Intermediate Microeconomics — Week 9

Professor Boyd **October 18 & 20, 2022**

QUIZ: Thursday's quiz covers weeks 7 and 8, including all of Chapter 6 and Chapter 7 through section 7.4 (pp. 181–237).

7.5 Long Run Costs

When necessary, we indicate long-run or short-run costs by prepending LR or SR to the name of the cost. This creates combinations such as LRMC, SRMC, LRATC, and SRATC. The distinction between fixed and variable cost only exists in the short run, so we do not need to write things like SRAFC as the average fixed cost AFC is necessarily a short run cost.

Our main focus here will be the comparison between long and short run cost curves. For that, we will mainly base the discussion on Cobb-Douglas production. The quantitative results will not apply in general, but the qualitative results will.

For use in macro models, Cobb-Douglas production is often enough due to averaging over the entire economy.

7.5.1 Cobb-Douglas Production and the Aggregate Economy

One striking fact about the US economy is that the shares of net income paid to capital and labor have remained remarkably constant over the last century. Capital gets about 30% of net business income while labor receives about 70% of net business income.

Here net business income means business income minus business taxes and depreciation of capital. This seems to be the correct way to measure this.[1]

If you measure the shares using gross business income, labor's share has been falling in recent decades.

[1] See Benjamin Bridgman (2018), Is labor's loss capital's gain? Gross versus net labor shares, *Macroeconomic Dynamics*, **22**, 2070–2087.

7.5.2 Cost Shares with Cobb-Douglas Production

This can be modelled by use a Cobb-Douglas production function of the form $F(L, K) = AL^{0.7}K^{0.3}$. To see how this works, consider the Cobb-Douglas production function $F(L, K) = AL^aK^b$.

We will assume constant returns to scale. That means $a + b = 1$, or $b = 1 - a$. Setting the marginal rate of technical substitution equal to the relative price, we obtain

$$\frac{w}{r} = \frac{a}{b}\left(\frac{K}{L}\right),$$

so

$$rK = \frac{b}{a}wL = \frac{1-a}{a}wL.$$

Then

$$\frac{wL}{C(q)} = \frac{wL}{wL + rK} = \frac{1}{1 + (1-a)/a} = a.$$

In other words, under constant returns to scale, the exponent on labor is labor's share of costs. Under constant returns to scale, profit maximization implies that income and cost are equal, so labor income is also a share a of firm income, meaning income net of business taxes and depreciation.

When $a = .7$, that means that labor gets paid $a = .7 = 70\%$ of firm income.

7.5.3 Cobb-Douglas: Long Run Cost

Since all Cobb-Douglas functions work in a similar way, we will use a Cobb-Douglas model with numbers that have been picked to simplify calculation rather than $a = 0.7$ and $b = 0.3$. We will use $A = 1$, $a = b = 0.5$, wage rate $w = 25$, and capital rental rate $r = 100$. Then

$$F(L, K) = \sqrt{LK}$$

and the optimality condition is

$$\frac{K}{L} = \frac{w}{r} = \frac{1}{4}$$

meaning that $K = L/4$. Now $F(L, K) = q$, so

$$q = \sqrt{LK} = \sqrt{L^2/4} = L/2.$$

The long run demands are $K^* = q/2$ and $L^* = 2q$ and long run cost is $wL^* + rK^* = 25(2q) + 100(q/2) = 100q$.

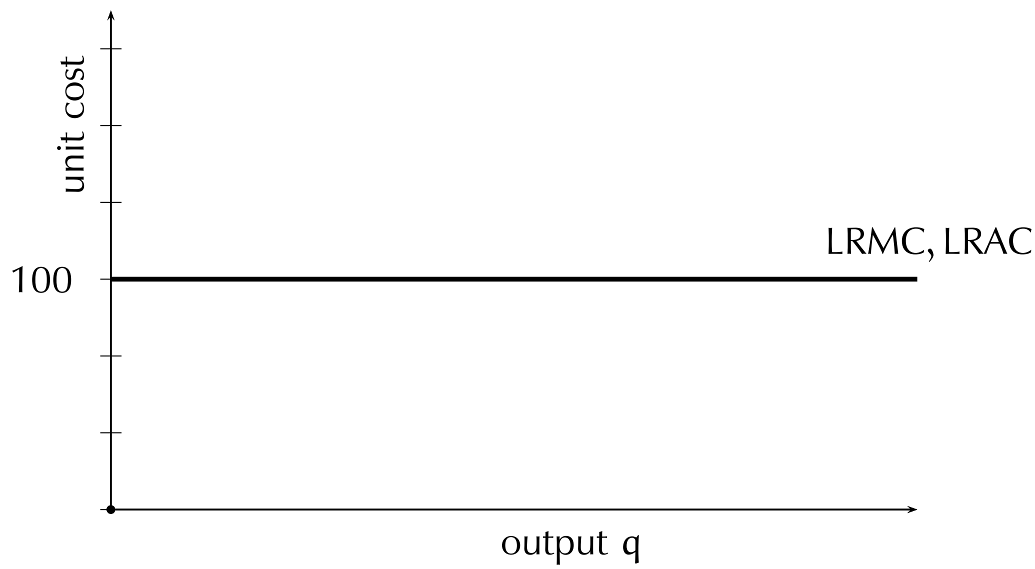
Long run average cost is LRAC = 100 and is equal to long run marginal cost of LRMC = 100.

The constant returns to scale are responsible for the fact LRMC = LRAC.

7.5.4 Graph of Long Run Cobb-Douglas Costs

Using the values $w = 25$ and $r = 100$, long run average and marginal costs are the same, LRMC = LRAC = 100.

LRAC and LRMC for Cobb-Douglas



Whenever production has constant returns to scale, long run marginal and average costs will be constant and equal.

7.5.5 Cobb-Douglas: Short Run Costs I

If we have a fixed capital stock of $\bar{K}$, $q = \sqrt{\bar{K}L}$. The labor input required to produce quantity q is $L(q) = q^2/\bar{K}$ and variable cost is

$$VC(q) = wL(q) = \frac{wq^2}{\bar{K}}.$$

Fixed cost is $FC = r\bar{K}$, so total cost is

$$TC(q) = r\bar{K} + VC(q) = r\bar{K} + \frac{wq^2}{\bar{K}}.$$

We divide by q to obtain average total cost,

$$ATC(q) = \frac{r\bar{K}}{q} + \frac{wq}{\bar{K}} = \frac{100\bar{K}}{q} + \frac{25q}{\bar{K}}$$

where $AFC = r\bar{K}/q = 100\bar{K}/q$ and $AVC = wq/\bar{K} = 25q/\bar{K}$.

We compute marginal cost by taking the derivative of variable cost:

$$MC(q) = \frac{2wq}{\bar{K}} = \frac{50q}{\bar{K}}.$$

In such models, the short run marginal cost is twice the average varible cost.

$$MC(q) = \frac{2wq}{\bar{K}} = 2\frac{wq}{\bar{K}} = 2\,AVC(q).$$

In Cobb-Douglas models, both the short run marginal and average variable cost are linear functions. The marginal cost has twice the slope as average variable cost. Both are inversely related to the fixed capital stock.

7.5.6 Cobb-Douglas: Short Run Costs II

We find the quantity q^* that minimizes short run average total cost by finding the intersection of(short-run) marginal cost and (short-run) average total cost. That is, we set

$$\mathrm{SRMC}(q^*) = \mathrm{SRATC}(q^*)$$

and then solve for q^*:

$$\frac{2wq^*}{\bar{K}} = \frac{r\bar{K}}{q^*} + \frac{wq^*}{\bar{K}}.$$

Subtracting $wq^*/\bar{K}$ from both sides yields

$$\frac{wq^*}{\bar{K}} = \frac{r\bar{K}}{q^*},$$

so that

$$w(q^*)^2 = r\bar{K}^2. \quad \text{In other words} \quad (q^*)^2 = \frac{r}{w}\bar{K}^2.$$

