Intermediate Microeconomics — Week 6

Professor Boyd **Sept. 27 & 29, 2022**

5.0 Answers to Selected Questions from Quiz #2 09/27/22

Due to impeding closure of FIU for Hurrican Ian, only a small group attended class today. We started by going over a selection of questions from last Thursday's quiz.

5.0.1 Question #2

The correct answer to question #2 is **C**, that Figure 2.4 violates the assumption of transitivity.

Answers **A**, **B**, and **D** are incorrect. The indifference curves are convex to the origin, so answer **A** cannot be correct. The fact that we can form an indifference map shows that all bundles are rankable, so **B** fails. Finally, option **D** describes a diminishing marginal rate of substitution, which follows from the convexity of the indifference curves.

Comparing bundles **C** and **B**, we find that bundle **C** is better than **B** by monotonicity. But bundle **B** is indifferent to $\boldsymbol{A}$. If preferences are transitive, it follows that bundle **C** is better than $\boldsymbol{A}$. But bundles $\boldsymbol{A}$ and **C** are both on the same U_b indifference curve. Neither is better than the other. This shows that transitivity is violated. Answer **C** is the correct one

5.0.2 Question #3

In each of I, II, and III, Carleton is able to rank silk ties and cufflinks. For III, considering them tied, equally valuable is also a ranking. Thus answer **A** is correct.

5.0.3 Question #4

The book handles taxes a bit differently from the way we did in class. It vertically shifts the supply curve by the amount of the tax instead of using a tax wedge.

We use the intersection of the shifted supply curve and unshifted demand curve to find the buyer's price \$315 and quantity 800. The seller's after-tax price must be less by the amount of the tax. In other words, it is the price on the unshifted supply curve at $q = 800$, \$125. The tax is the difference between the two prices $\$315 - \$125 = \$190$, which is answer **B**.

5.0.4 Question #6

Kyle's utility function is $U(X, Y) = 4X + 3.7Y$. The marginal utility is the partial derivative of utility with respect to X,

$$\frac{\partial U}{\partial X} = 4.$$

Alternatively, we may compute

$$\begin{aligned}
\Delta U &= U(X + \Delta X, Y) - U(X, Y) \\
&= 4(X + \Delta X) + 3.7Y - 4X - 3.7Y \\
&= 4\Delta X.
\end{aligned}$$

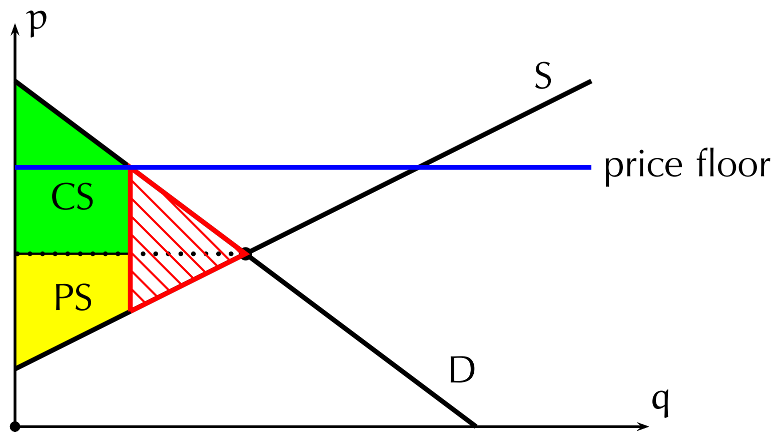
Then

$$MU_X = \frac{\Delta U}{\Delta X} = 4.$$

Either way, we get $MU_X = 4$, which is answer **D**.

5.0.5 Question #7 I

I found it helpful to sketch a diagram illustrating the situation.



To find the area of the red deadweight loss triangle, we need to know the price floor itself ($15,000), the quantity demanded at the price floor, the supply price for that quantity demanded, and the equilibrium quantity.

The demand equation is $Q^D = 12,000 - 0.4P$. Setting $P = \$15,000$, we find $Q^D = 12,000 - 6,000 = 6,000$. We substitute this quantity in the supply equation to find the corresponding supply price. Since $Q^S = 0.1P + 5,000$, we se $6,000 = 0.1P + 5,000$, so $P = \$10,000$ is the supply price. This tells that the (vertical) base of the deadweight loss triangle is $\$15,000 - \$10,000 = \$5,000$.

5.0.6 Question #7 II

To find the equilibrium quantity, we set $Q^S = Q^D$ and solve for P. Thus

$$\begin{aligned} 12,000 - 0.4P &= 0.1P + 5,000 \\ 7,000 &= 0.5P \\ 14,000 &= P \end{aligned}$$

Substituting back in the supply equation, $Q^* = 0.1(14,000) + 5,000 = 6,400$. The height of the deadweight loss triangle is $6,400 - 6,000 = 400$.

The deadweight loss is half the base times the height

$$\frac{1}{2}400 \times 5,000 = 200 \times 5,000 = 1,000,000.$$

The correct answer is **D**.

5.1.12 Price Indices in Practice

Repeated from 9/22/22

The problem with the true cost-of-living index is twofold. We can't observe utility levels, and different consumers have different preferences (implying different price indices). In an attempt to deal with the latter problem, the Bureau of Labor Statistics (BLS) computes several different consumer price indices for consumers in different situations.

Although we can't observe utility, we can observe consumption bundles. It's even easier to observe average consumption bundles. The consumer price index uses an average consumption bundle. We measure its cost at the old prices and new, and then take the ratio to get the consumer price index.

That is roughly how the consumer price index is calculated, but the calculation also involves refinements that get us closer to the true cost-of-living index. Some of these, such as the treatment of housing prices are controversial among the public, but are justified by economic theory. Housing prices are ignored, and housing is valued at the rental price for comparable units. Another issue that has attracted the attention of non-BLS economists is how to adjust the price index when old goods are replaced by new goods.

The consumption bundle is updated periodically to better approximate the average cost minimizing bundle.

The Bureau of Labor Statistics calculates various price indices. As part of that calculation they have to figure out an average consumption bundle. They collect data on all sorts of consumer expenditures to do this. A recent example can be found at

https://www.bls.gov/news.release/pdf/cpi.pdf

Check out the tables near the end of the pdf to see how spending is broken down.

5.2 Demand: Income and Substitution Effects 09/29/22

We're now ready to decompose the effect of a price change into two parts: a pure price change, and an induced income change.

To do that, we first consider the effect of a price change, holding **real income** constant. By that, we mean that we adjust income so that the consumer can just afford to buy a consumption bundle that is equally good as the old consumption bundle, but no better.

