

## Intermediate Microeconomics — Week 8

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Professor Boyd

October 11 & 13, 2022

### 6.1 2022 Nobel Memorial Prize in Economic Science

Here are the winners.

- Ben Bernanke
- Douglas Diamond
- Phil Dybvig

#### 6.1.1 Ben Bernanke

Ben Bernanke was Chairman of the Federal Reserve from 2006–2014. Led the Fed during the financial crisis that began in 2007, and the following “Great Recession”.

Bernanke was well-known for his work on the Great Depression of 1929–1939. Years ago, in their book *A Monetary History of the United States, 1867–1960*, Milton Friedman and Anna Schwartz had argued that the collapse of the money supply played an important role in creating the Great Depression. Bernanke argued that the collapse in credit was also very important in his 1983 paper, “Monetary Effects of the Financial Crisis in the Propagation of the Great Depression” (1983), *American Econ. Review* **73**, 257–276.) As a result, it was not enough to just print more money. The Fed needed to avoid a potential credit shortage too.

This made Bernanke uniquely qualified to deal with the “Great Recession”, which involved the first widespread bank runs since the 1930s. Although the bank runs were not runs on the commercial banking system, they were runs on what has been called the shadow banking system. Bernanke was able to recognize this, and acted to counter-act the contraction in both money supply and credit.

### **6.1.2 Ben Bernanke on Irreversible Investments**

Another major paper of Bernanke's is "Irreversibility, Uncertainty, and Cyclical Investment", (1983), *Quarterly Jour. Economics* **98**, 85–106. This paper argues we should think about real investments as options. Once the investment is made, the option is gone, but beforehand investors must balance the value of increased information from waiting to invest vs. the extra returns that early commitment can bring.

## 6.2 Diamond and Dybvig: Intermediation

Last week, when presenting the basic borrowing/lending model I used the word “intermediation” at least twice. It refers to the fact that lenders often don’t directly lend to borrowers, but do so through intermediaries, often banks. Diamond and Dybvig are responsible for the basic microeconomic model of banks as intermediaries.

The original Diamond and Dybvig paper is “Bank Runs, Deposit Insurance, and Liquidity”, (1983) *Journal of Political Economy* **91**, 401–419.

The idea is that lenders often do not want to tie up their money. Rather, they want it in a liquid asset—an asset that can easily be exchanged for money at little cost. Many borrowers want to get longer-term loans, where they don’t have to pay back the loan immediately. They pay it back over a period of years (e.g., 30-year mortgages).

Banks are in the business of borrowing money in a liquid fashion (e.g., the money you put in a savings or money market account) and lending it illiquidly, as 30-year mortgages. Diamond and Dybvig developed a relatively simple model that allowed them to analyze bank runs and the effect of deposit insurance.

A simpler exposition of their model is in a Richmond Fed publication, Douglas W. Diamond, “Banks and Liquidity Creation: A Simple Exposition of the Diamond-Dybvig Model” (2007), *Economic Quarterly* **93**, 189–200.

### 6.3.8 Prices and Capital-Labor Ratio

When the marginal rate of technical substitution is constant along rays through the origin, the cost minimizing bundle of inputs will have the same capital-labor ratio as long the input prices remain unchanged. This follows from the fact that

$$\frac{w}{r} = \text{MRTS} \left( \frac{K}{L} \right),$$

so the MRTS remains the same as long as the capital-labor ratio is unchanged. Of course, for many production functions, the MRTS is **not** a function of  $K/L$ .

## 6.4 Short-run Production Costs

We will work through a short-run cost problem using the Cobb-Douglas production function  $F(L, K) = \sqrt{KL}$  with factor prices  $r$  and  $w$  and fixed capital input of  $\bar{K} = 100$ .

The short-run production function is

$$Q(L) = F(L, \bar{K}) = \sqrt{L\bar{K}} = 10\sqrt{L}.$$

To find the cost, we first determine the labor input  $L$  required to produce quantity  $q$ . That is we solve  $q = Q(L)$  for  $L$ . Now

$$q = 10\sqrt{L}.$$

We square this to find

$$q^2 = 100L,$$

and then solve for  $L$  as a function of  $q$ . That gives us the short-run labor demand

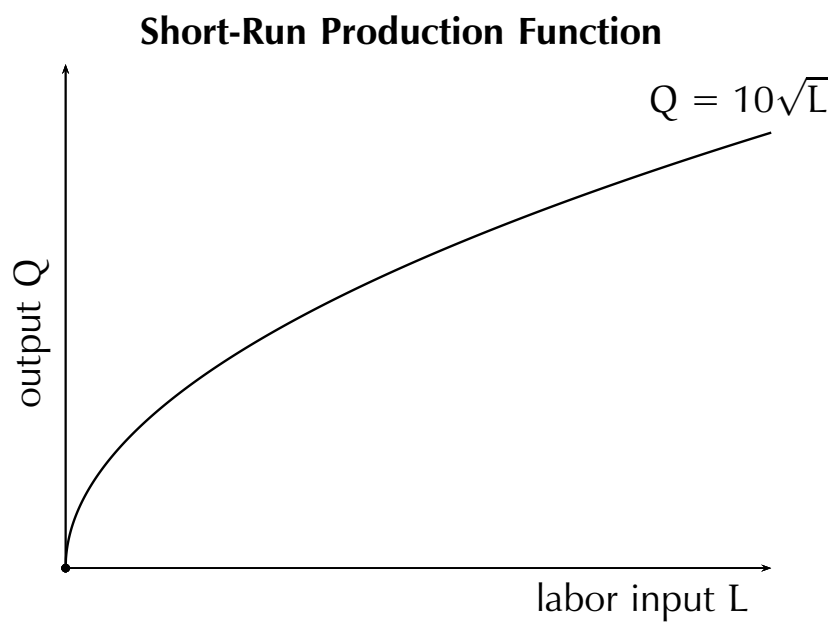
$$L(q) = \frac{q^2}{100}.$$

We then use the equation  $C = rK + wL$ , so the short-run cost  $C_S$  is

$$C_S(q) = r\bar{K} + wL(q) = 100r + \frac{wq^2}{100}.$$

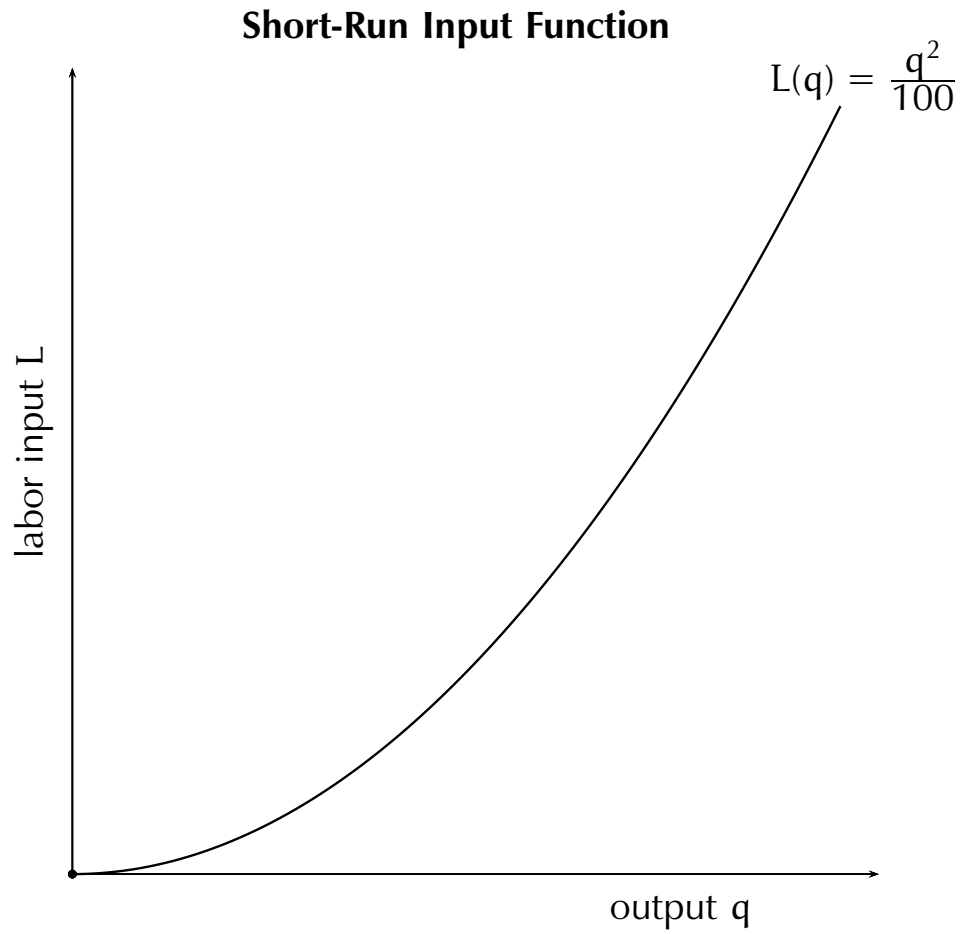
### 6.4.1 Graphical Derivation of Short-run Cost I

Graphically, we start with the short-run production function.