Then short-run cost is at a minimum at q^* given by

$$q^* = \bar{K}\sqrt{\frac{r}{w}}.$$

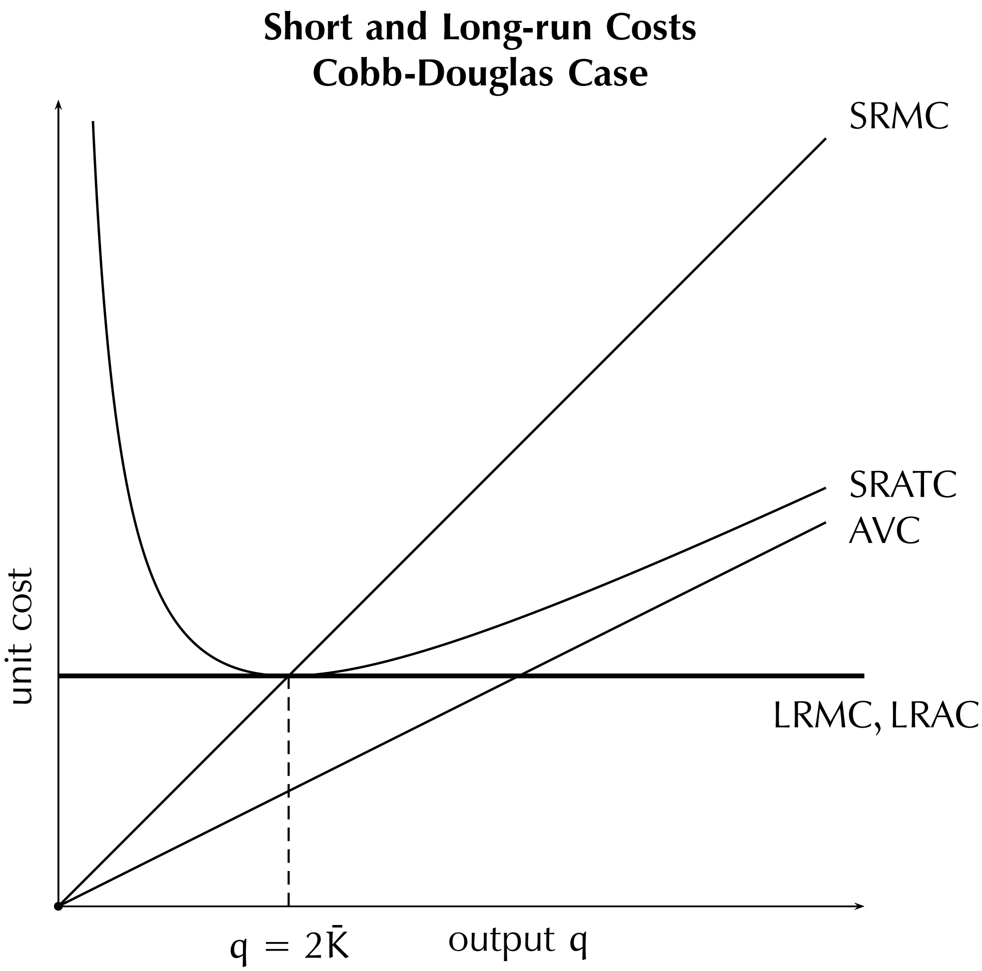
It follows that the minimum short-run average total cost (SRATC) is

$$\mathrm{ATC}(q^*) = \mathrm{SRMC}(q^*) = \frac{2wq^*}{\bar{K}} = 2w\sqrt{\frac{r}{w}} = 2\sqrt{rw},$$

which is the same as the long run average cost. For Cobb-Douglas, the minimim of SRATC is always LRAC. That is true not just of Cobb-Douglas or constant elasticity of substitution production functions, but whenever the production function is homogeneous of degree one (i.e., constant returns to scale).

7.5.7 Graphing Short and Long Run Cobb-Douglas Costs

Using the values $w = 25$ and $r = 100$, long run average and marginal costs are the same, LRMC = LRAC = 100. Now suppose that fixed capital is $\bar{K} = 300$. We add the short-run cost curves to the graph.

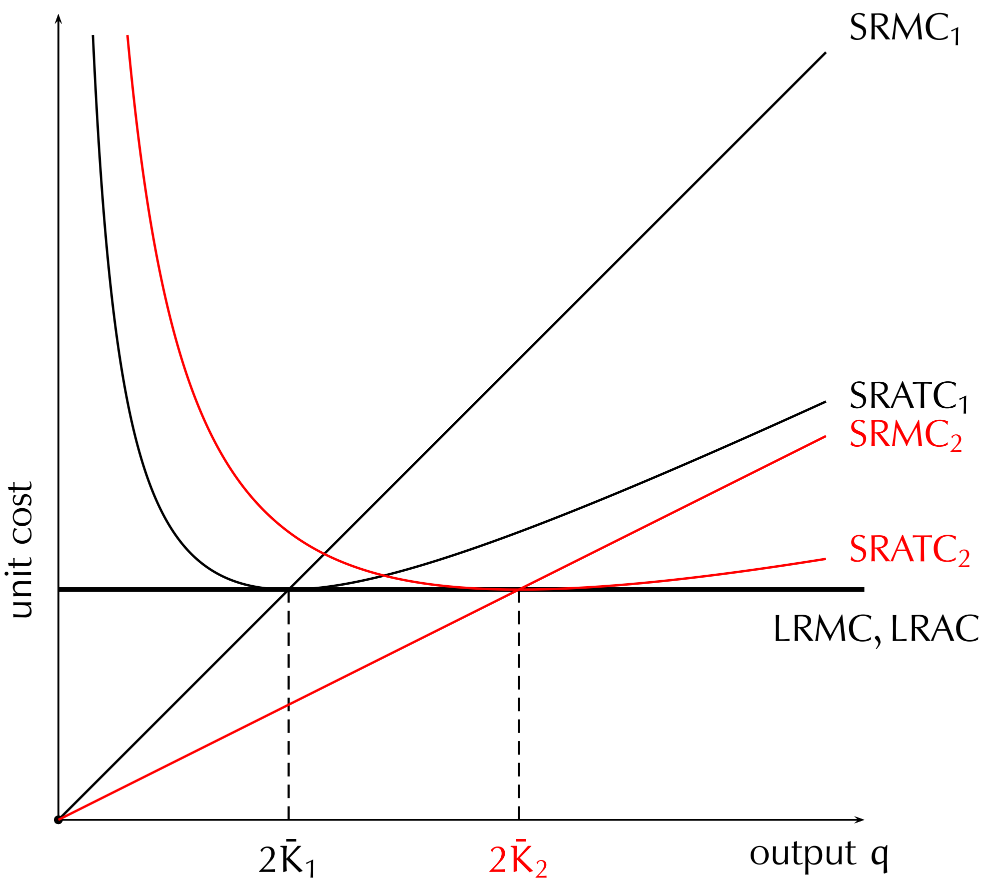


Whenever production has constant returns to scale, long run marginal and average costs will be constant and equal. Notice that the short run and long run costs are only equal when $\bar{K} = q/2$, when the fixed capital stock is exactly equal to the long run optimal quantity.

7.5.8 Changing the Fixed Capital Stock

Increasing fixed capital from K_1 to K_2, increases the optimal production level, and leads to a new set of short run cost curves. Depending on the output level, it may be cheaper to have capital stock K_1 or K_2.

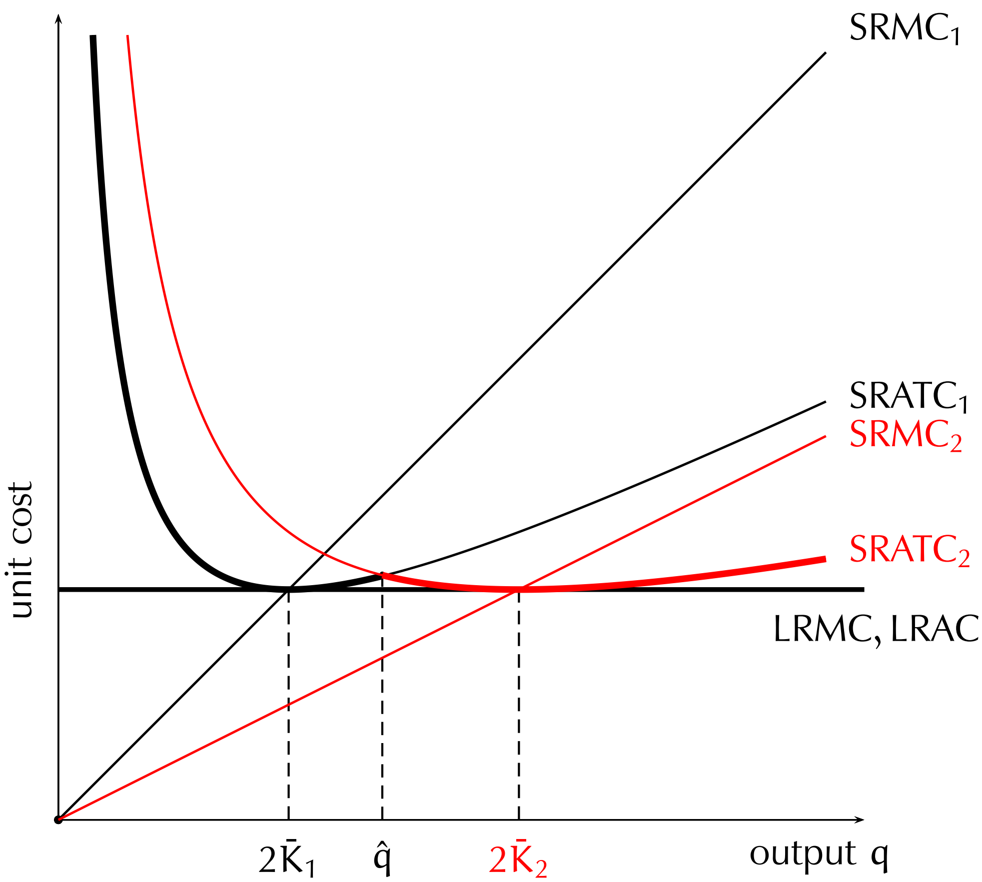
Effect of Fixed Capital Stock on Cost



7.5.9 Cost Comparison for two Fixed Capital Stocks II

The output lever $\hat{q}$ determines whether the firm will be able to produce at lower cost with $K = \bar{K}_1$ or $K = \bar{K}_2$. The heavier line indicates whether $SRATC_1$ or $SRATC_2$ is lower cost at any q.