The expenditure function can be used to describe this. If the consumer had utility u under the old prices (p_x, p_y), income after prices change to $\mathbf{p}'$ must be the minimum expenditure function $e(p'_x, p'_y, u)$.[1] We look for the tangency between the budget line with income $e(p'_x, p'_y, u)$ and the indifference curve u. That bundle is referred to as the **compenstated** or **Hicksian demand bundle**.[2]

The change from the original consumption bundle to the compensated demand bundle, where we used the new prices and income, but old utility, defines the **substitution effect**. To get the actual demand point, we let income adjust from $e(p'_x, p'_y, u)$ back to its original level m. The resulting change in consumption is the **income effect**.

This decomposition of the effect of a price change into substitution and income effects is called the **Slutsky decomposition**.[3]

[1] The minimum expenditure function was the last major piece of modern consumer theory to be developed. It was introduced by Lionel McKenzie (1919–2010) in his 1957 paper in the *Review of Economic Studies*, "Demand Theory without a Utility Index".

[2] Sir John Hicks (1904–1989) received the Nobel prize in 1972. His 1939 book, *Value and Capital* does consumer theory in the modern fashion, using indifference curves. Hicks was also responsible for creating the IS-LM model in his 1937 paper in *Econometrica*, "Mr. Keynes and the 'Classics': A Suggested Interpretation".

[3] Named after the Russian economist Evgeny (Eugen) Slutsky (1880–1948) who was the first to study this decomposition.

5.3 The Effect of a Price Increase

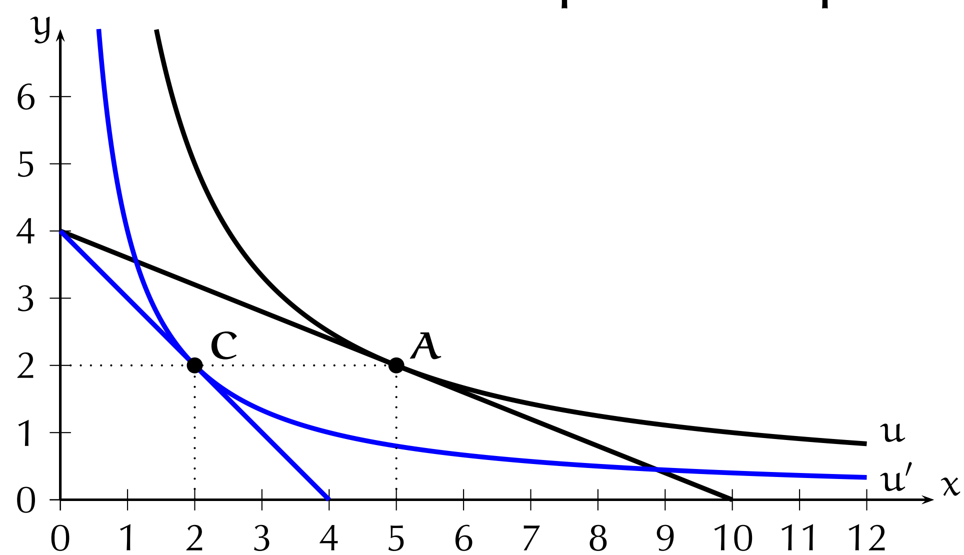
We start with a utility maximization problem where $p_x = \$4$, $p_y = \$10$ and $m = \$40$. Utility $u(x, y) = xy$ is maximized at bundle $\boldsymbol{A} = (5, 2)$ where MRS $= p_x/p_y = 2/5$. Utility at $\boldsymbol{A}$ is $u = 10$.

Then we increase the price of X to $p'_x = \$10$, leaving both $p_y = \$10$ and $m = \$40$ unchanged. The utility maximum moves to bundle $\mathbf{C} = (2, 2)$ where MRS $= p'_x/p_y = 1$. The utility level after the price change

$$u' = u(\mathbf{C}) = 2 \times 2 = 4$$

is less than utility before the price increase ($u = 10$), reflecting the fact that real income has fallen.

Price Increase affects Optimal Consumption



The quantity of X demanded has fallen from 5 to 2. Since utility is Cobb-Douglas, the change in the price of X does not affect the quantity of Y demanded, which remains at 2.

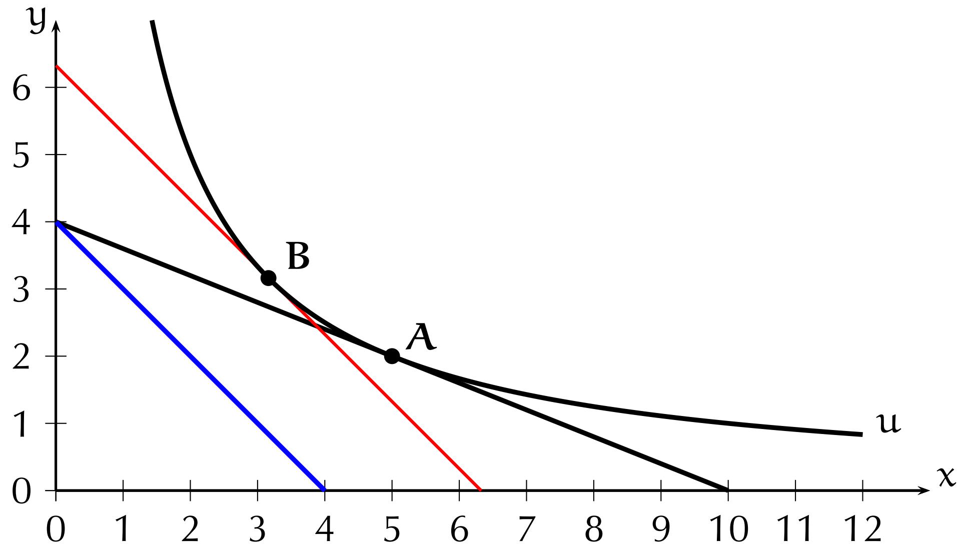
5.3.1 The Substitution Effect

To find the substitution effect, we hold real income constant by staying on the indifference curve u, but use the new prices.

We look for the point on the indifference curve through bundle $\mathbf{A}$ where the marginal rate of substitution is the same new relative price $p'_x/p_y = 10/10 = 1$. That is an increase over the old relative price of $2/5$, so we **must move up and left** from $\mathbf{A}$ to bundle $\mathbf{B}$.