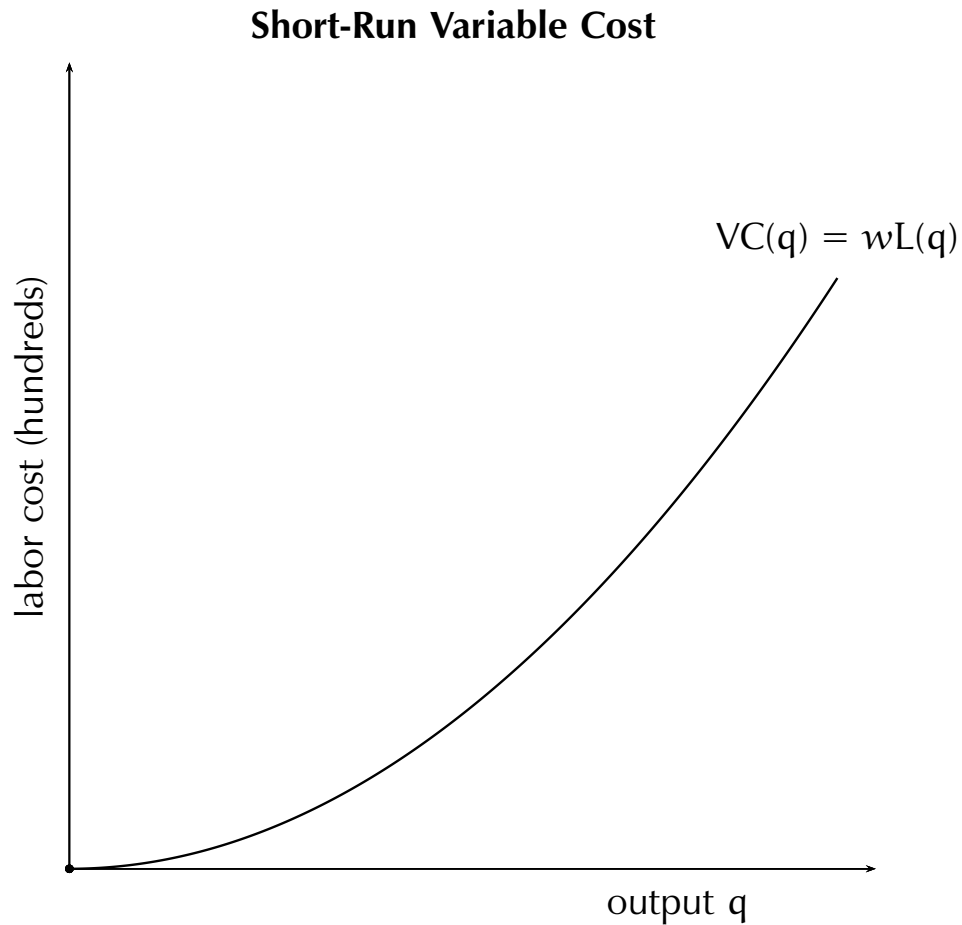
### 6.4.2 Graphical Derivation of Short-run Cost II

We then invert it by flipping the axes to get input as a function of output



### 6.4.3 Graphical Derivation of Short-run Cost III

Step three is to multiply by the wage rate  $w$ , which changes the scale, but not the shape. It's still a parabola.



This graph shows the cost of the variable input, labor, as a function of output. It shows the **variable cost** function  $VC = wL(q)$ .

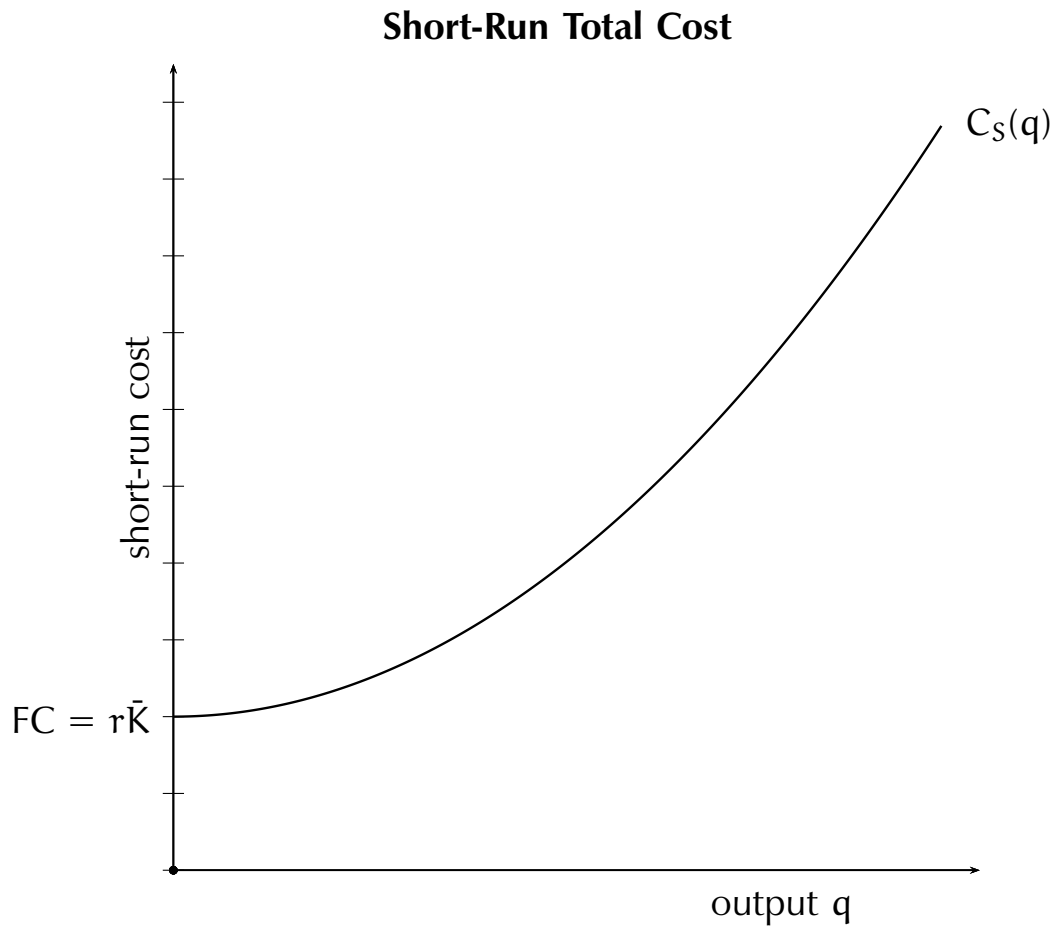


#### 6.4.4 Graphical Derivation of Short-run Cost IV

Finally, recalling that

$$\text{Cost} = rK + wL,$$

we add in the cost of the fixed capital  $r\bar{K}$  to obtain the short-run total cost. This raises the cost curve by the cost of the fixed capital input—the **fixed cost**  $FC = r\bar{K}$ .