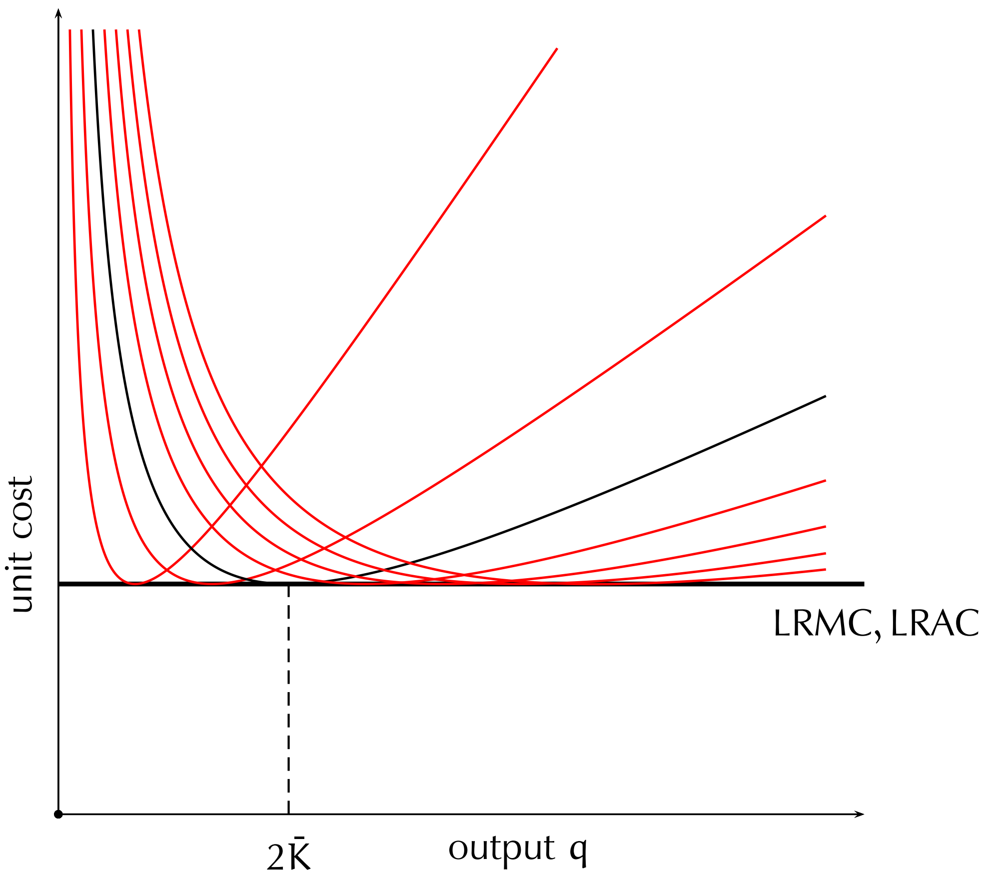
Cost with Two Different Fixed Capital Stocks



7.5.10 Multiple Short Run Cost Curves

The red curves are short run cost curves for several different capital stocks, ranging from $\bar{K}/3$ to $7\bar{K}/3$ in increments of $\bar{K}/3$. The previous SRATC is shown in black, with a capital stock of $\bar{K}$ and optimal production level (lowest ATC) of $q^* = 2\bar{K}$. As you can see, the LRAC curve is the lower limit of all of the SRATC curves.

Cost with Many Different Fixed Capital Stocks

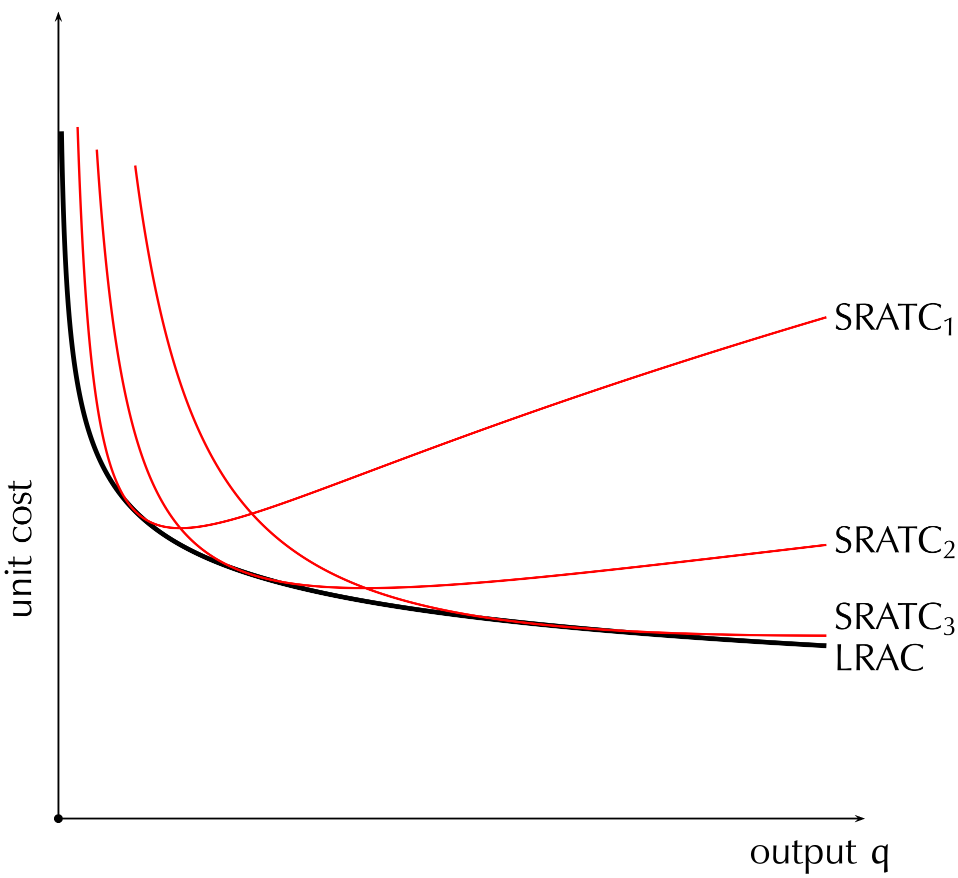


7.5.11 What if there are Increasing or Decreasing Returns

Once again, the LRAC curve is the lower limit of the SRATC curves, but trends downward under increasing returns and upward under decreasing returns.

Here's an example based on $F(L, K) = L^{2/3}K^{2/3}$. The red curves are various short run average cost curves. The heavy black curve is long run average total cost.

