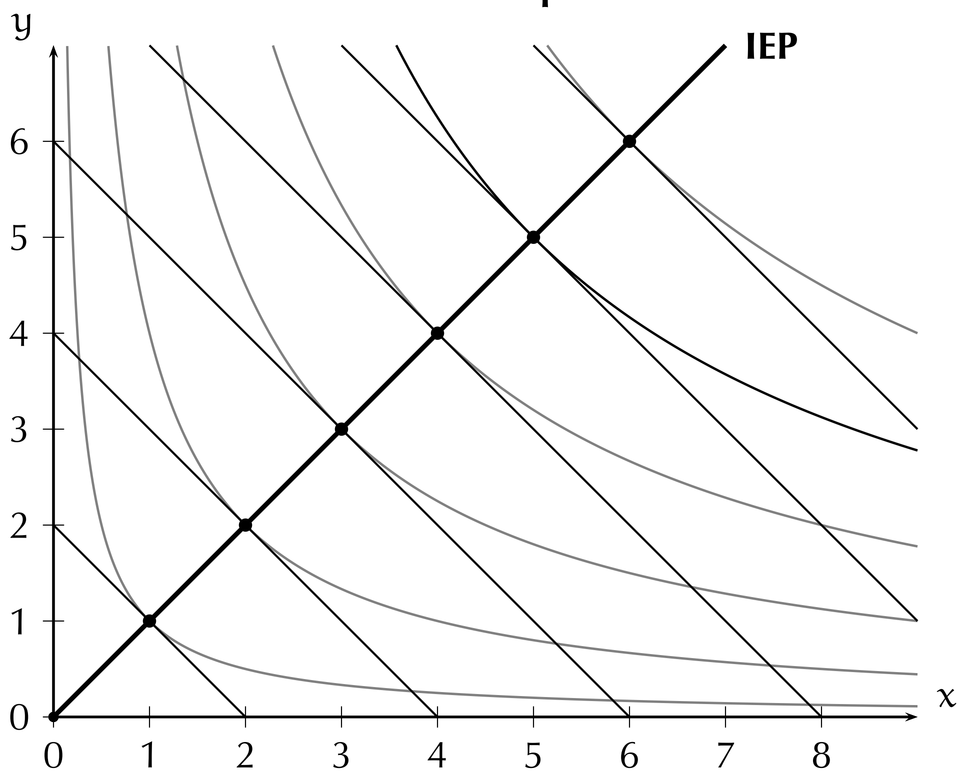


5.1.5 Income Expansion Paths

A second way of portraying income effects is the **income expansion path (IEP)**. The income expansion path is graphed in X−Y space. It is the locus of optimal consumption choices as income changes, for fixed prices and a fixed indifference map.

Below, we use a Cobb-Douglas indifference map and prices $p_x = p_y = 1$. The dots indicate the optimal choices for incomes of \$2, \$4, \$6, \$8, \$10, and \$12.