**6.4.5 Short-run Cost Function with Numbers**

Let's set  $r = \$60/\text{per hour}$  and  $w = \$15/\text{per hour}$ . Then

$$\begin{aligned}C_S(q) &= 100r + \frac{wq^2}{100} \\ &= 6000 + \frac{15q^2}{100} \\ &= 6000 + \frac{3q^2}{20}\end{aligned}$$

is the short-run cost function.

## 6.5 Long Run Cost Revisited

We will solve the long-run cost minimization problem for the same production function and prices:

$$F(L, K) = \sqrt{LK}$$

$$r = 60$$

$$w = 15$$

$$q = 200$$

In the long run case,  $K$  is free to vary, rather than being fixed at  $\bar{K}$ . In our previous example, we set  $\bar{K} = 100$ .

We know that the optimal  $K$  and  $L$  must satisfy

$$q = F(L, K) \quad \text{Production equation}$$

$$\text{MRTS} = \frac{w}{r} \quad \text{Optimality equation.}$$

**6.5.1 Minimizing Long-Run Cost: An Example**

In our case,  $w = 15$  and  $r = 60$ , so  $w/r = 1/4$ . Moreover,  $MRTS = K/L$ , and  $q = 200$ , so we have

$$200 = \sqrt{LK} \quad \text{Production equation}$$

$$\frac{K}{L} = \frac{1}{4} \quad \text{Optimality equation.}$$

The optimality equation can be rewritten

$$4K = L.$$

Then we substitute  $4K$  for  $L$  in the production equation

$$200 = \sqrt{LK} = \sqrt{(4K)K} = \sqrt{4K^2} = 2K$$

so  $K^* = 100$  is the **optimal capital input** or **capital demand**. Then  $L^* = 4K^* = 400$  is the **optimal labor input** or amount of **labor demanded**.

### 6.5.2 Comparing Long-Run and Short-run Cost

Let's take a look at the cost in both the long and short run. We find the long-run cost by using the cost equation and factor demand. The long-run cost is

$$rK^* + wL^* = 60(100) + 15(400) = 12,000.$$

The short-run problem we examined used the same production function and the same factor prices ( $r = 60$  and  $w = 15$ ). We also used a fixed stock of capital of  $\bar{K} = 100$ , the same as the optimal long-run capital demand (at the current wage and capital rental rates).

The short-run cost function is

$$C_S(q) = 6000 + \frac{3}{20}q^2.$$

Substituting  $q = 200$ , the optimal long-run quantity, we find that the short-run cost is

$$C_S(200) = 6000 + \frac{3(200)^2}{20} = 12,000.$$

It's the same as the long-run cost when producing the long-run optimal amount. This is not a coincidence, but a consequence of using the same technology and prices, and also setting the short-run capital stock  $\bar{K}$  equal to the long-run optimum.

**6.5.3 Minimizing Cost: More General Case**

We use a variation on the same production function  $F(L, K) = A\sqrt{LK}$ , but allow prices and quantity produced to vary. The parameter  $A > 0$  will allow us to change productivity. Using this production function, we can calculate the long-run cost and long-run demand for labor and capital.

We know that the optimal  $K$  and  $L$  must satisfy

$$q = A\sqrt{LK} \quad \text{Production equation}$$
$$\frac{K}{L} = \text{MRTS} = \frac{w}{r} \quad \text{Optimality equation.}$$

**6.5.4 Minimizing Cost: Optimality Equation**

We again use the optimality equation to relate K and L. Now

$$\frac{K}{L} = \frac{w}{r}$$

so

$$K = \left(\frac{w}{r}\right) L.$$

### 6.5.5 Minimizing Cost: Production Equation

We substitute  $K = (w/r)L$  in the production equation to find

$$q = A\sqrt{LK} = A\sqrt{\left(\frac{w}{r}\right)L^2} = AL\sqrt{\frac{w}{r}}.$$

Then

$$L^* = \frac{q}{A}\sqrt{\frac{r}{w}} \quad \text{and} \quad K^* = \frac{q}{A}\sqrt{\frac{w}{r}}.$$

These are the conditional capital and labor demand functions. They depend on both the quantity produced and factor prices. They are **conditional** because they depend on the **quantity produced**. Keep in mind that we still **have not solved the firm's profit maximization problem**.

All other things equal, increases in the capital rental rate  $r$  increase demand for labor and decrease demand for capital. Changes in the wage rate  $w$  work in the opposite direction, where increased  $w$  reduces demand for labor and increases demand for capital. With this technology, labor and capital are (imperfect) substitutes.



### 6.5.6 Effect of the Productivity Parameter

The parameter  $A$  affects **total factor productivity**. It is a type of change in productivity that affects both factors equally.

Examining the demand equations,

$$L^* = \frac{q}{A} \sqrt{\frac{r}{w}} \quad \text{and} \quad K^* = \frac{q}{A} \sqrt{\frac{w}{r}},$$

we see that increases in productivity (bigger  $A$ ) reduce demand for both labor and capital. We need less of each to produce the same amount of output when productivity increases.

### 6.5.7 The Long-run Cost Function

We're not done yet. We still need to find the cost function. To do that, we substitute the labor and capital supply functions in our cost expression to find the long-run cost function  $C(q)$ .

$$\begin{aligned}C(q) &= rK^* + wL^* \\&= r\frac{q}{A}\sqrt{\frac{w}{r}} + w\frac{q}{A}\sqrt{\frac{r}{w}} \\&= \frac{q}{A}\sqrt{\frac{r^2w}{r}} + \frac{q}{A}\sqrt{\frac{rw^2}{w}} \\&= 2\frac{q}{A}\sqrt{rw}\end{aligned}$$

When  $A = 1$ ,  $r = 60$  and  $w = 15$ , we have

$$\begin{aligned}L^* &= q\sqrt{60/15} = 2q \\K^* &= q\sqrt{15/60} = q/2.\end{aligned}$$

and the long-run cost is  $C(q) = 2q\sqrt{900} = 60q$ .

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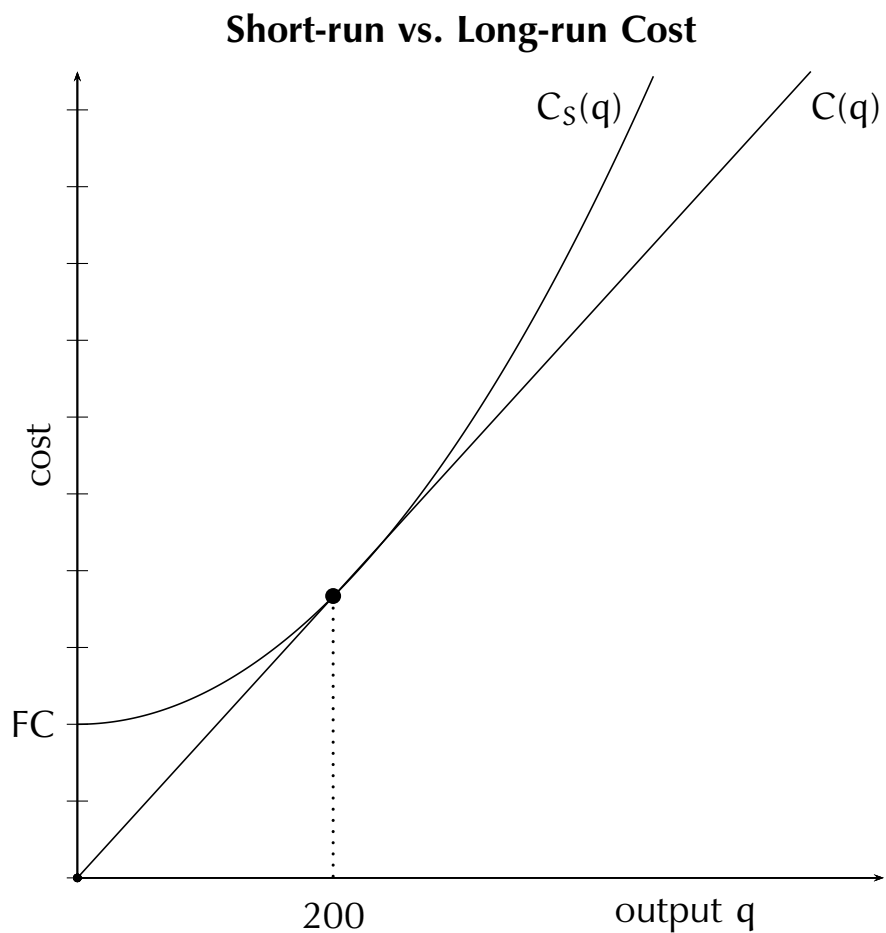
**6.5.8 Properties of Long-run Cost**

The long-run cost function is linear in the quantity produced ( $q$ ). This is due to a constant returns to scale technology (more on that later).