Short and Long Run Cost with Increasing Returns to Scale

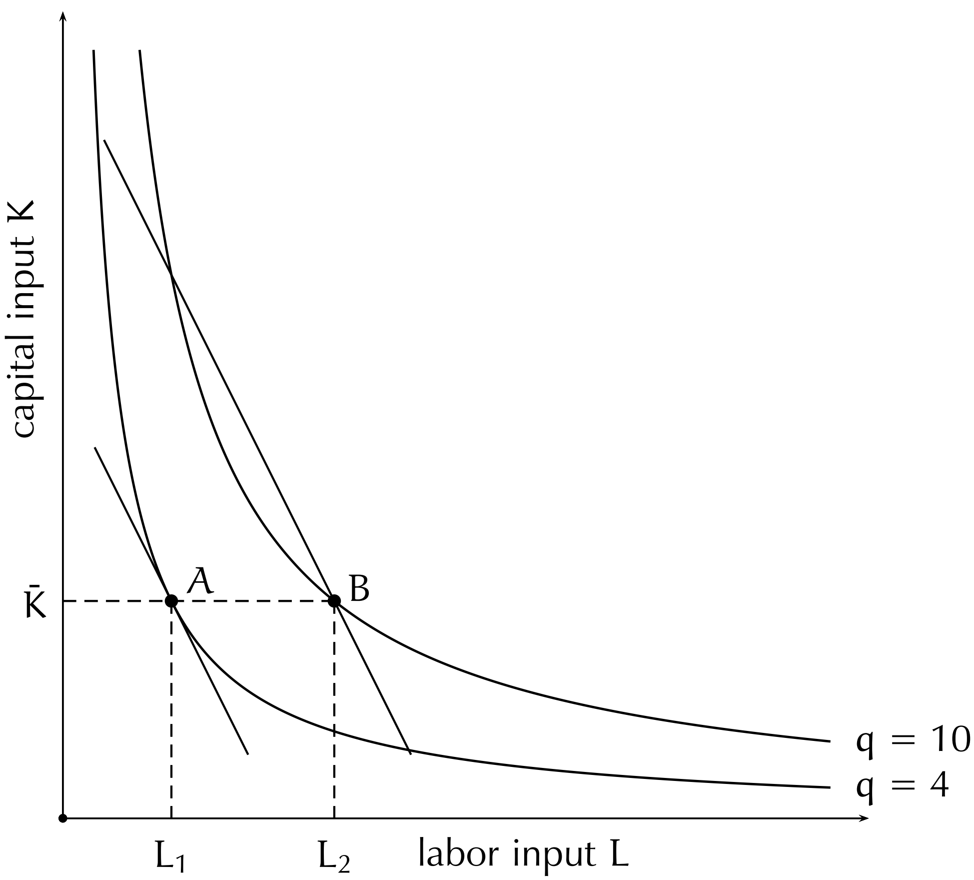


7.5.12 Short and Long Run Cost with Isoquants I

In the short run, capital is fixed at $\bar{K}$. We start at the long run optimum point A, where the marginal rate of technical substitution is equal to the relative price of labor (2), as shown by the tangent line at A.

In the short run, the only way to increase production from $q = 4$ to $q = 10$ is to increase the labor input from L_1 to L_2. That is, we move from A to B. However, this is not optimal in the long run. You can see that the line with slope -2 through B is not tangent to the $q = 10$ isoquant.

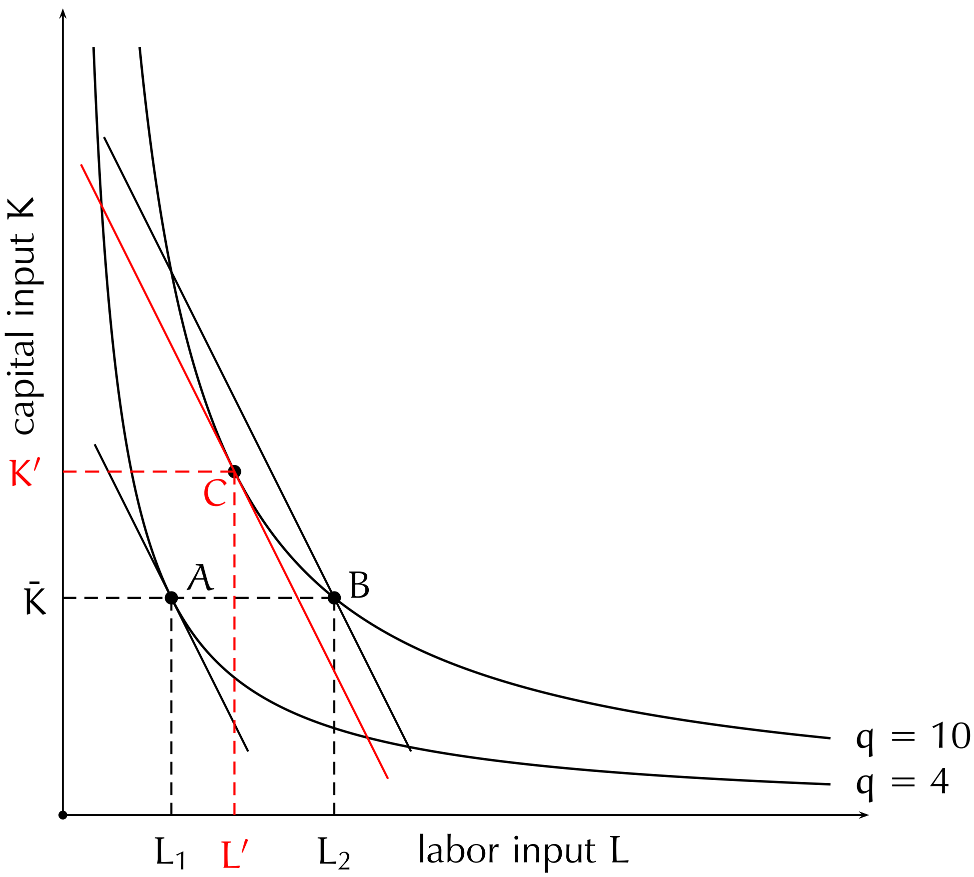
Comparison of Short and Long Run Cost using Isoquants



7.5.13 Short and Long Run Cost with Isoquants II

The long run optimal point for $q = 10$ is at $C = (L', K')$, where cost is lower than at B (compare the height of the isocost lines through B and C). Comparing points B and C, the substitution of capital for labor allows for lower costs at C.

Comparison of Short and Long Run Cost using Isoquants



7.5.14 Long Run Marginal Cost I

We have two different ways to compute marginal cost in the long run. We can increase production by using more labor or by using more capital.

If we use more labor,

$$\frac{\Delta q}{\Delta L} \approx MP_L \, .$$

Then

$$\Delta L \approx \frac{\Delta q}{MP_L}$$

so the cost of the additional labor is

$$w \Delta L \approx w \frac{\Delta q}{MP_L}.$$

The marginal cost using labor is

$$MC = w \frac{\Delta L}{\Delta q} \approx \frac{w}{MP_L}.$$

7.5.15 Long Run Marginal Cost II

A similar calculation using capital tells us the marginal cost is

$$MC = \frac{r}{MP_K}.$$

We have two different-looking expressions for the marginal cost.

Fortunately, they are the same. At the long run cost minimizing point,

$$MRTS = \frac{w}{r}.$$

But

$$MRTS = \frac{MP_L}{MP_K},$$

so

$$\frac{MP_L}{MP_K} = \frac{w}{r}.$$

Divide by MP_L and multiply by r to find

$$\frac{r}{MP_K} = \frac{w}{MP_L}. \tag{7.5.1}$$

This shows that the two different ways of computing long run marginal cost are the same!

Equation (7.5.1) is written in marginal cost form. It can also be inverted to write it in terms of marginal product per dollar of input (bang for the buck).

$$\frac{MP_K}{r} = \frac{MP_L}{w}. \tag{7.5.2}$$

7.5.16 Optimality with Many Inputs

We can use equation (7.5.2) to rewrite the optimality condition to handle the case of many inputs. Optimality requires that all inputs K_i and L_j that are actually used have the same "bang for the buck". That is, for all i and j.

$$\frac{MP_{K_i}}{r_i} = \frac{MP_{L_j}}{w_j}$$

and moreover, if K_i or L_j is not used in production, their "bang for the buck" is never more than the "bang for the buck" of the goods that are used in production (more likely less rather than equal).

8.1 Chapter 8: Supply in a Competitive Market

Production and pricing decisions are affected by the type of market a firm operates in. In this course, we will consider 4 basic types of markets, Perfect Competition, Monopolistic Competition, Oligopoly, and Monopoly.

- Monopoly = One seller
- Oligopoly = a few sellers
- Perfect and Monopolistic Competition have many sellers

These markets are distinguished by the number of firms, how similar their products are (product differentiation), and the size of any barriers to entry or exit.

8.1.1 Four Model Markets

We will focus on four types of model markets. These don't exhaust all of the possibilities, but cover a good variety of them.

Type of Market	Number of Firms	Differentiation	Barriers to Entry/Exit
Perfect Competition	Many	None	None
Monopolistic Competition	Many	Yes	Maybe
Oligopoly	A Few	Maybe	Some
Monopoly	One	Unique	A lot

In this chapter, we focus on the first case, perfect competition.

8.1.2 Perfect Competition: Supply and Demand

The perfectly competitive model is the supply/demand model. There are many firms producing what is essentially the same product, and firms face no barriers to entry or exit from that market. We will also presume that all of the firms are relatively small. That makes them price-takers. They accept the market price and do not attempt to change it. Due to their relatively small size, they can't change it.

Price taking is sometimes practiced by firms that are large in an absolute sense. Many, many years ago I watched part of a congressional hearing on the petroleum industry. The part I recall best focused on how firms decided what wholesale price of gasoline to charge.

They questioned an Amoco executive about pricing. At the time, Amoco (Standard Oil of Indiana, now part of BP) controlled about 10% of the wholesale market. That meant they were a big firm in an absolute sense, but not so much as a share of the market. Not small, but not really big either.