The red line has the same slope as the new (blue) budget line, and is tangent to the indifference curve through $\mathbf{A}$. The movement from $\mathbf{A}$ to $\mathbf{B}$ is the **substitution effect**.

Substitution Effect of a Price Increase

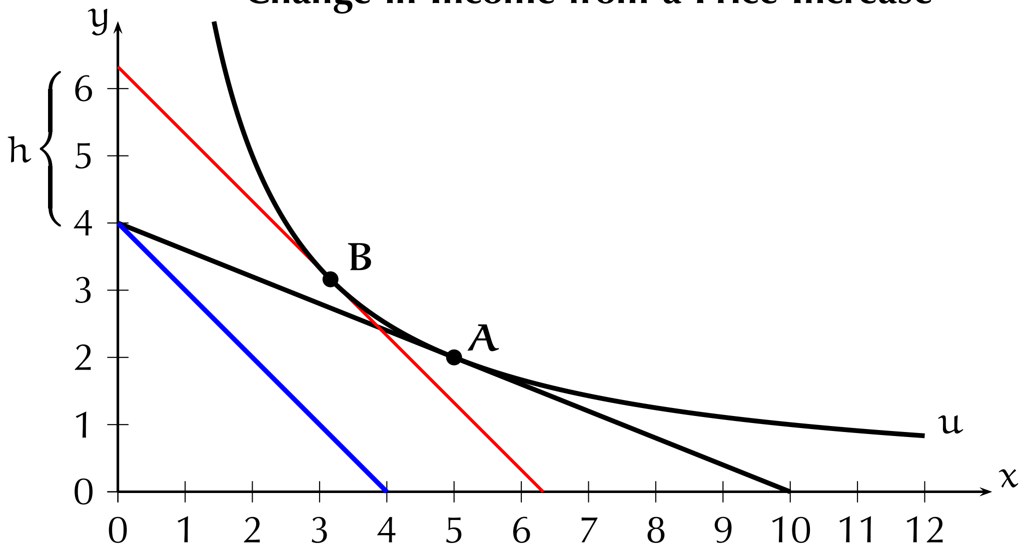


5.3.2 The Change in Real Income

We can now measure the change in real income. We measure it along the Y-axis because the price of Y has not changed. The change in real income is the vertical distance h between the two budget lines times the price of y. I.e., the increase in the price of X means that real income falls by hp_y (to the blue budget line).

The red budget line has income $\$40 + hp_y$. We have compensated the consumer for the lost real income. Because of this, bundle **B** is called the **compensated demand point** or **Hicksian demand point**.

Change in Income from a Price Increase

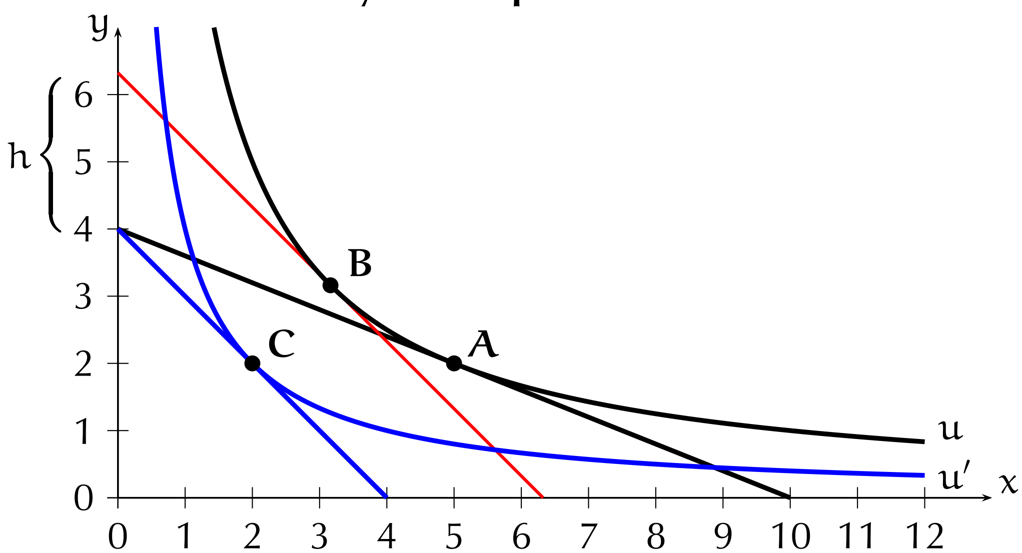


5.3.3 The Income Effect

We have seen that the increase in the price of X has reduced real income. To find the full effect of the change in price, we have to include the change in real income. We reduce income by hp_y, moving down to the blue budget line, where the optimal choice is bundle **C**. The movement from bundle **B** to **C** is the **income effect**.

We have decomposed the effects of the price change, which moves the demand point from bundle **A** to **C** into the substitution effect, $\mathbf{A} \rightarrow \mathbf{B}$, and the income effect, $\mathbf{B} \rightarrow \mathbf{C}$.

Slutsky Decomposition: Price Increase



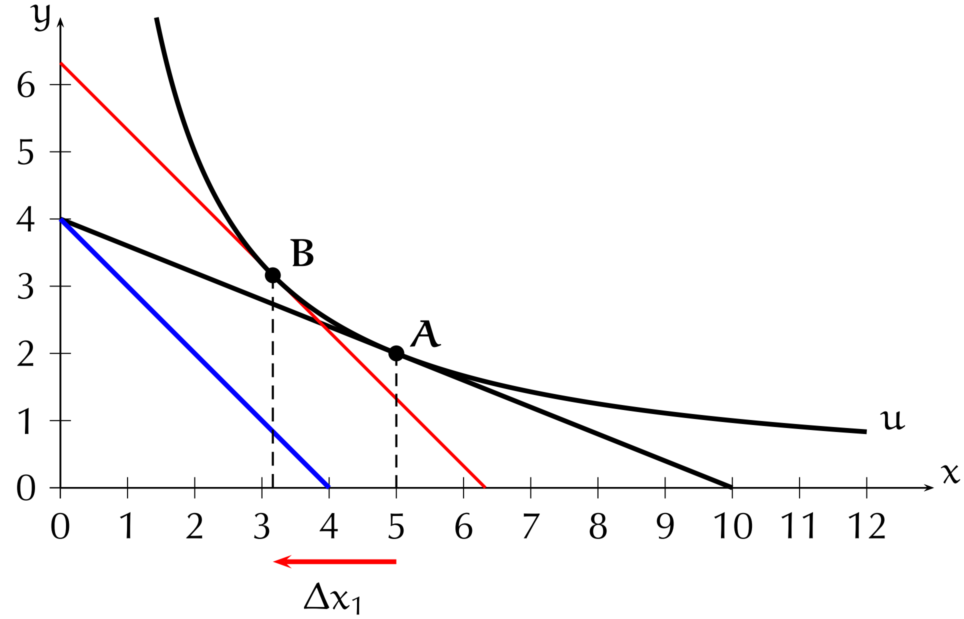
Our plan is to analyze the income and substitution effects separately, and then put them together. We start with the substitution effect and take a look at compensated demand.