Long-run cost increases if either of the factor prices increases, and cost decreases if productivity increases.

### 6.5.9 Comparison of Long-run and Short-run Cost

In most cases, the short and long-run cost functions are only equal when  $K^*(q) = \bar{K}$ . Since  $K^*(q) = q/2$ , this happens when  $q = 200$ . This is illustrated below.



## 6.6 Returns to Scale

Returns to scale describes how output is affected by a proportional change in inputs. **Scaling** inputs by a factor  $t$  means replacing  $(L, K)$  by  $(tL, tK)$ , leaving  $K/L$  unchanged. Returns to scale is all about how  $F(L, K)$  and  $F(tL, tK)$  relate to one another.

**Constant Returns to Scale:** A production function has constant returns to scale if whenever all inputs are increased by the same factor  $t$ , the output is also increased by the factor  $t$ .

**Increasing Returns to Scale:** A production function has increasing returns to scale if whenever all inputs are increased by the same factor  $t$ , the output is increased by more than the factor  $t$ .

**Decreasing Returns to Scale:** A production function has decreasing returns to scale if whenever all inputs are increased by the same factor  $t$ , the output is increased by less than the factor  $t$ .

**6.6.1 Cobb-Douglas and Returns to Scale I**

Let's take a general Cobb-Douglas production function

$$F(L, K) = AL^aK^b$$

with  $A, a, b > 0$ .

This has constant returns to scale when  $a + b = 1$ , increasing returns to scale when  $a + b > 1$  and decreasing returns to scale when  $a + b < 1$ .

### 6.6.2 Cobb-Douglas and Returns to Scale II

To see this, compute

$$\begin{aligned} F(tL, tK) &= A(tL)^a(tK)^b \\ &= At^at^bL^aK^b \\ &= t^{a+b}AL^aK^b \\ &= t^{a+b}F(L, K). \end{aligned}$$

Since we are interested in increases in inputs, we take  $t > 1$ .

When  $a + b = 1$ ,  $F(tL, tK) = tF(L, K)$  for  $t > 1$ , the constant returns to scale case.

When  $a + b > 1$ ,  $F(tL, tK) = t^{a+b}F(L, K) > tF(L, K)$  for  $t > 1$ , the increasing returns to scale case.

Finally, when  $a + b < 1$ ,  $F(tL, tK) = t^{a+b}F(L, K) < tF(L, K)$  for  $t > 1$ , the decreasing returns to scale case.

**6.6.3 Increasing Returns and Diminishing Marginal Product**

It is perfectly possible to have increasing returns to scale and a diminishing marginal product. Consider the production function

$$F(L, K) = L^{2/3}K^{2/3}$$

As  $2/3 + 2/3 = 4/3 > 1$ , we have increasing returns to scale.

However, the marginal products are

$$MP_L = \frac{2}{3}L^{-1/3}K^{2/3} = \frac{2}{3} \frac{K^{2/3}}{L^{1/3}}$$

and

$$MP_K = \frac{2}{3}L^{2/3}K^{-1/3} = \frac{2}{3} \frac{L^{2/3}}{K^{1/3}}.$$

The marginal product of labor is diminishing in labor input  $L$  and the marginal product of capital is diminishing in capital input  $K$ .



### 6.6.4 Homogeneous Functions

We say a production function is **homogeneous** of degree  $\gamma$  if there is an exponent  $\gamma > 0$  with  $F(tL, tK) = t^\gamma F(L, K)$ .

We have actually shown that Cobb-Douglas production is homogeneous of degree  $a + b$ . Homogeneous production is more general than constant elasticity of substitution production, which is a special case.

Homogeneous production also has the property that the marginal rate of technical substitution is constant along each ray through the origin.

If we have to model production without that property (constant MRTS), we must use a production function that is not homogeneous, such as

$$F(L, K) = 2L^2 + \sqrt{LK}.$$

In this case  $MP_L = 4L + \frac{1}{2}\sqrt{K/L}$  and  $MP_K = \frac{1}{2}\sqrt{L/K}$ , so

$$MRTS = \frac{MP_L}{MP_K} = 4L\sqrt{\frac{K}{L}} + \frac{K}{L}$$

which is not a function solely of  $K/L$ .

### 6.6.5 Microsoft Windows

One product that doesn't fit well into our production function framework is Microsoft Windows. Nonetheless, we can still figure out what the cost function looks like.

Producing a new version of Microsoft Windows is expensive. The biggest single item is producing the code for the operating system and its drivers. One estimate I found online suggested that Windows 8 cost roughly \$20 billion to produce (\$20,000,000,000). We'll take that as a starting point.

Windows 11 was likely more expensive to create. Unfortunately, a quick search didn't turn up plausible information on its cost.

Besides the cost of developing the software, Microsoft also has to pay the expensive of delivering it. In the beginning, Windows 1.0 came in a nice box, containing a stack of 5 1/4" floppy disks (!) and a number of manuals and other materials to tell you what you needed to know to install it on your computer and then make use of it. I know you don't remember a time when computer icons and a mouse were unfamiliar, but with Windows 1.0, they were unfamiliar to most computer users. Macintosh users were an exception, but IBM computer users, their target market, was generally unfamiliar with the concept. For that matter, most were also unfamiliar with the computer mouse. As for touchpads, forget it! They were unknown to the average user.<sup>1</sup>

Today, the process has been streamlined. The manuals have disappeared, the floppy disks replaced by preinstallation or downloading from a server. The server often does not belong to Microsoft. If a corporation is involved, you download from the corporation's servers, at no cost to Microsoft, and the corporation handles support too (that's how it is at FIU). Still, Microsoft had to supply the original copies, usually electronically. The cost of distributing Windows is now very small, but not zero.

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<sup>1</sup> The whole package, plus networking, was available on the Xerox Star and Alto workstations that inspired both Apple's Steve Jobs and Microsoft's Bill Gates. They also both raided Xerox PARC for employees.

### 6.6.6 Cost Function for Intellectual Property

So we have a start-up cost of \$20 billion or so to produce the original copy, and distribution and support costs that depend on the number of copies sold. In other words, the cost of producing and selling additional copies of Microsoft Windows is small, perhaps \$10 each. That means the cost of  $q$  copies is something like

$$C(q) = 2 \times 10^{10} + 10q.$$

Windows is not the only product with this type of cost structure. It is typical of anything falling into the broad category of intellectual property (IP). Intellectual property is often relatively expensive to produce, and very cheap to duplicate. Movies, music, books, software, all involve intellectual property and have similar cost structures.

This type of cost function also doesn't fit well in competitive markets. It's what we call a natural monopoly. We will analyze such products later when we consider monopoly markets.

### 6.6.7 Changes in Total Factor Productivity

**10/13/22**

Much of the increase in per capita incomes in the USA is due to increases in total factor productivity (TFP). This indicates that improved technology is the primary source of long-term economic growth.

The simplest type of increase in total factor productivity is to multiply a production function by a constant  $A$ . When  $A > 1$  we have an increase in total factor productivity, while if  $A < 1$ , total factor productivity decreases.

When  $F(L, K)$  is replaced by  $AF(L, K)$ , we can produce an amount  $Aq$  using the same inputs that used to produce only  $q$ . When  $F$  has constant elasticity of substitution, the change in productivity will be neutral—it affects all inputs in the same way.