8.1.3 Wholesale Gasoline Pricing

So how did Amoco decide how much to charge? When the executive got up in the morning, he called a contact familiar with the current state of the spot gasoline market, to find out how much wholesale gasoline was selling for. Once he knew, he made another call, instructing Amoco to use that price for the day. This is classic price-taking behavior. Find out the market price, and then use it.

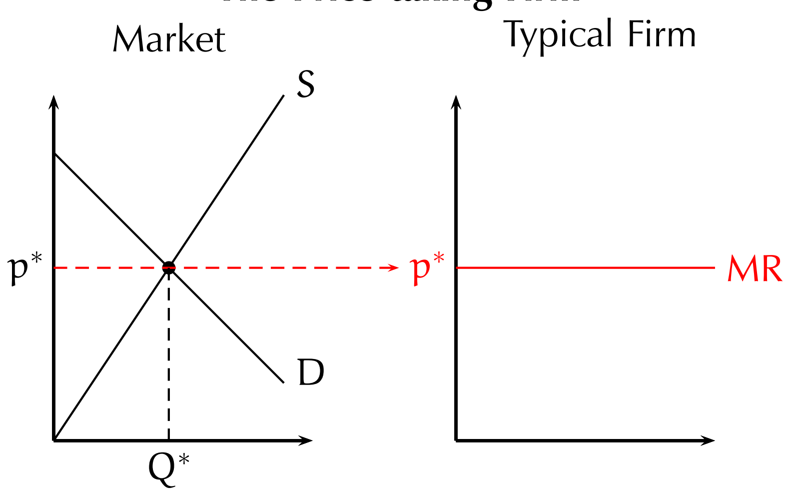
Amoco had no incentive to lower the price, as they could sell what they wanted at the market price. And if they tried to raise the price, they would lose too many customers—maybe all of them—for the day. So they changed the market price—no more, no less.

8.1.4 The Price-taking Firm

To model the price-taking firm, we use a double diagram. The left side is a supply and demand model that tells us what is happening in the market as a whole. The right side uses the price from the supply/demand model and examines how a typical firm responds.

For the moment, the right part of the diagram simply includes the price used by the price-taking firm.

The Price-taking Firm



Here Q^* is the market quantity supplied.

8.1.5 Profit for a Price-taking Firm

The price-taking firm will maximize profit. Of course, profit is the difference between revenue and (economic) costs. For a price-taking firm, revenue is p^*q where q is the firm's output. Cost is given by the cost function, so it maximizes

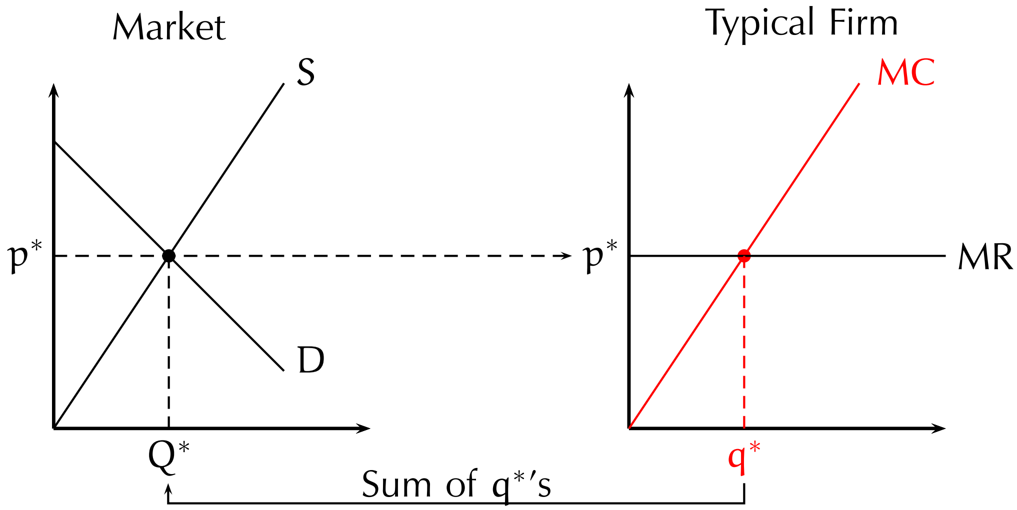
$$p^*q - C(q) = R(q) - C(q)$$

where $R(q) = p^*q$ is revenue.

8.1.6 More about Profit Maximization

Already on day 1, we saw that maximization requires marginal benefit equal marginal cost. For a price-taking firm, the marginal benefit (marginal revenue) is $MR = p^*$. We find the firm's profit-maximizing output q^* by setting $p^* = MC(q^*)$. For other types of firm, we still set $MR = MC$, but marginal revenue differs from price.

The Price-taking Firm and the Market



We find the market supply Q^* by adding together the supplies q^* by the individual firms.

8.1.7 Market vs. Firm Supply

If there were N identical firms, each would supply q^* for a total supply of Nq^* at price p^*. Changes in the number of firms shift the supply curve in the left hand diagram.

Interestingly, the number of firms doesn't affect the elasticity of supply.

To see this, take N firms, each with the same supply function $q(p)$. Then market supply is $Q(p) = Nq(p)$.

The market elasticity of supply is

$$\begin{aligned} \frac{p}{Q(p)}\frac{dQ}{dp} &= \frac{p}{Nq(p)}\left(N\frac{dq}{dp}\right) \\ &= \frac{p}{q(p)}\frac{dq}{dp}, \end{aligned}$$

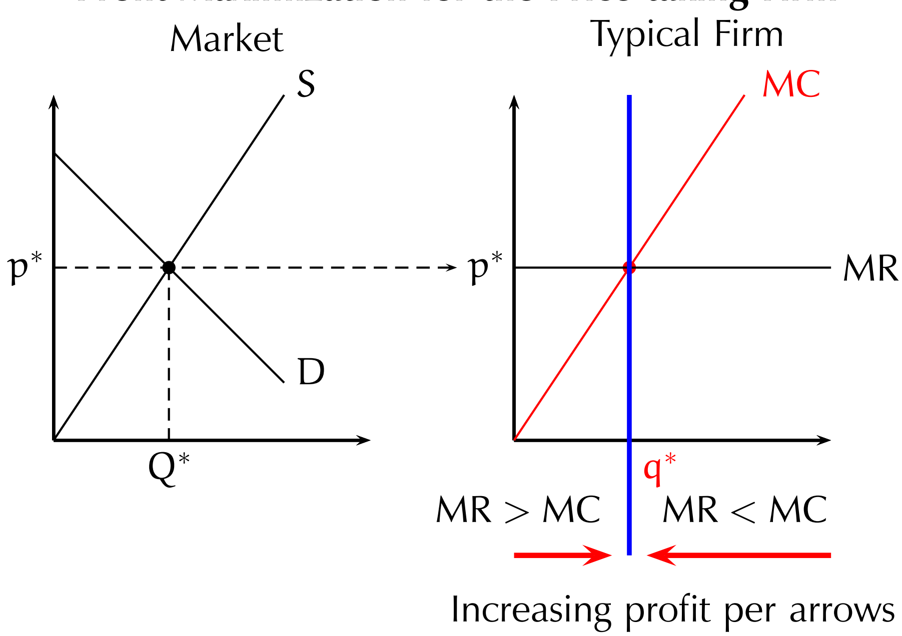
which is the elasticity of supply of each of the identical firms.

8.1.8 Profit Maximization for a Price-taking Firm

To understand better why profit is maximized at q^*, we consider what happens away from q^*. To the left of q^*, $p^* = MR > MC$, meaning that increasing output increases profit. To the right of q^*, $p^* = MR < MC$, so increasing output decreases profit. The red arrows indicate the direction that profit increases.

This means that profit increases as we move rightward to q^*, then decreases. In other words, q^* maximizes profit.

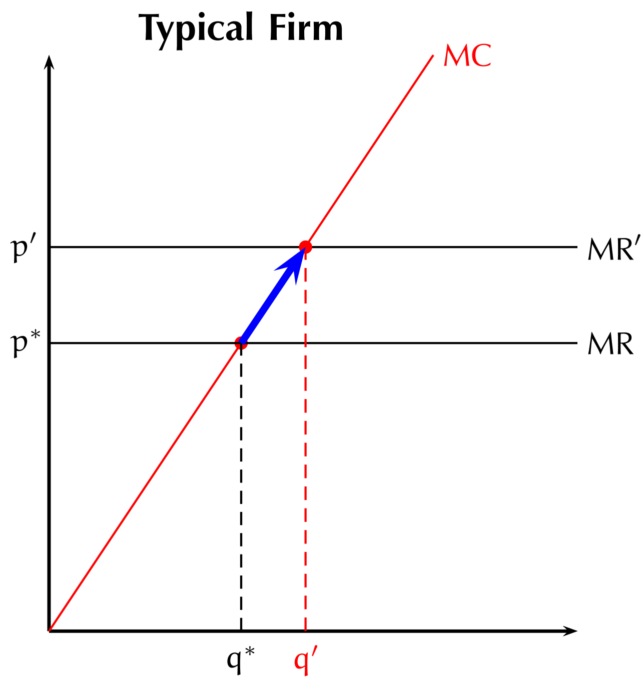
Profit-Maximization for the Price-taking Firm



8.1.9 Effect of a Price Increase

10/20/22

How does the typical firm react to an increase in price? To answer this, we use the firm's diagram and boost the price to p'. The increased prices causes the firm to increase output to q'. The firm marches up the marginal cost curve.