5.3.4 Substitution and the Law of Demand

The substitution effect lies at the heart of the Law of Demand. When the price of X increases, we must move up and left from point **A** to find the new tangency. This works because the marginal rate of substitution diminishes as we move down to the right, and increases as we move up to the left. Since we move left, the quantity of X demanded falls by $|\Delta x_1|$.

Had the price of X fallen,, we would move down to the right, and the quantity of X demanded would increase. This tells us that compensated demand has a negative slope. It obeys the Law of Demand.

Substitution and the Law of Demand

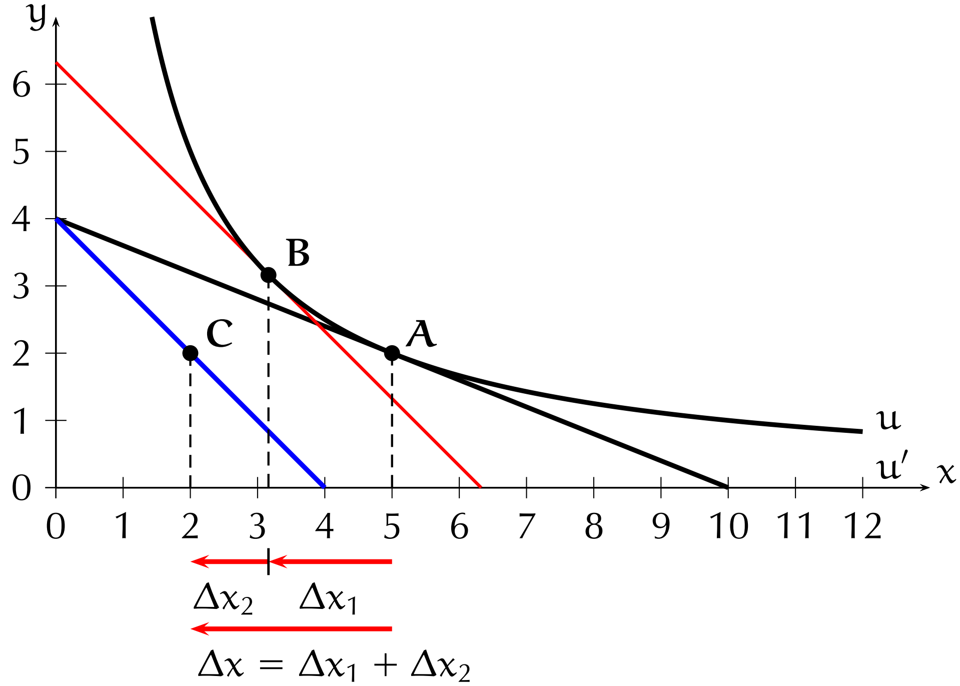


5.3.5 The Law of Demand for a Normal Good

Suppose X is a normal good. Then the move from bundle **B** to **C** would also reduce quantity demanded because income is lower. With X normal, the lower income reduces quantity demand by $|\Delta x_2|$.

Both the substitution effect and income effect reduce demand in this case. When we put them together, they reinforce each other. The quantity demanded falls, twice. In such cases an increase in the price of X leads to a smaller quantity of X demanded, and a decrease in price increases the quantity demanded. The Law of Demand holds.

The Law of Demand when X is Normal



5.3.6 The Income Effect for an Inferior Good

But what if X is an inferior good? Then the move from bundle **B** to **C** would **increase quantity demanded** due to a reduction in income. As long as that increase in quantity demanded is smaller than the reduction from the substitution effect, the combined effect still reduces quantity demanded. The Law of Demand still holds.

5.3.7 The Income Effect for a Strongly Inferior Good

But if the income effect on X is bigger than the substitution effect, we would have a problem. The overall effect on quantity of X demanded would **increase** quantity demanded! Can that really happen?

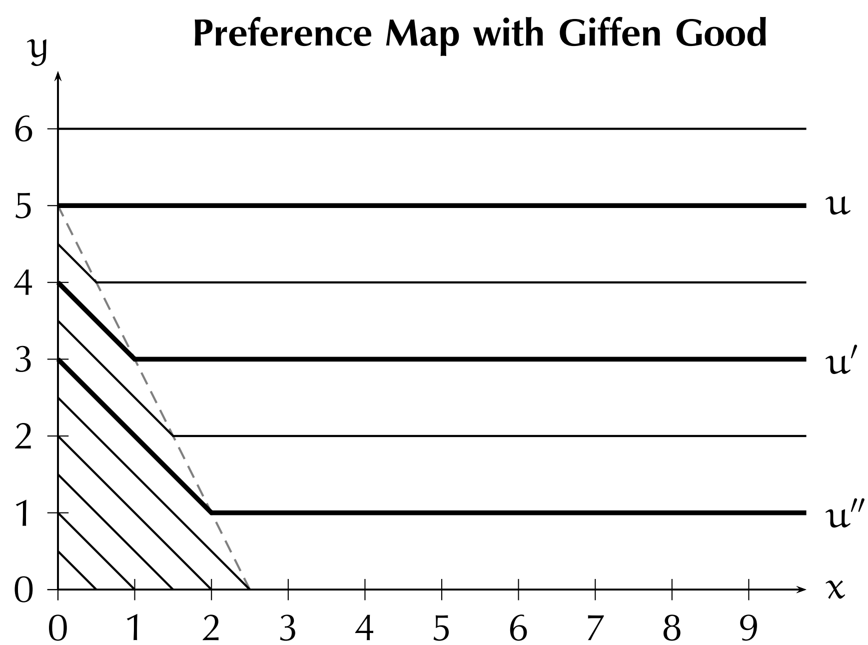
Yes it can!

There are preferences where a price increase can lead to an increase in quantity demanded of X. When that happens, we say that X is a **Giffen good**.[4]

[4] They are named after Sir Robert Giffen (1837–1910). He was a Scottish statistician and economist who collected data on spending by the poor during the Victorian era. That data suggested demand could have a positive slope for certain prices. Potato demand during the Irish Potato Famine (1845–1852) has sometimes been considered an example of a Giffen good, but the most recent studies suggest it is not a Giffen good.