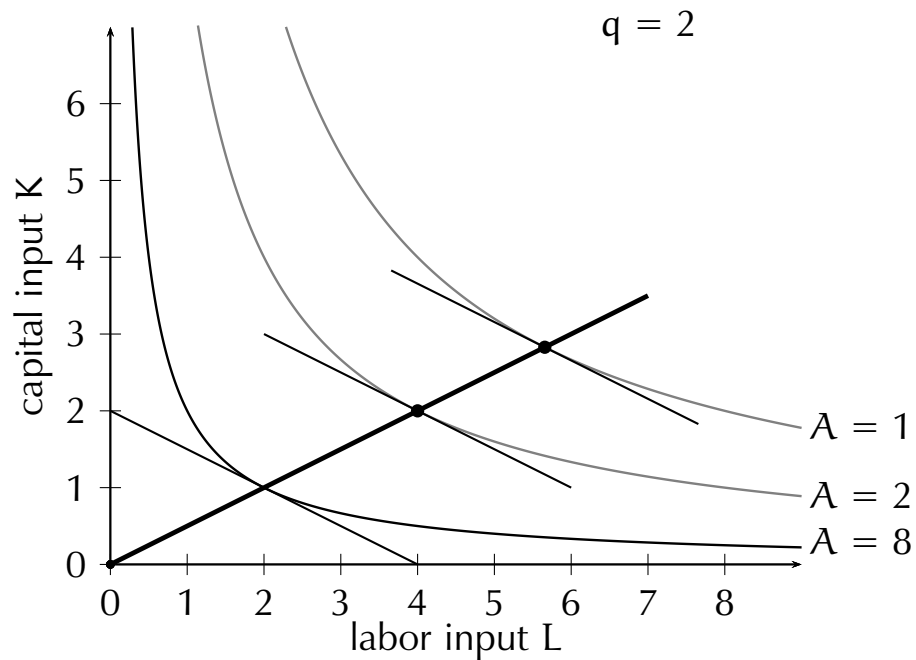
Increases in total factor productivity are largely responsible for the increases in wealth we've seen during the 20<sup>th</sup> and early 21<sup>st</sup> centuries (and probably earlier). The rate of TFP growth has slowed in the 21<sup>st</sup> century, as has economic growth. Although there's been substantial evidence that total factor productivity increased at a constant rate (exponential growth), that has recently been called into question.<sup>2</sup> It may be that economic growth will be slower in the future.

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<sup>2</sup> Thomas Philippon (2022), "Additive Growth", NBER Working Paper #29950. <http://www.nber.org/papers/w29950>

### 6.6.8 Increase in Total Factor Productivity: Cobb-Douglas

Below, we illustrate an increase in  $A$  for Cobb-Douglas production for several values of  $A$ . Here  $w/r = 1/2$ , so the minima are where  $MRTS = 1/2$ .



The  $A = 2$  curve allows us to produce the same amount ( $q = 16$ ) with half of the input and cost used when  $A = 1$ . The  $A = 8$  curve reduces the cost of production of  $q = 16$  to  $\frac{1}{8}$  the original value as the inputs fall from  $(4\sqrt{2}, 2\sqrt{2})$  to  $(2, 1)$ .

## 7.1 Chapter 7: Costs

We investigate a number of different types of costs in this chapter. These include fixed cost, variable cost, total cost, short-run costs, long-run costs, average costs, marginal costs, accounting costs, opportunity costs, sunk costs, and economic costs.

We start by focusing on costs relevant to decision-making. We consider a sole proprietorship, a firm owned by an individual. We'll start with data on costs and revenue, and ask whether the owner should continue to run his firm, or sell out and do something else.

**7.1.1 Cost Accounting Example**

Here's the data for the firm.

Revenues	\$ 1,000,000
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Supplies	\$ 300,000
Water & Power	\$ 30,000
Salaries	\$ 200,000
Rent (own bldg.)	\$ 0
Can rent out for	\$ 250,000
Paid for bldg. in 2001	\$ 1,500,000
Owner's Salary	\$ 120,000
Job Offer #1	\$ 90,000
Job Offer #2	\$ 150,000
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The owner has three assets, the firm itself, the building, and his own labor. How can he maximize the value of these assets? Is using his building and labor in the firm his best use of those assets?

### 7.1.2 Profit Received from the Firm

**Accounting costs** are payments made. These include payments made to the owner of the firm.

No rent is received, so it doesn't count. The cost of the building was paid long ago, and doesn't affect the current year. The job offers aren't taken, so they aren't included. Every other cost in the table represents payments made. They count, and are marked in red. The accounting costs total to **\$650,000**.

**Accounting profit** is revenue minus accounting costs. Based on the table below, the accounting profit is **\$350,000**.

Revenues	\$1,000,000
<hr/>	
Supplies	\$ 300,000
Water & Power	\$ 30,000
Salaries	\$ 200,000
Rent (own bldg.)	\$ 0
Can rent out:	\$ 250,000
Paid for bldg. in 2001	\$ 1,500,000
Owner's Salary	\$ 120,000
Job Offer #1	\$ 90,000
Job Offer #2	\$ 150,000
<hr/>	
Accounting Costs	\$ 650,000

From this, we conclude that the accounting profits are \$650,000.



### 7.1.3 Best Alternative

From the previous page, we find that if the owner continues to operate his firm, he gets \$350,000 accounting profit.

Revenues	\$1,000,000
– Accounting Costs	<b>\$ 650,000</b>
Accounting Profit	<b>\$ 350,000</b>

The owner also gets his \$120,000 salary his firm pays him. His assets, including his own labor, generate annual income of \$470,000 if he uses them in his own firm. In this case he collects no rent.

If the owner closes his firm, takes the best job offer (\$150,000), and rents out the building to someone else (\$250,000), he earns income of \$400,000.

The owner is better off by  $\$470,000 - 400,000 = \$70,000$  if he continues to operate his firm. Since the firm pays more by \$70,000 than the next best use of his assets, we say he earns an **economic profit** of \$70,000. The economic profit is the difference between all the owner's income earned when operating the firm and the income that would be earned in the next best alternative (e.g., taking the \$150,000 job).

### 7.1.4 Three Types of Costs

Another way of looking at this problem involves classifying costs into three categories:

1. Accounting costs = amounts paid to others
2. Opportunity costs = value of best alternative use for assets owned by the firm or its owners.
3. Sunk costs = costs paid in the past.

The presence of opportunity costs should not be a surprise as our definition of economic profit involved alternative uses of resources.

We then define **economic costs**,

$$\text{Economic costs} = \text{Accounting costs} + \text{Opportunity costs}$$

and **economic profit**,

$$\text{Economic profit} = \text{Revenue} - \text{Economic costs}.$$

### 7.1.5 Sunk Costs

**Sunk costs** are costs that were incurred in the past and cannot be recovered. Since they are not affected by current decisions, they do not affect the comparison between choices made today. As the slogan says, sunk costs are sunk!

In our example, the price paid to buy the building in 2001 is a sunk cost. It doesn't reflect the current value, which we already have in the form of current rental value.

We will usually ignore sunk costs. They may be useful to know when projecting future costs. In this case, the historical cost of \$1,500,000 for the building doesn't give us any useful information once we know the rental value.

### 7.1.6 Updated Cost and Revenue List

We remove the sunk cost, the price paid for the building from the list so that it doesn't distract us. The portions marked in red are accounting costs, monies actually paid out by the firm.

Revenues	\$1,000,000
<hr/>	
Supplies	\$ 300,000
Water & Power	\$ 30,000
Salaries	\$ 200,000
Rent (own bldg.)	\$ 0
Can rent out:	\$ 250,000
Owner's Salary	\$ 120,000
Job Offer #1	\$ 90,000
Job Offer #2	\$ 150,000
<hr/>	
Accounting Costs	\$ 650,000

The other three costs, the rental value of the building and the job offers foregone, are opportunity costs.

### 7.1.7 Opportunity Costs

The **opportunity cost** of any action is the next best alternative—the thing an agent would otherwise choose.

The concept of opportunity cost dates to Frederic Bastiat (1850), in the essay “What is seen and what is not seen”. Opportunity costs are the “not seen” part, what doesn’t happen because you did something else.