Recall that the supply curve tells us for each price, how much the firm wants to supply at that price. That is exactly what the marginal cost curve is doing here. The marginal cost curve **is** the supply curve for the individual firm, and market supply is the horizontal sum of marginal cost curves of the firms. It represents marginal cost at each point on it. Supply is marginal cost.

8.1.10 Derived Demand

The fact that marginal cost equals price tells us more. We know that in both short and long run,

$$MC = \frac{w}{MP_L}$$

Since firm chooses q^* with $p = MC(q^*)$,

$$p = \frac{w}{MP_L}.$$

We can rearrange this equation to read

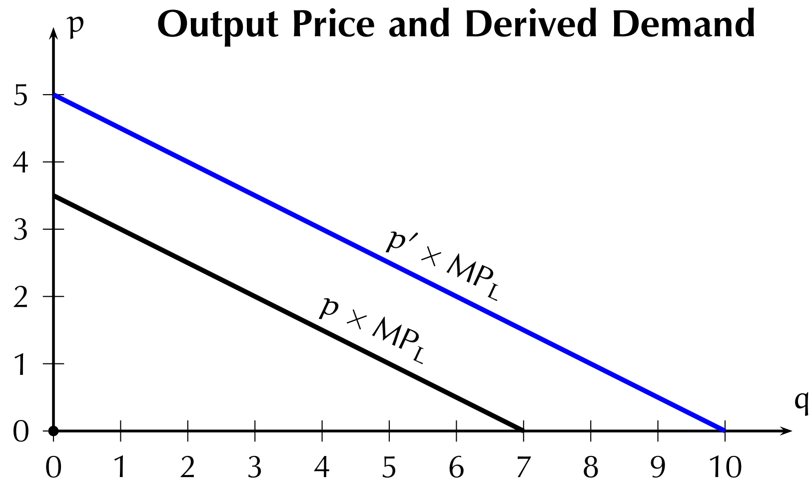
$$w = p \times MP_L(L).$$

This allows us to read the demand for labor off the profit maximization equation. For each wage rate w, we can solve for labor demanded L.

This is a **derived demand** in that demand for labor by the firm depends on the demand for the firm's product via the product price p. The right-hand side is called the **value of marginal product of labor**, VMP_L.

8.1.11 Effect of Product Price on Demand for Factors

Changes in the price of a product will affect demand for the factors used to produce that product. Suppose $p' > p$. Then the demand for labor increases in the same proportion as the output price. This is illustrated below.



There is more about derived demand and labor markets in Chapter 13.

8.2 Costs and Supply

We previously wrote profit (π) in terms of average total cost:

$$\pi = q^* \left[p^* - \text{ATC}(q^*) \right].$$

This let us illustrate profit or loss as a box on the firm's diagram.

8.2.1 Short-run Profit and Average Total Cost

We can rewrite the expression for a price-taking firm's profit in terms of average total cost. Suppose the market price is p^*, and profit is maximized at q^*, where $p^* = MC(q^*)$.

Profit (π) is revenue minus cost:

$$\pi = p^* q^* - C(q^*).$$

We rewrite by factoring out q^*,

$$\begin{aligned}\pi &= q^* \left[p^* - \frac{C(q^*)}{q^*}\right] \\ &= q^* \big[p^* - ATC(q^*)\big]. \end{aligned} \tag{8.2.1}$$

As a result

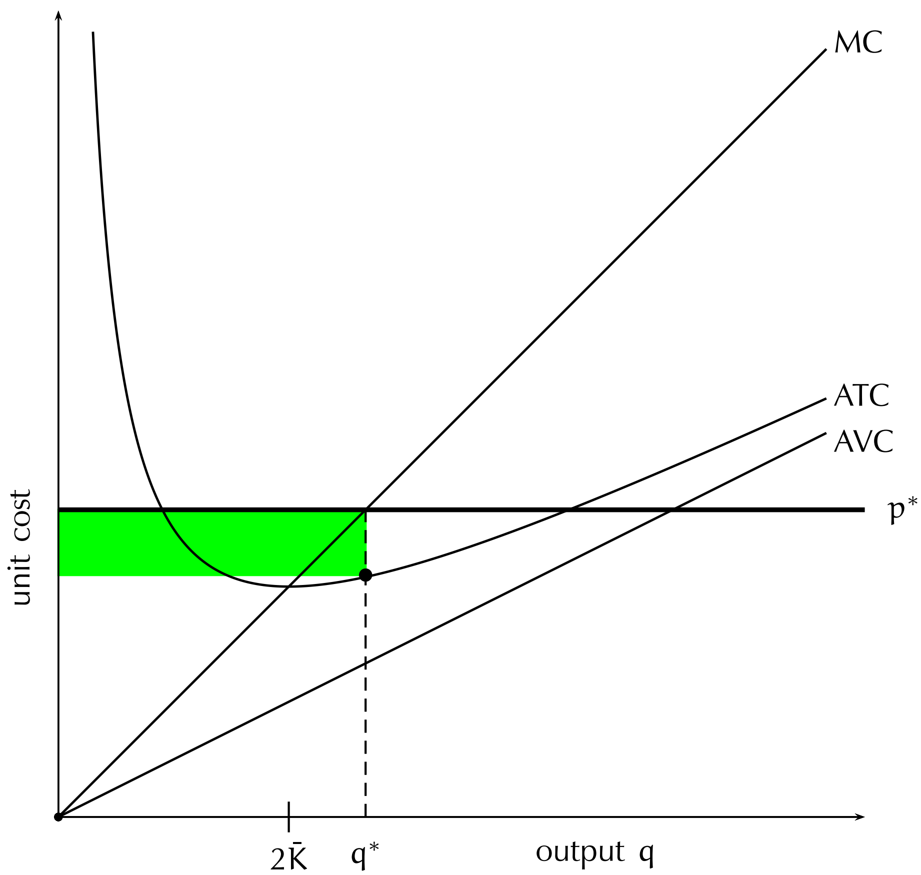
- $\pi > 0$ if and only if $p^* > ATC(q^*)$
- $\pi < 0$ if and only if $p^* < ATC(q^*)$
- $\pi = 0$ if and only if $p^* = ATC(q^*)$

8.2.2 Positive Profit

We can illustrate profit on the firm's graph by using a profit box. By equation (8.2.1) the amount of profit is a box with base q^* and height $p^* - \text{ATC}(q^*)$. The top is at p^* and the bottom at $\text{ATC}(q^*)$. If profit is negative, the top is below the bottom.

Here's a case with positive profit. The profit is the area of the green box. The height of the black dot indicate the average total cost at the profit-maximizing quantity q^*. ATC will be different at other quantities.

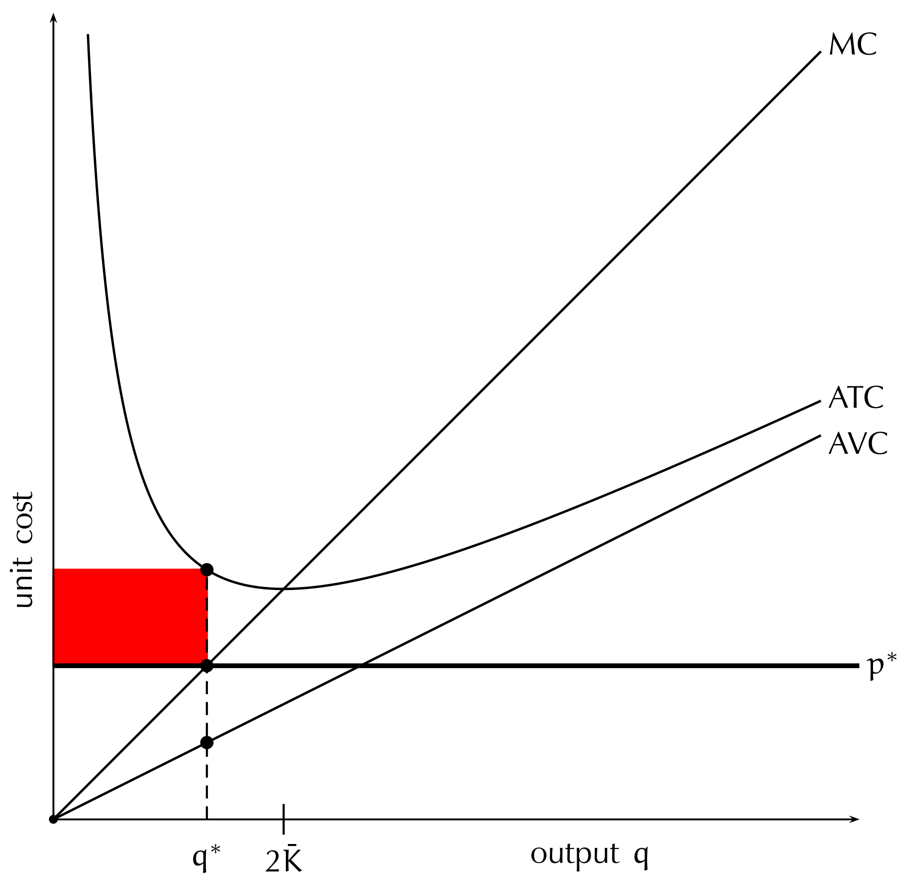
A Profit-Making Firm



8.2.3 Loss

Here's a case with a loss. The loss is the area of the red box.