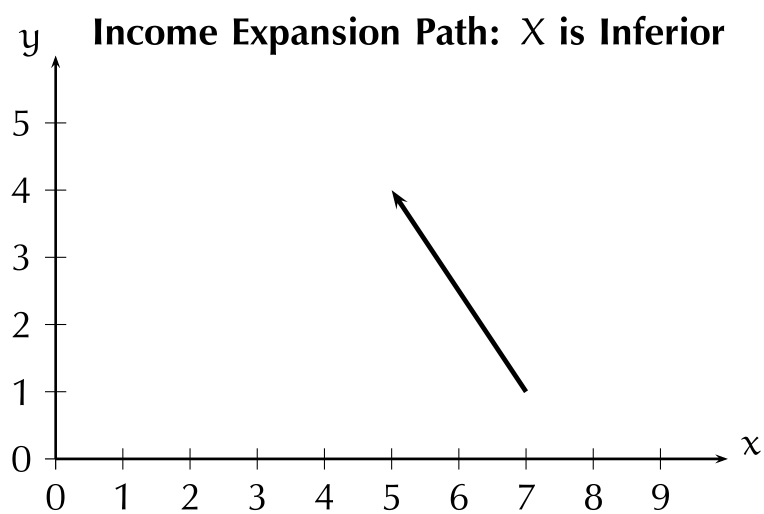
An Income Expansion Path



5.1.6 Interpreting Income Expansion Paths

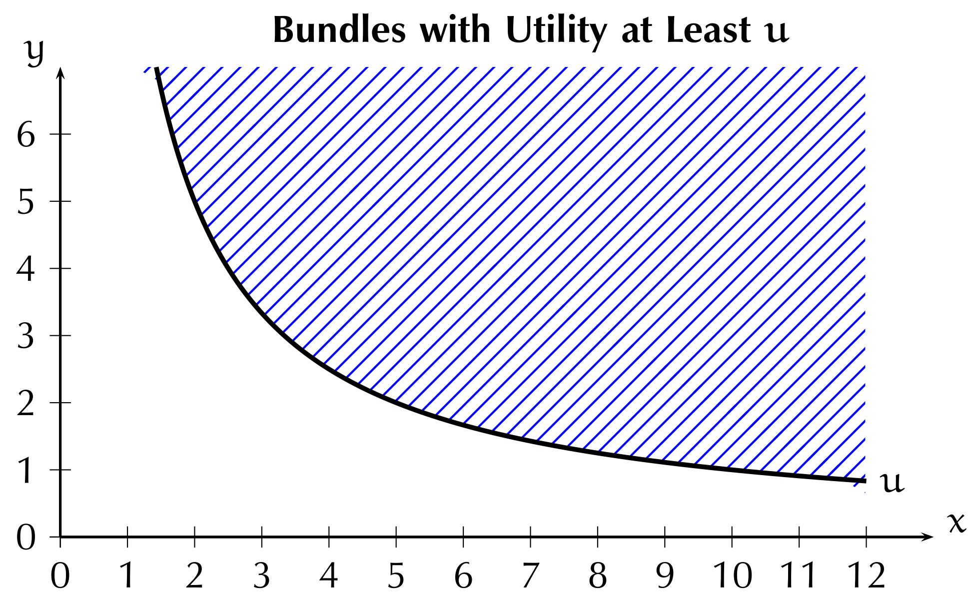
The income expansion path is positively sloped at some income level whenever both goods are normal there. If the slope is negative, one of the goods must be inferior. To tell which, you have to follow the path as income increases. If the path moves toward the vertical axis, X is inferior. If the path moves toward the horizontal axis, Y is inferior.

The diagram shows a portion of an income expansion path where X is inferior. If the arrow were reversed, Y would be inferior.



5.1.7 Expenditure Minimization I

To deal with the issue of changes in real income, we turn the utility maximization problem on its head. Instead of maximizing utility over a budget set, we try to minimize cost over the set of better consumption bundles. To do this, we start with a utility level u and consider all consumption bundles that give the consumer at least utility u.



Then we try to find the **cheapest** such consumption bundle, given prices p_x and p_y, rather than the **most valuable** consumption bundle, as with utility maximization. Here we try to minimize consumer expenditure, the cost to the consumer.