5.3.8 A Giffen Preference Map

The indifference map below is the indifference map of a Giffen good. They are horizontal to the right of the dashed line ($y = 5 - 2x$) and have slope -1 to its left.

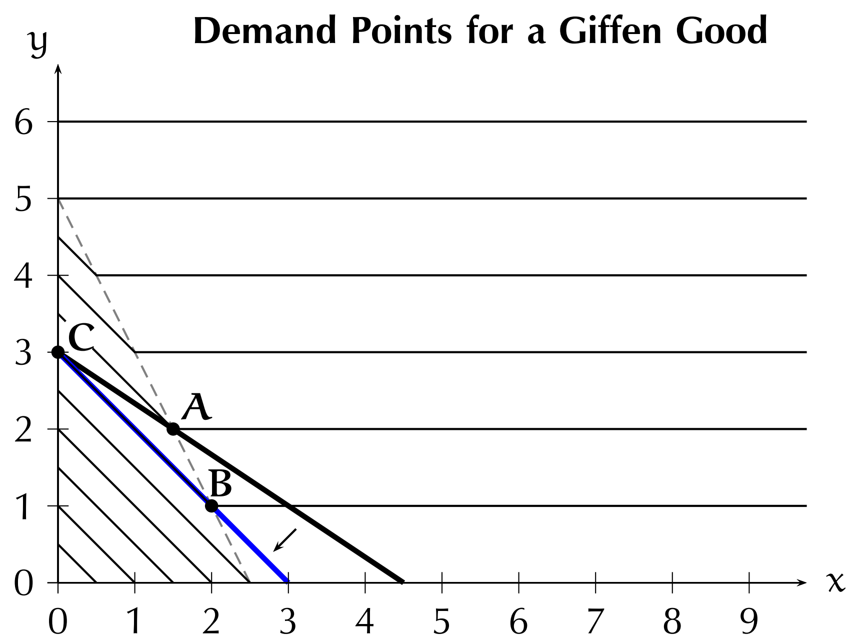


5.3.9 Demand Points for a Giffen Good

The diagram shows the solution to the consumer's problem for two different prices of X. Here income is \$3 and $p_y = \$1$.

The lighter lines are the indifference curves and the heavier lines are budget lines for $p_x = 2/3$ and $p_x = 1$. When the price of X increases from 2/3 to 1, the budget line rotates downward and the optimal consumption bundle moves from $\mathbf{A}$ to $\mathbf{B}$. Demand for X increases from 1.5 to 2.

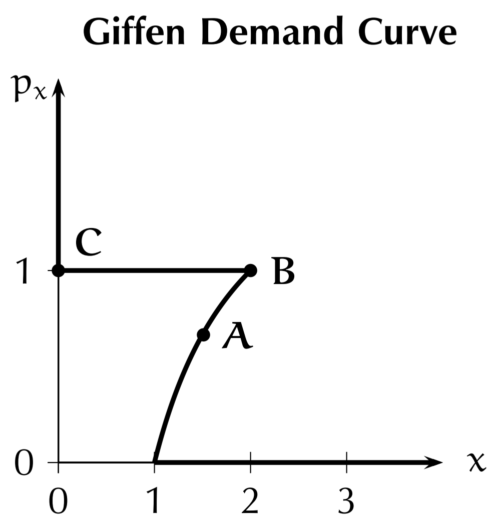
Optimal consumption slides down the dashed line as p_x increases from 0 to 1. After that, we are in perfect substitutes territory. Demand for X is anything in the interval $[0, 2]$ when $p_x = 1$, and drops to zero when $p_x > 1$.



5.3.10 A Giffen Demand Curve

The diagram plots demand in the usual way, with the price of X on the vertical axis. The interesting bit is where we slide down the dashed line. Demand on that segment is $x = 2/(2-p_x)$. This works for p_x between 0 and 1. The corresponding inverse demand is defined on for $1 \leq x \leq 2$. It is $p_x = 2 - 2/x$.

When $p_x = 1$, the quantity demanded is anything between 0 and 2. For $p_x > 1$, the quantity of X demanded is zero. Here's the resulting demand curve.



5.3.11 From Individual to Market Demand

In practice, Giffen goods have proven hard to find. Some analyses suggested that the potatoes were a Giffen good during the Irish potato famine. However, that now appears incorrect. So why don't we see Giffen goods in practice?

To understand that we need to consider how individual demand curves are combined to form market demand curves. Individual demand curves tell us the quantity demanded by an individual at each price. Market demand is the total demanded by all individuals in the market at each price. We get market demand by adding the individual demands horizontally. At each price, we add the quantities demanded.

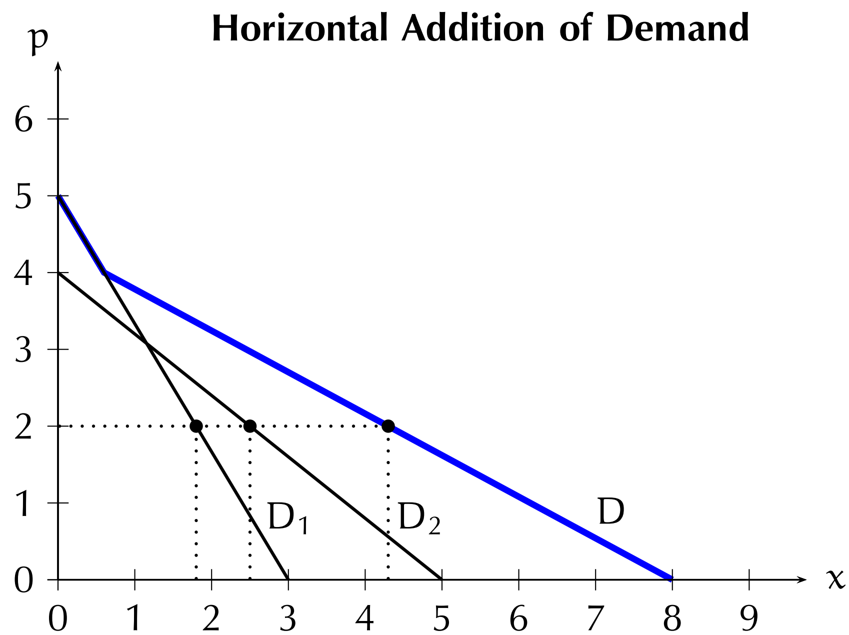
Thus if there are three individuals in a market with demands $q_1(p) = 10/p$, $q_2(p) = 5/p$, and $q_3(p) = 20/p^2$, the market demand is

$$\begin{aligned} q(p) &= q_1(p) + q_2(p) + q_3(p) \\ &= 10/p + 5/p + 20/p^2 \\ &= 15/p + 20/p^2. \end{aligned}$$

5.3.12 Horizontal Addition of Demand

If we graph the addition, we can see why it's called horizontal addition.