A Firm Making a Loss

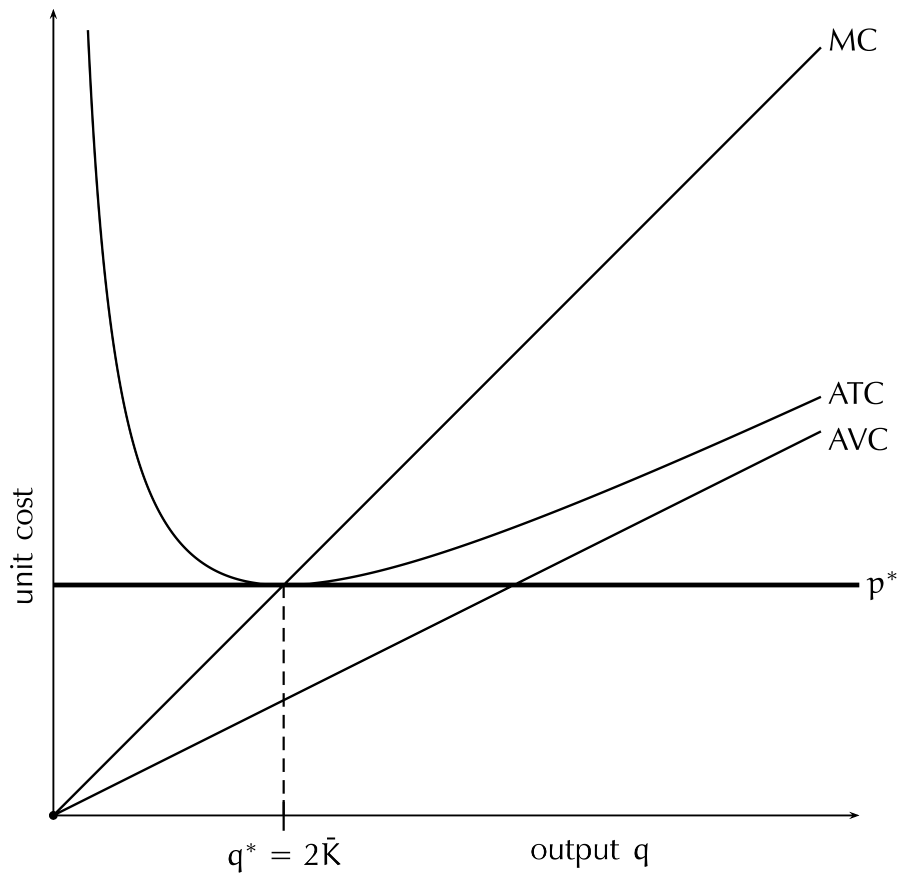


From top to bottom, the three dots at q^* represent ATC(q^*), MC(q^*), and AVC(q^*).

8.2.4 Zero Profit

Finally, when maximum profit is zero, $p^* = ATC(q^*)$ and the box disappears.

Zero Profit



One interesting thing about the zero profit case under constant returns to scale (including Cobb-Douglas) is that the market price is also the minimum average total cost, where $MC(q^*) = ATC(q^*)$

When a firm earns zero profits, it is also producing at the lowest possible average total cost. This indicates the firm is using resources efficiently.

8.2.5 Long Run Behavior of Typical Firms

We are not yet done with profit. If a firms profit is not zero, the firm's owners will have an incentive to change what they are doing.

If the firm is making a loss, it will need to restructure in the long run, to end the losses. It can do this in two possible ways: it can close (free exit), or it can downsize.

This happens not just for our firm, but others in the industry, and the mass behavior of firms shifts the supply curve inward.

If the firm is making a profit, that means they are making an above-market return on their investment. They may exploit this by expanding, and other entrepreneurs that have not entered this market have an incentive to do so. Under free entry, they are able to do so.

This expansion and entry shifts the supply curve outward.

The market is not in long run equilibrium when firms' profits are not zero.

8.2.6 Consequences of Losses

If the firm is losing money, it needs to close or restructure in long run. But that brings up another question. Should it even be producing now? Should the firm close in the short run?

For that, we have to compare the alternatives.

If the firm closes in the short run, quantity produced is zero, revenue is zero, and variable cost is zero. However, the firm cannot escape its fixed costs in the short run, and incurs a loss equal to its fixed cost. That is,

$$\pi_0 = -\,FC\,.$$

If they stay in business in the short run, they also make a loss,

$$\pi = p^* q^* - TC(q^*) < 0.$$

The question is how the loss from staying in business in the short run compares to losing the fixed cost.

That is, what is the difference in profit? We compute it:

$$\pi - \pi_0 = \left[p^* q^* - TC(q^*)\right] - (-\,FC).$$

8.2.7 Comparison of Losses

Now

$$\begin{aligned}
\pi - \pi_0 &= \pi - (-\,\text{FC}) \\
&= \pi + \text{FC} \\
&= \left[p^* q^* - \text{TC}(q^*)\right] + \text{FC} \\
&= \left[p^* q^* - \text{VC}(q^*) - \text{FC}\,\right] + \text{FC} \\
&= \left[p^* q^* - \text{VC}(q^*)\right] \\
&= q^* \left[p^* - \frac{\text{VC}(q^*)}{q^*}\right] \\
&= q^* \left[p^* - \text{AVC}(q^*)\right].
\end{aligned}$$

Our calculation shows that the firm will be better off operating in the short run if $p^* > \text{AVC}(q^*)$ and will be better off closing immediately if $p^* < \text{AVC}(q^*)$.

8.2.8 Comparison of Losses: Cobb-Douglas

Suppose a firm has the Cobb-Douglas production function $F(L, K) = AL^a K^b$ with $A, a, b > 0$ and $a, b < 1$. Fix capital at $\bar{K}$ so that $q = AL^a \bar{K}^b$. Now solve for L:

$$L = \left(\frac{q}{A\bar{K}^b}\right)^{1/a}.$$

Variable cost is then

$$VC(q) = wL = w\left(\frac{q}{A\bar{K}^b}\right)^{1/a} = w\frac{q^{1/a}}{A^{1/a}\bar{K}^{b/a}}.$$

That means average variable cost is

$$AVC(q) = w\frac{q^{1/a-1}}{A^{1/a}\bar{K}^{b/a}}$$

and marginal cost is

$$MC(q) = \frac{w}{a}\frac{q^{1/a-1}}{A^{1/a}\bar{K}^{b/a}} > w\frac{q^{1/a-1}}{A^{1/a}\bar{K}^{b/a}} = AVC(q)$$

since $0 < a < 1$ implies $1/a > 1$.