When dealing with firms, we think of opportunity costs as dollar value foregone.

For a firm that owns and uses a building, the opportunity cost is usually the money they could have earned by selling the building or renting it out to others (whichever is higher in a present value sense).

For the owner of a firm who works there, the opportunity cost of labor time is the value could have been earned working elsewhere on other projects or in another line of work. Here the owner has two job offers. Only the best counts as an opportunity cost.

### 7.1.8 Accounting for Opportunity Costs

There are two assets belonging to the owner where we have information on opportunity costs—the building and the owner’s labor.

The building could be rented out, earning \$250,000, if it were not used by the firm. Since the firm is paying no rent, that would represent an additional \$250,000 income. There is \$250,000 opportunity cost from not renting out the building.

The owner has two job offers. If he takes either, he can’t earn his salary \$120,000 from the firm. That means job offer #1 would lose him  $\$30,000 = 120,000 - 90,000$ , but job offer #2 would gain him  $\$30,000 = 150,000 - 120,000$ . The opportunity cost here is \$30,000 from not taking job offer #2.

### 7.1.9 Accounting and Opportunity Costs

**Accounting costs** are payments made. These include payments made to the owner of the firm. They are shown in red. Opportunity costs are shown in green.

Revenues	\$1,000,000
<hr/>	
Supplies	\$ 300,000
Water & Power	\$ 30,000
Salaries	\$ 200,000
Rent (own bldg.)	\$ 0
Can rent out:	\$ 200,000
Owner's Salary	\$ 120,000
Job Offer #1	\$ 90,000
Job Offer #2	\$ 150,000
<hr/>	
Accounting Costs	\$ 650,000

From this, we conclude that the accounting profits, revenue minus accounting costs, are

Revenues	\$1,000,000
-Accounting Costs	\$ 650,000
<hr/>	
Accounting Profit	\$ 350,000

### 7.1.10 Economic Profit of the Firm

**Economic costs** are the sum of accounting and net opportunity costs.

**Economic profit** is revenue minus economic costs.

Supplies	\$ 300,000
Water & Power	\$ 30,000
Salaries	\$ 200,000
Rent (own bldg.)	\$ 0
Can rent out:	\$ 250,000
Owner's Salary	\$ 120,000
Job Offer #1	-\$30,000
Job Offer #2	+\$30,000
<hr/>	
Revenues	\$1,000,000
-Accounting Costs	\$ 650,000
<hr/>	
Accounting Profit	\$ 350,000
<hr/>	
Revenues	\$1,000,000
-Accounting Costs	\$ 650,000
-Net Opportunity Costs	\$ 280,000
<hr/>	
Economic Profit	\$ 70,000



**7.1.11 What if the Owner's Salary Changes?**

Changing the owner's salary does not affect economic profits. Adding \$1 to the owner's salary reduces accounting profits by \$1. The owner suffers a loss of \$1 in cash profits, but gains it back by getting \$1 more in salary. The point here is that the owner gets both profit and his salary, so moving a dollar from one category to the other doesn't affect what the owner gets.

Since the two types of income may be taxed differently, the decision about how big a salary the owner takes often depends on what leads to the smallest tax burden.

## 7.2 Costs and Profits

So far, we've focused on costs related to economic profit—accounting costs, opportunity costs, and sunk costs. The sunk costs are ignored, but we pay attention to accounting and opportunity costs, which together form economic costs.

In this chapter, we examine many types of costs. One way to help keep them straight is to remember that each one has a purpose. Economic costs are used to calculate economic profit. This makes it the key to understanding producer behavior.

We study economic profit because it tells us whether resources are being used wisely, in the way most useful to their owner(s). Accounting costs, opportunity costs, and sunk costs are tools we use to compute economic profit.

### 7.2.1 Zero Economic Profit

**Zero economic profit** has a special status. If you earn zero economic profit, it means that your investments of time, money, and property in your firm pay off exactly as well as in the best alternative. At this point your gains are not affected by whether or not you stay in business.

**Positive economic profit** means a firm is doing the best they can with their resources, and so should stay in business.

**Negative economic profit** (an economic loss) means a firm would do better investing elsewhere. The firm should close and move on. This will be important later on when we consider long-run firm behavior.

### 7.3 Short-run Costs: Fixed, Variable, and Total

We divide our analysis of firm behavior into two parts: short-run and long-run. This recognizes that producers may respond quite differently to changes in market conditions when they have ample time to adjust.

We do this in a simple way, by assuming that one (or more factors) can be adjusted quickly. These are called **variable factors**, represented by labor in the two factor model. Other factors require considerable time to adjust. These are called **fixed factors**, represented by capital in the two factor model.

Fixed costs only occur in the short run. In the long run, **all factors are variable**, meaning that **all costs are variable**.

In the short run, some factors of production are fixed, unchanging. Fixed factors typically include plant and capital equipment.

Other factors are variable. These usually include hourly labor, electricity, and other factors whose use changes with output (e.g., pizza dough at a pizza restaurant).

### 7.3.1 Fixed, Variable, and Total Cost

In our two factor production model, where the factors are  $K$  (capital) and  $L$  (labor), we take  $K$  to be fixed in the short run, but not in the long run.

The **fixed cost** and **variable cost** are the costs of using the fixed and variable factors.

Total cost (TC) in the two factor production model is the sum of fixed and variable costs:

$$TC = r\bar{K} + wL$$

where  $r$  is the hourly capital rental rate and  $w$  is the hourly wage rate.

Of these costs, the variable cost (VC) is the cost of the variable input, labor,  $wL$ .

The fixed cost (FC) is the cost of fixed capital,  $r\bar{K}$ . We then have the basic relation:

$$TC = r\bar{K} + wL = FC + VC.$$

**7.3.2 Fixed and Variable Cost with Cobb-Douglas**

Let's go back to the Cobb-Douglas production function,  $f(L, K) = \sqrt{LK}$ .

Suppose capital is fixed at  $K = 400$ , with a rental rate of \$50/hour and a wage rate of \$20/hour. The fixed cost is then  $FC = (50)(400) = \$20,000$ .

The variable cost will depend on how much of the variable factor ( $L$ ) is used. If we produce a quantity  $q$ , we set  $q = f(L, 400) = 20\sqrt{L}$  and then solve for  $L$ .

This yields short run labor demand of  $L = q^2/400$ . We multiply by the wage rate to get the variable cost of

$$VC = \frac{wq^2}{400} = \frac{20q^2}{400} = \frac{q^2}{20}.$$

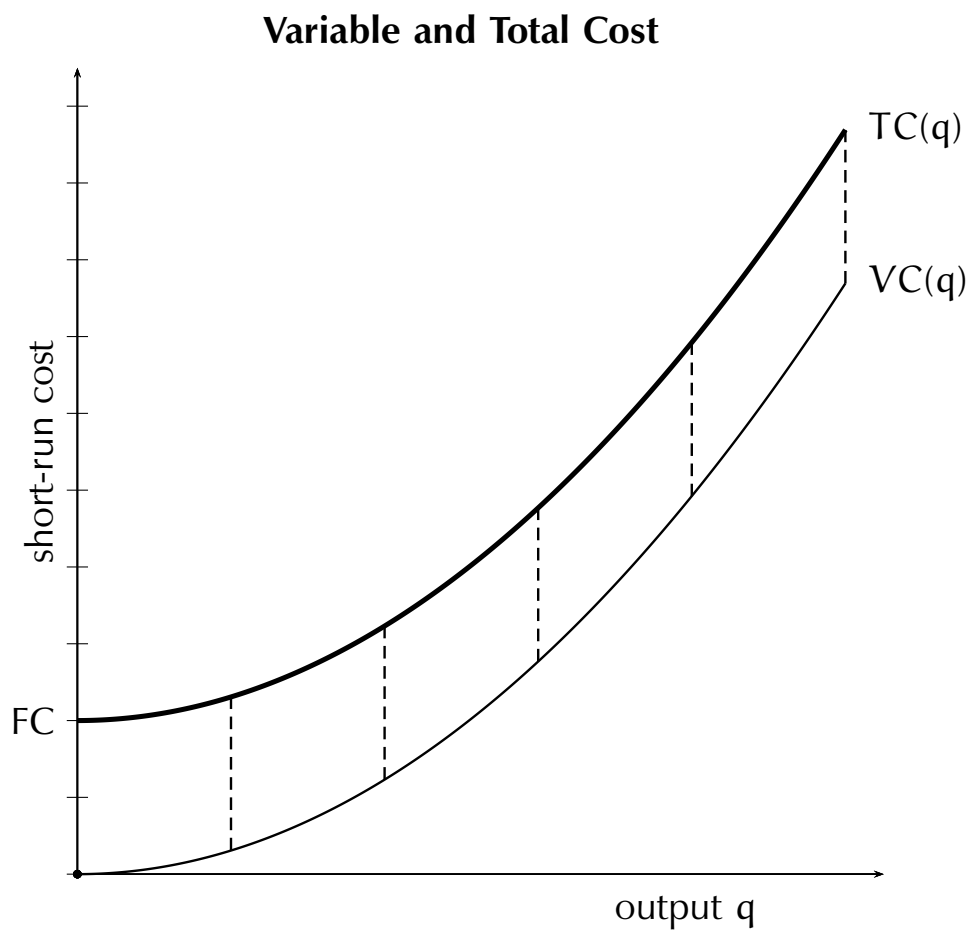
Total cost is then fixed plus variable cost:

$$TC = 20,000 + \frac{q^2}{20}.$$

### 7.3.3 Graphing Short-run Cost

The following diagram illustrates fixed, variable, and total short-run costs for Cobb-Douglas Production. The vertical distance between the variable and total cost curves is always the fixed cost.

The dashed vertical lines all have length equal to the fixed cost. Yes, there is a bit of an optical illusion. If you measure them, you can verify that they all have the same length.



## 7.4 Short-run Average and Marginal Costs

Every type of cost has an average cost and a marginal cost associated with it. Average costs are used to determine profit per unit (unit profit). We will compare prices and marginal costs to find how to maximize profit.

It's possible to put prices, marginal costs, and average costs on the same graph because they all are measured using the same units, dollars per unit of output, e.g., dollars per pound.

### 7.4.1 Average Costs

We find any average cost by dividing the cost by the quantity produced. We indicate this by putting an A in front of the cost symbol. E.g.,

$$ATC = TC / q, \quad AVC = VC / q, \quad AFC = FC / q.$$

Now  $ATC = AFC + AVC$  because  $TC = FC + VC$ . To see this apply the definition of average cost and separate terms, as below:

$$\begin{aligned} ATC &= \frac{TC}{q} = \frac{FC + VC}{q} \\ &= \frac{FC}{q} + \frac{VC}{q} \\ &= AFC + AVC. \end{aligned}$$



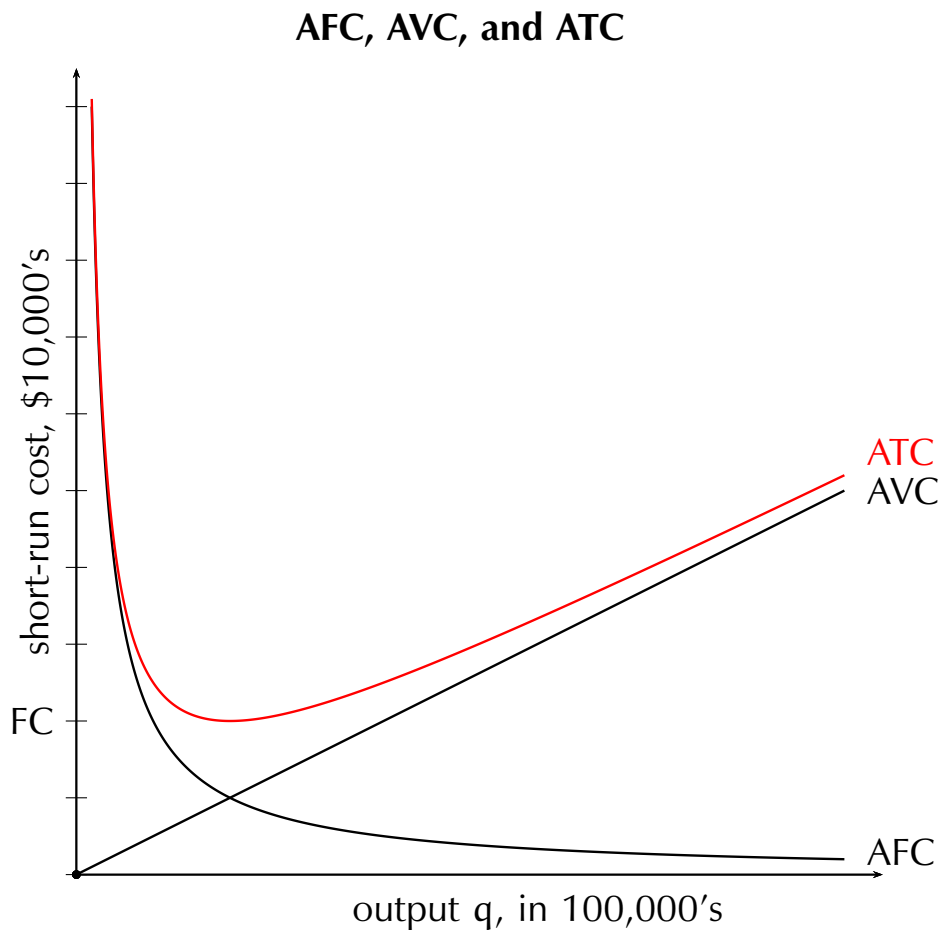
### 7.4.2 Calculating Average Costs

In our previous example,  $FC = \$20,000$  and  $VC = q^2/20$ . It follows that

$$AFC = 20,000/q, \quad AVC = q/20, \quad ATC = 20,000/q + q/20.$$

Notice that when  $q = 0$ ,  $AFC(q) = +\infty$ . This is always true when the fixed cost is non-zero. When the fixed cost is 0,  $AFC(0) = 0/0$ —an undefined quotient. However, is it best to interpret it as zero in this case.

### 7.4.3 A Graph of Average Costs



The average total cost curve has a typical U-shape. This comes from a combination of declining average fixed cost and an average variable cost that increases.

The **ATC** can also have a U-shape if the **AVC** decreases and then increases, although the rates matter then.

#### 7.4.4 Marginal Cost

The marginal cost is the change in cost divided by the change in quantity of output. Marginal cost is the slope of the cost curve. There are two ways to calculate it, using finite differences (chords) or calculus (tangents). Thus

$$MC = \frac{\Delta C}{\Delta q} \quad \text{or} \quad MC = \frac{dC}{dq}$$

where  $C$  is the cost function in question.

If we apply this to the fixed cost,  $\Delta FC$  is always zero because the fixed cost doesn't change. The marginal fixed cost is always zero. That means that the marginal total cost and marginal variable cost are the same. To see this,