We take two linear demand curves, D_1 and D_2. At a price of \$2, we add the quantities of 1.8 from D_1 and 2.5 from D_2 to obtain 4.3, which is the market quantity demanded. Notice that above the choke price for D_2, market demand coincides with D_1.



5.3.13 Why do Market Demand Curves Slope Downward?

When we add market demand curves we are adding both their substitution effects and income effects. The substitution effects all slope downwards. Income effects vary. Sometimes they reinforce the substitution effects, sometimes moderate them, and more rarely, overturn them.

When we add them together, the income effects most likely average normal and provide some reinforcement to the aggregate substitution effect. Even if the average income effect is not normal, it is unlikely to be terribly large because we are adding both positive and negative income effects, and some of them cancel out. Either way, we likely get the Law of Demand.

5.4 Slutsky Decomposition: Labor Supply

We examine an application of consumer theory to labor supply.

5.4.1 Two Approaches to Labor Supply

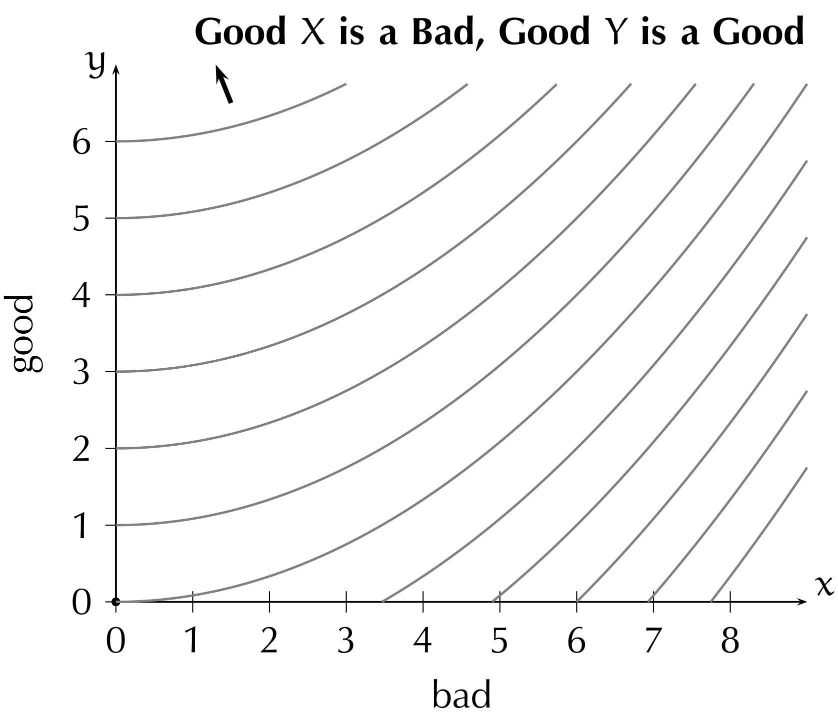
There are two ways to model labor supply. One of them treats labor as a bad, rather than a good. The idea is that labor itself is unpleasant, or at least not what we want to be doing. We work to earn money that we can use to buy goods we desire.

The other method treats the main cost of labor as time–time that could be used in other beneficial ways. This means we think of the cost of labor as primarily involving an opportunity cost.

5.4.2 Indifference Curves when One Good is a Bad

When modelling preferences, the difference between a good and a bad is that for goods, more is better. For bads, more is worse.

The diagram below shows an indifference map containing a good and a bad. Higher indifference curves are better because consumption of the good (Y) is higher or because consumption of the bad (X) is lower.



5.4.3 Consumption-Leisure Choice

The second approach, which we will use, is to focus on time allocation, where the individual allocates time between labor (which generates income) and other purposes, which we will call "leisure". Here "leisure" encompasses all other activities—sleeping, eating, time spent with friends, watching tv, etc., etc. Fancier models break this into categories, but a simple model will suffice for an introduction to the problem.

We have a certain amount of time to allocated. On a daily basis, 24 hours. Weekly, it's 168 hours; in the average month, just over 730 hours; in the average year, just under 8766 hours. Call the time available T.

The individual then chooses how much leisure (ℓ) to take and how many labor hours to supply (L), with $\ell + L = T$. On a daily basis (and here we think of an average day), $\ell + L = 24$.

5.4.4 We Supply Labor so we can Consume

The point of working in this model is to earn income. As a worker, our individual earns a wage rate w, so income from working L hours is $w \times L$. The consumer then spends that income on consumption.

To keep the model simple we model consumption as a single good. Then $c = wL$. In the end, the individual chooses a bundle (ℓ, c) which implicitly defines labor time $L = T - \ell$. This choice is subject to a budget constraint:

$$c \leq wL = w(T - \ell).$$

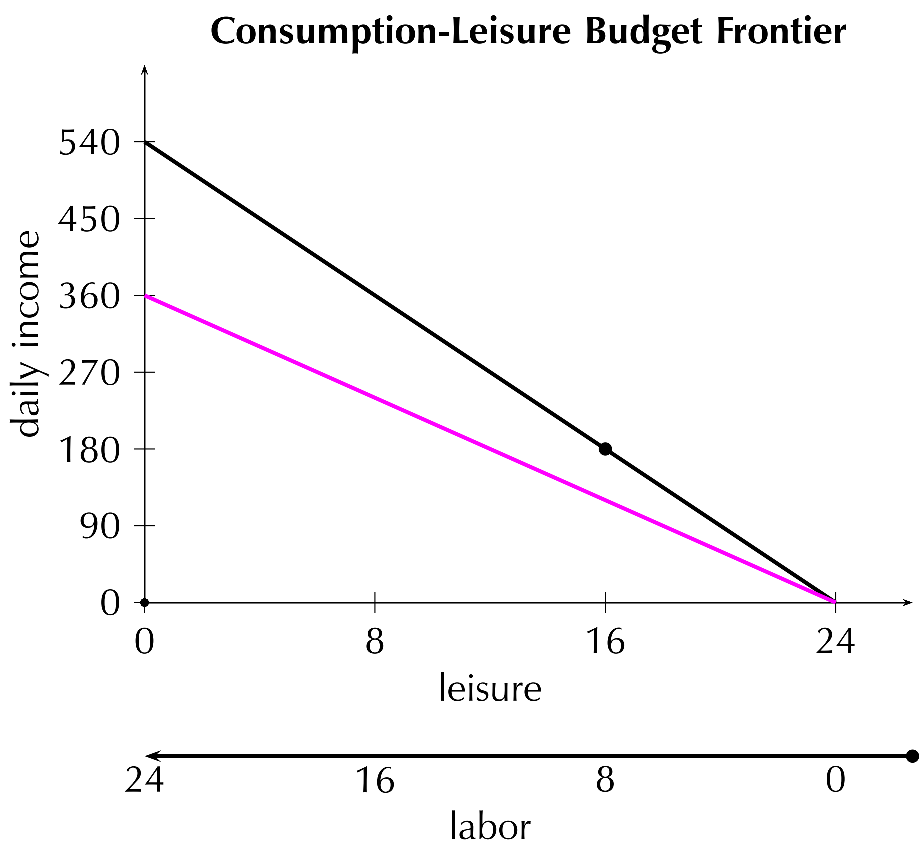
If we wish to consider inflation, we would modify this model by including a price level p for consumption, in which case the budget constraint becomes $pc \leq w(T - \ell)$.[5]