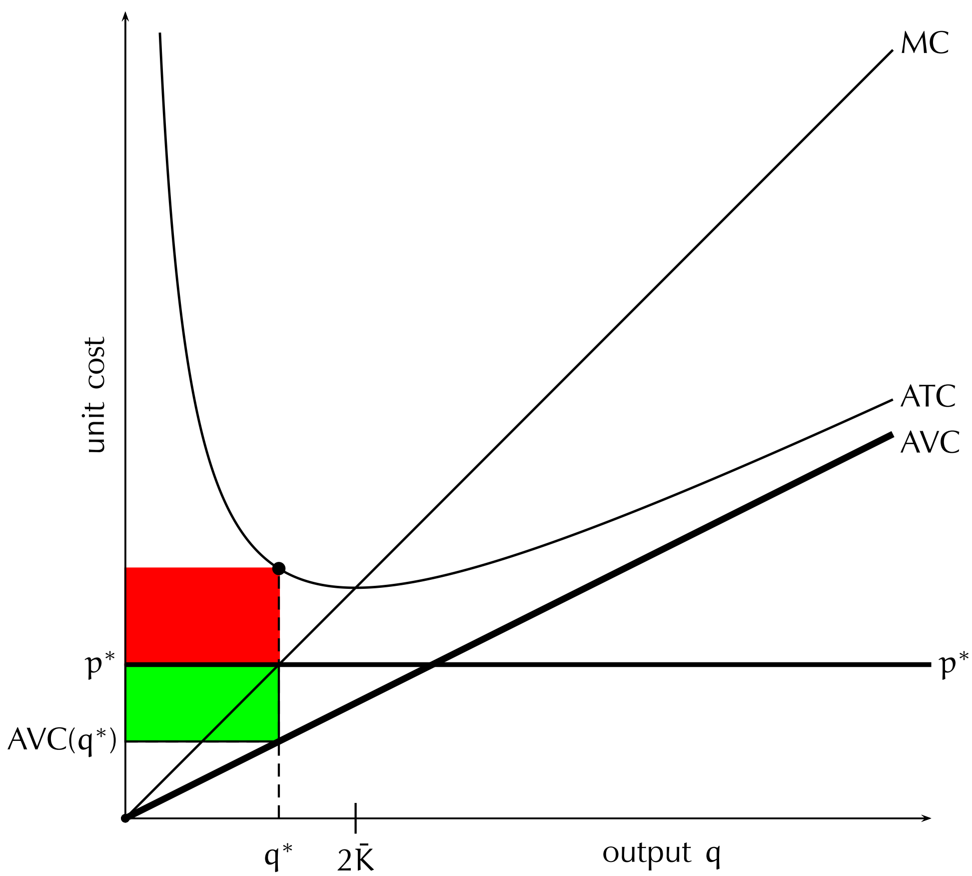
But then $p^* = MC(q^*) > AVC(q^*)$, so the firm will not close in the short run.

8.2.9 Cobb-Douglas Losses

The average total cost is above the price, so the firm will suffer losses in the long-run, shown by the red box.

When $a = 1/2$, the average variable cost is linear in output q. This is illustrated in the diagram below. In such cases, the average variable cost line (the heavy line in the diagram) lies below marginal cost. At q^*, $p^* = MC(q^*) > AVC(q^*)$. The firm more than covers its variable cost and is better off continuing to operate in the short run. The green box shows the gain compared to closing immediately.

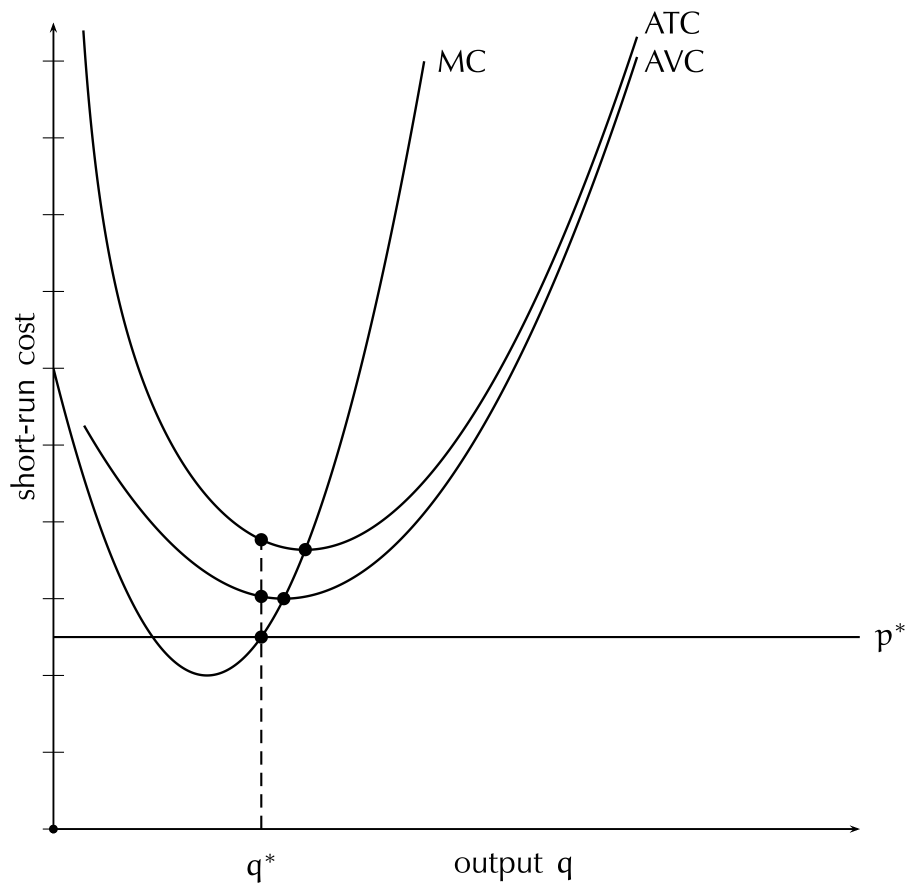
Cobb-Douglas: Short-run Gain and Long-run Loss



8.2.10 Another Comparison of Losses

We earlier considered (page 7.4.8) an alternative type of cost function where marginal cost is U-shaped. If the price is below the minimum average **variable** cost, it may close in the short run. This case is illustrated below.

Firm Closes Immediately



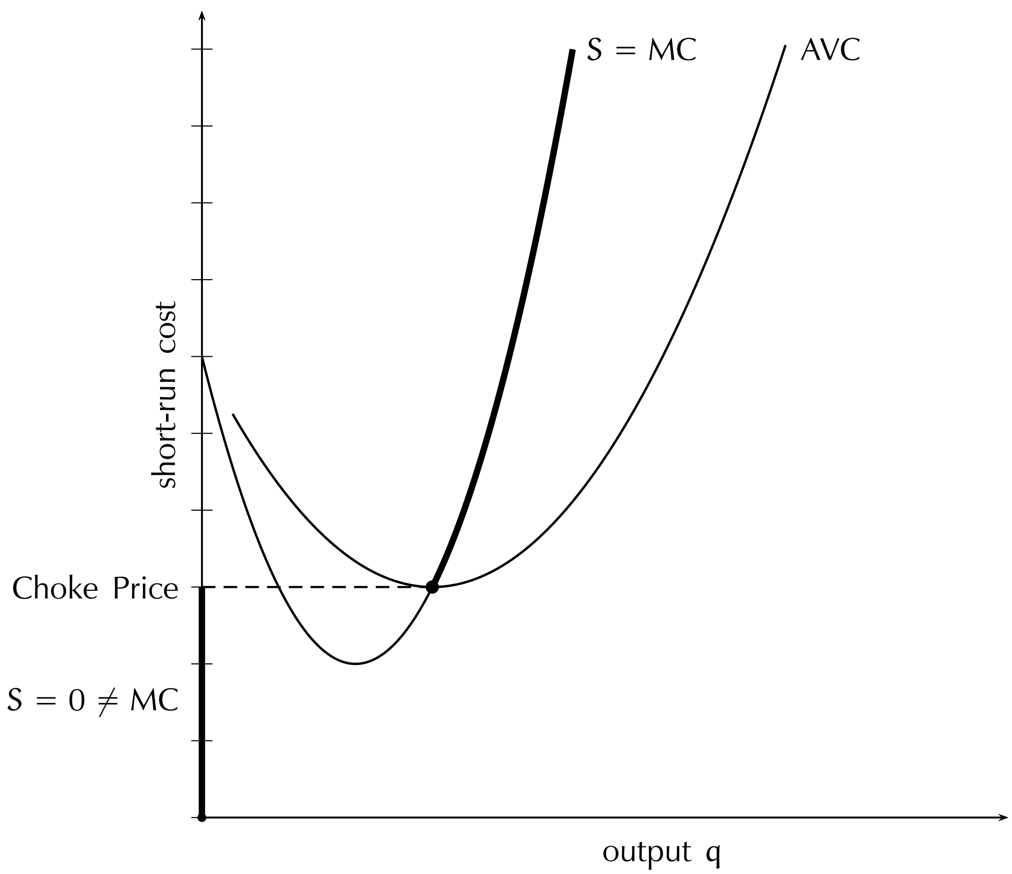
As you can see, at q^*, $p^* < \text{AVC}(q^*) < \text{ATC}(q^*)$ indicating that the firm loses money in both the short and long run. It should close immediately.

8.2.11 AVC Cutoff

One consequence of this is that when it is possible to have $MC(q) < AVC(q)$ for some q, the quantity supplied can be zero. This happens whenever $p^* < AVC(q)$.

In that case, the AVC cuts off the supply curve. At lower prices, the firm closes in the short run. Quantity supplied is zero. This is illustrated in the diagram where both heavy lines together constitute supply.

Quantity Supplied is Zero Below the Minimum AVC



When the production function has the Cobb-Douglas form, the only time $MC = AVC$ is when production is already zero. The cutoff has no effect on Cobb-Douglas production functions.

October 20, 2022