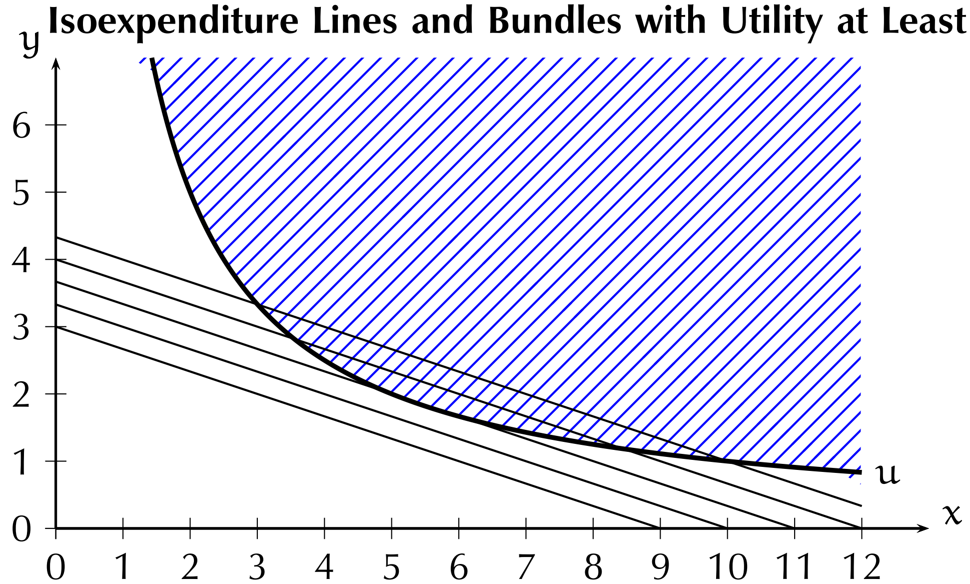
This sort of trick, where we replace maximization problems by closely related minimization problems, and vice-versa, is one of the most useful in the microeconomist's toolbox. There are many variations on it and we could spend the rest of the semester exploring these variations. We don't have time for that, so we will just focus on one such problem, cost minimization.

5.1.8 Expenditure Minimization II

Let $p_x = 2$ and $p_y = 6$, yielding a relative price of $1/3$. If we take any line of slope $-1/3$, all of the consumption bundles on that line cost the same. We refer to these budget-like lines as **isocost lines** or **isoexpenditure lines**. The expenditure associated with an isoexpenditure line is p_x times its horizontal intercept, or p_y times the vertical intercept.

It's pretty clear that we can do better than the top two isoexpenditure lines by moving to a lower isoexpenditure line. The bottom two do not give us enough utility, and the middle one comes very close to solving the problem. It almost does.

Isoexpenditure Lines and Bundles with Utility at Least u



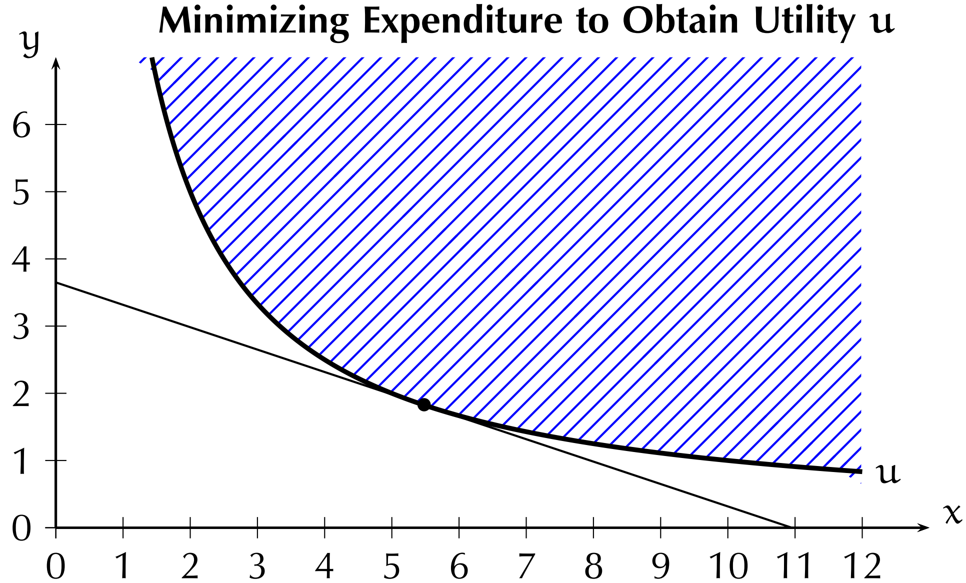
5.1.9 Cost Minimization: Solution

As you've problem guessed from the previous diagram, the solution involves a tangency between the isoexpenditure line and the indifference curve. It also has to yield utility u.