[5] From August 2021 to August 2022, inflation increased prices by 8.2%, while average hourly earnings increased by just 5.2%, meaning that real average hourly earnings (w/p) fell by 2.8%. Here the relevant calculation is $1.052/1.082 = 0.972$, yielding a decrease of 2.8%. We also note that average weekly hours are falling. See https://www.bls.gov/news.release/realer.t01.htm

5.4.5 The Consumption-Leisure Budget Frontier

When the wage rate changes, the horizontal anchor point remains fixed at 24 hours leisure. The black line is the budget line for a \$22.50 wage, the magenta line is for a \$15 wage. One way to see this is that the vertical anchor point is $w \times T = 24w$. Here $22.50 \times 24 = 540$ and $15 \times 24 = 360$ are the anchor points for \$22.50 and \$15, respectively.

The lower scale shows how leisure translates into labor supplied.

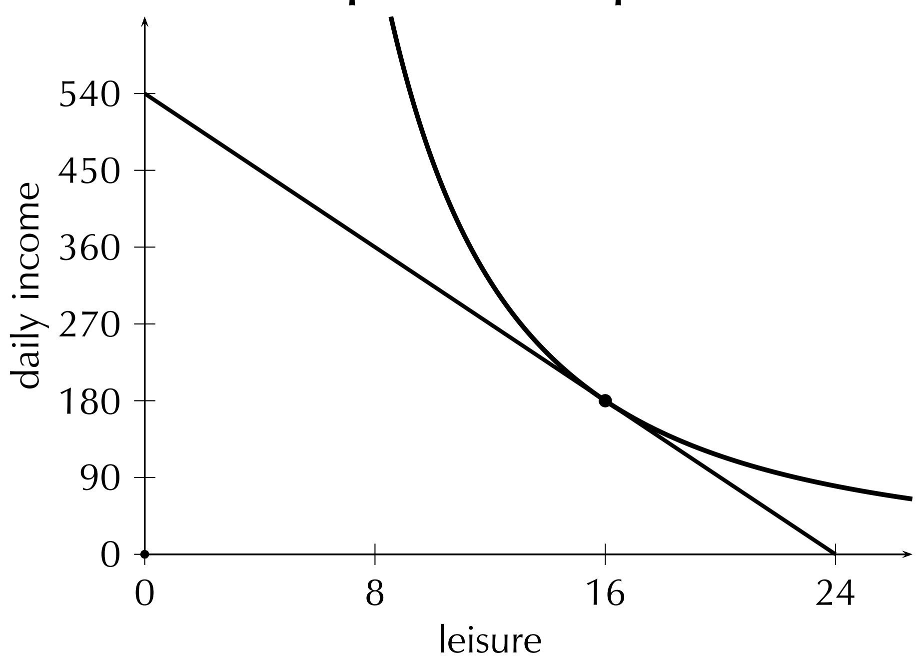


5.4.6 Optimal Consumption-Leisure Choice

As with standard consumer theory, the optimum is the point on the budget frontier where the marginal rate of substitution is equal to the relative price.

The optimal choice in the diagram below is to take 16 hours of leisure. That means supplying 8 hours of labor. At $22.5 per hour, that translates into income (and consumption) $\$22.5 \times 8 = \$180$.

Consumption-Leisure Optimal Choice



5.4.7 What is the Slope of Labor Supply?

We typically think of both consumption and leisure as normal goods as there is a substantial amount of evidence in favor of it.

If the wage rate increases to $30, the substitution effect causes a decline in leisure, equivalently, an increase in labor supply. However, the increase in the wage rate makes this individual richer. Leisure is a normal good, and we expect an increase in leisure (decrease in labor supply) to result. The overall effect on labor supplied is ambiguous. It cannot be determined using general principles. Data is required.

For "prime-age" males (25-55 years old), labor supply is quite inelastic, the income and substitution effects nearly cancel.[6] For women, the income effect is weaker than the substitution effect, and labor supply is upward sloping, as we expect a supply curve to be.

The situation with labor supply is different from a Giffen good. Giffen goods must be strongly inferior. In constrast, inelastic or even downward-sloping labor supply occurs because of consumption being normal.

[6] Prime-age workers are mostly old enough to have completed their education and yet young enough that few have retired. FYI. The average retirement age is about 62, and substantial numbers of workers retire before age 60.