$$\Delta TC = \Delta VC + \Delta FC = \Delta VC.$$

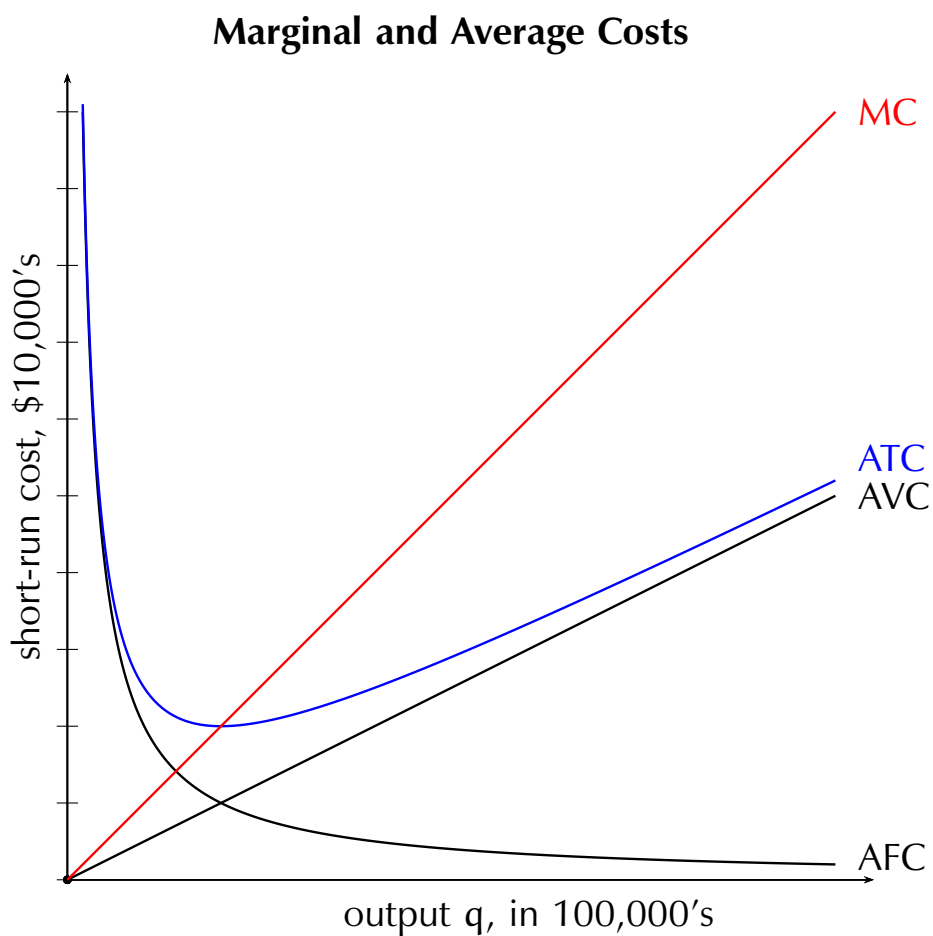
Dividing by  $\Delta q$  shows the two marginal costs are the same.

$$\frac{\Delta TC}{\Delta q} = \frac{\Delta VC}{\Delta q} + \frac{\Delta FC}{\Delta q} = \frac{\Delta VC}{\Delta q}.$$

Since these marginal costs are the same, we use the term **marginal cost** (MC) for their common value.

### 7.4.5 A Graph of Marginal and Average Costs

In our previous example,  $TC = 20,000 + q^2/20$ , so marginal cost is  $MC = q/10$ . The slope of marginal cost in the graph is one because I have adjusted the scales.



Notice that the marginal cost curve intersects both average variable cost and average total cost at their minimum points. This is not a coincidence! It is a property of many cost functions and their marginal costs. It even works when average variable cost is linear, as it is here. The marginal cost is the same for both cost functions.

### 7.4.6 More on Marginal Costs

The marginal cost (MC) is the rate at which cost changes when quantity changes. In calculus terms, marginal cost is the derivative  $MC = dC/dq$ , the slope of the cost function. This can be approximated by using finite changes,  $MC = \Delta C/\Delta Q$ .

We saw before that when

- $MC > AC$ , AC is rising with  $q$
- $MC < AC$ , AC is falling with  $q$
- $MC = AC$ , AC is at a minimum.

**7.4.7 Marginal and Average Cost: Calculus Version**

Consider the derivative of any average cost

$$AC = \frac{C(q)}{q}.$$

We have

$$\begin{aligned}\frac{d}{dq}(AC) &= \frac{dC/dq}{q} - \frac{C(q)}{q^2} \\ &= \frac{1}{q} \left[ \frac{dC}{dq} - \frac{C(q)}{q} \right] \\ &= \frac{1}{q} [MC - AC].\end{aligned}$$

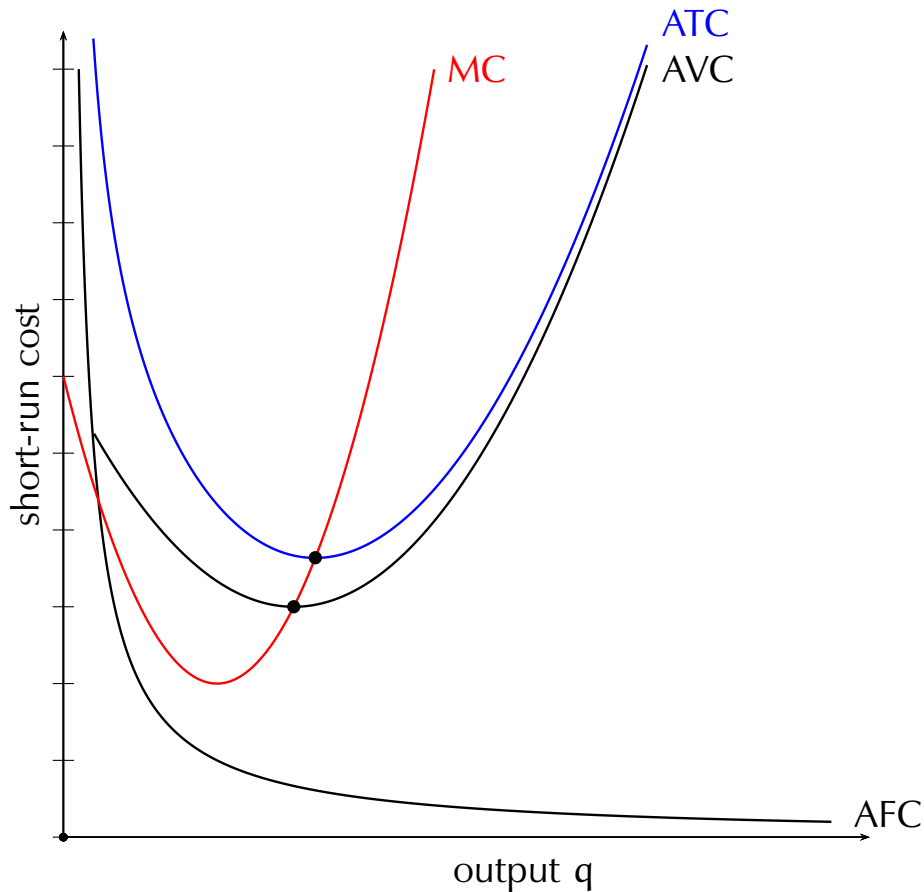
This gives us the same relationships.

- $MC > AC$ ,  $AC$  is rising with  $q$
- $MC < AC$ ,  $AC$  is falling with  $q$
- $MC = AC$ ,  $AC$  is at a minimum.

### 7.4.8 Another Short-run Cost Function

Here  $MC = q^2 - 4q + 6$ ,  $TC = q^3/3 - 2q^2 + 6q + 2$  where  $FC = 2$ . Then  $AVC = q^2/3 - 2q + 6$  and  $ATC = q^2/3 - 2q + 6 + 2/q$ . Again, the marginal cost cuts both  $AVC$  and  $ATC$  at their minima.

### Average and Marginal Costs Compared



Again, the average total and variable cost curves are U-shaped, with minima where the marginal cost intersects them. The vertical distance between the two average cost curves is the average fixed cost. It may not appear that way on the graph, but this is an optical illusion. If you measure it, you will see that.

### 7.4.9 Why use a U-shaped Marginal Cost Curve?

You'll notice the shape of the marginal cost was different in the previous diagram. It is often the case that firms will have increasing returns to scale when production is low. This is because their operation is designed to function properly only when throughput is sufficiently high.

Increasing returns mean decreasing average costs, hence the region of decreasing costs. In competitive markets, these increasing returns will run out before the firm has reached limits of demand. Otherwise, we have conditions leading to monopoly or oligopoly, which we will start analyzing in a couple of weeks.

Eventually, the presence of fixed factors of production brings the Law of Diminishing Returns into play. Even if we merely end up with constant returns to scale, the presence of a fixed factor requires that the variable factors have diminishing returns to scale.

We've seen this when using the two factor Cobb-Douglas production function, where marginal costs are linear in the variable factor. This is true of any constant returns to scale production function with at least one fixed factor and one or more variable factors.

*October 14, 2022*