$$\begin{aligned} MRS(x,y) = \frac{MU_x}{MU_y} &= \frac{p_x}{p_y} && \text{Optimality equation.} \\ u &= u(x,y) && \text{Utility constraint} \end{aligned}$$

The expenditure minimizing point is $(x^*, y^*) = (\sqrt{30}, \sqrt{30}/3)$, which is approximately $(5.48, 1.83)$ and the corresponding expenditure is $2\sqrt{30} + 6\sqrt{30}/3 = 4\sqrt{30} \approx 21.91$.

Minimizing Expenditure to Obtain Utility u



5.1.10 Minimum Expenditure Function

The minimum expenditure we calculated depends on prices p_x and p_y, and on the utility level u. We use the notation $e(p_x, p_y, u)$ for the minimum expenditure.

Here e stands for expenditure. We prefer to not use the word "cost" here since we want to reserve the term "cost function" for firms, not consumers. Using different terms helps avoid confusion. In fact, a firm's cost function will be calculated the same way. We will see how to do the same kind of calculation later for firms.

5.1.11 The True Cost-of-Living Index

We can use the expenditure function to define a price index, the **true cost-of-living index**. Here we have two sets of prices, (p_x, p_y) and (p'_x, p'_y). We want to measure how much prices have risen or fallen.

If both prices increase by the same percentage, its easy. Just use that percentage. In the real world, with many prices, this just doesn't happen. When prices in general rise, some prices may actually go down, some go up, some go up a lot. In August 2022, the price of food went up 0.8% while the price of gasoline went down 12.2%. How can we determine what happened to prices in general? We use a price index to measure this.[2]

We're interested in how price changes affect consumers. The most natural measure is to ask how much a consumer's income would have to rise to be able to maintain their standard of living, their utility level. We know it takes income of $e(p_x, p_y, u)$ to get utility u at prices (p_x, p_y) and that it takes income $e(p'_x, p'_y, u)$ to get the same utility at prices (p'_x, p'_y). So we just take the ratio. The true cost-of-living index P is defined by[3]

$$P = \frac{e(p'_x, p'_y, u)}{e(p_x, p_y, u)}.$$

[2] One commonly used price index is the consumer price index (CPI). The CPI was virtually unchanged in August. However, it's not a good measure of month-to-month changes because two parts of it, food and energy prices, are both highly volatile. A better measure for month-to-month changes is the price of all items except food and energy. The seasonally adjusted version should be slightly better still. That went up 0.6% in August, for an annual rate of about 7.4%. Inflation works like compound interest, so the calculation is $(1 + 0.6\%)^{12} = (1.006)^{12} \approx 1.0744 = 1 + 7.44\%$.

[3] The true cost-of-living index was introduced by Alexander A. Konüs in 1924.

5.1.12 Price Indices in Practice

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The problem with the true cost-of-living index is twofold. We can't observe utility levels, and different consumers have different preferences (implying different price indices).

We can observe consumption bundles, or at least average consumption bundles. The consumer price index uses an average consumption bundle. We measure its cost at the old prices and new, and then take the ratio to get the consumer price index. This is roughly how the consumer price index is calculated, but sometimes involves refinements that get us closer to the true cost-of-living index. Some of these, such as the treatment of housing prices are controversial among the public, but are justified by economic theory. (This is not to say they are perfect.)

The consumption bundle is updated frequently so that it will approximate the cost minimizing bundle.

The Bureau of Labor Statistics calculates various price indices. As part of that calculation they have to figure out an average consumption bundle. They collect data on all sorts of consumer expenditures. A recent example can be found at

https://www.bls.gov/news.release/pdf/cpi.pdf

Check out the tables near the end of the pdf to see how spending is broken down.

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