5.4.8 The New Jersey Income Maintenance Experiment

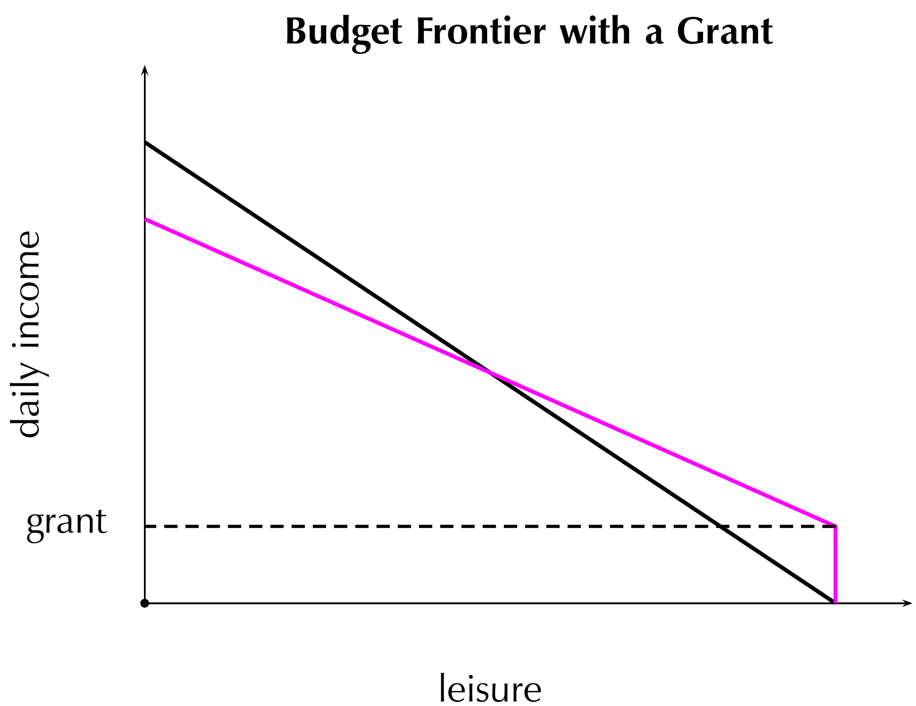
During the Nixon administration, what is now the Department of Health and Human Services, together with the states of New Jersey and Pennsylvania, ran an experiment studying the effects of a “negative income tax” on work incentives.[7] This program included poor families in four New Jersey cities (Trenton, Paterson, Passaic, and Jersey City) and Scranton, Pennsylvania. Other such experiments were conducted elsewhere. All this was in support of a Nixon administration attempt to reform the welfare system.

Participants were given a cash grant to raise their incomes. The grants ranged from 1/2 to 1 1/4 times the poverty line. Those that earned income also had to repay the grant at rates ranging from 30% to 70% of income.

[7] The Department of Health, Education, and Welfare was separated during the Carter administration into the Department of Education and the Department of Health and Human Services.

5.4.9 Cash Grants, Taxes, and Budget Constraints I

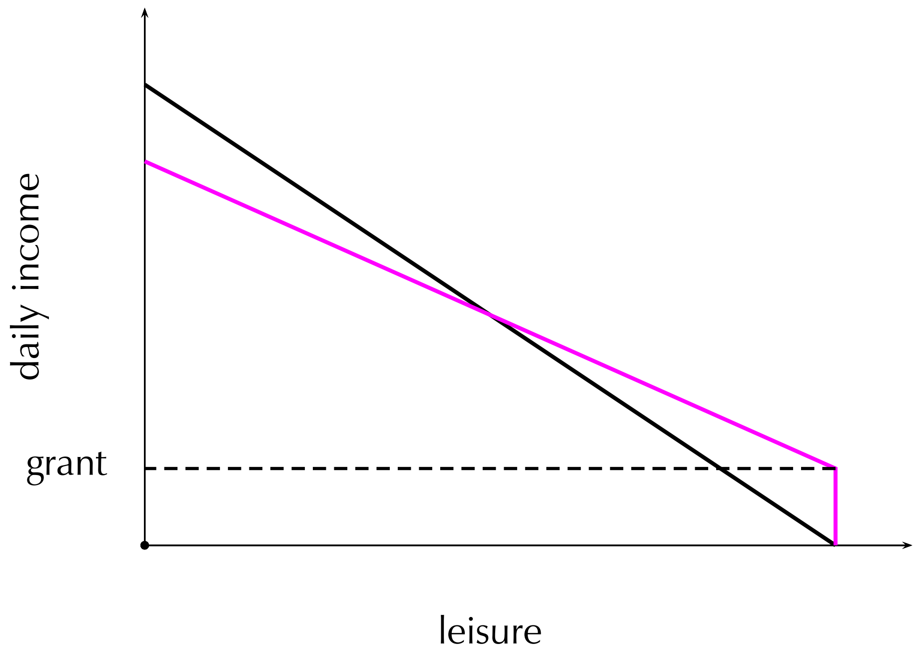
Let's see how this grant program affects budget constraint. The grant increases income when zero labor is supplied ($\ell = 24$). However, the tax portion reduces the wage rate. The magenta line shows the new budget frontier.



5.4.10 Cash Grants, Taxes, and Budget Constraints II

Unless work effort is high, income will increase. Because leisure is a normal good, work effort will fall. Moreover, the tax part of the program works through the substitution effect to further reduce labor effort. We expect work effort to fall, but income may rise (due to the cash grant).

Budget Frontier with a Grant



5.4.11 The New Jersey Income Maintenance Experiment: Results

Here's what happened in the first year, showing percentage changes.[8]

	Num. Emp. Per Family	Hours Per Fam.	Ave. $ per hour	Total Earnings
Family total	−12.2	−11.8	+9.8	− 3.2
Male Head	− 3.6	− 6.2	+7.7	+ 0.9
Female Spouse	−25.0	−25.2	+7.3	−20.0
Other Earner	−41.7	−41.1	+7.3	−28.9

[8] See https://www.jstor.org/stable/pdf/23858534.pdf

5.5 A Model of Borrowing and Lending

Another application of consumer theory is to borrowing and lending. In the latter case, the theory addresses consumers who are suppliers to the loan market—suppliers of loans. In other words, they are savers. It also is concerned with consumers who are demanders of loans (borrowers).

5.5.1 A Two-Period Model

We will use a simple model with two time periods, the present and the future, also known as today and tomorrow.

We distinguish consumption in each period. Consumption now is c_0 and future consumption is c_1. Both are assumed to be normal goods.

The consumer earns income in each period, w_0 now and w_1 in the future.

Finally, there is an interest rate $r > 0$. All borrowing and lending occurs at the interest rate r.

5.5.2 The Budget for Borrowers I

Suppose our consumer decides to borrow money from the bank today. Let A be the amount borrowed. Borrowing an amount A today (the principal) commits the consumer to paying the lender back in the future. The amount paid back consists of both the principal A plus interest $r \times A$. That means the borrower must pay back

$$A + rA = (1 + r)A$$

in period one.

Borrowing in period one isn't possible because there is no period two when it can be paid back.[9]

Having borrowed A, consumption in period zero can be increased from w_0 to $c_0 = w_0 + A$. That means that the consumer borrows

$$A = c_0 - w_0 > 0$$

in period zero and pays back

$$(1 + r)A = (1 + r)(c_0 - w_0)$$

in period one.

September 29, 2022

[9] More complex versions of this model allow for borrowing in many periods.