Intermediate Microeconomics — Week 10

Professor Boyd **October 25 & 27, 2022**

8.2.12 What is Revenue minus Variable Cost? 10/25/22

It's clear that the the difference between revenue and variable cost is important. What may not be so clear is that we already encountered it in Chapter 3!

8.2.13 Variable Cost, Sum of the Marginal Costs

Let's focus on the variable cost. Recall that the marginal cost is the rate of change in cost. Suppose we compute the marginal cost for unit changes in quantity:

$$MC(q) = \frac{\Delta TC}{\Delta q} = \frac{TC(q+1) - TC(q)}{(q+1) - q} = TC(q+1) - TC(q).$$

Let Q be a positive integer. Now we add up the marginal costs, starting at $q = 0$. Many of the terms cancel out. Everything but the TC(0) and TC(Q) terms cancel.

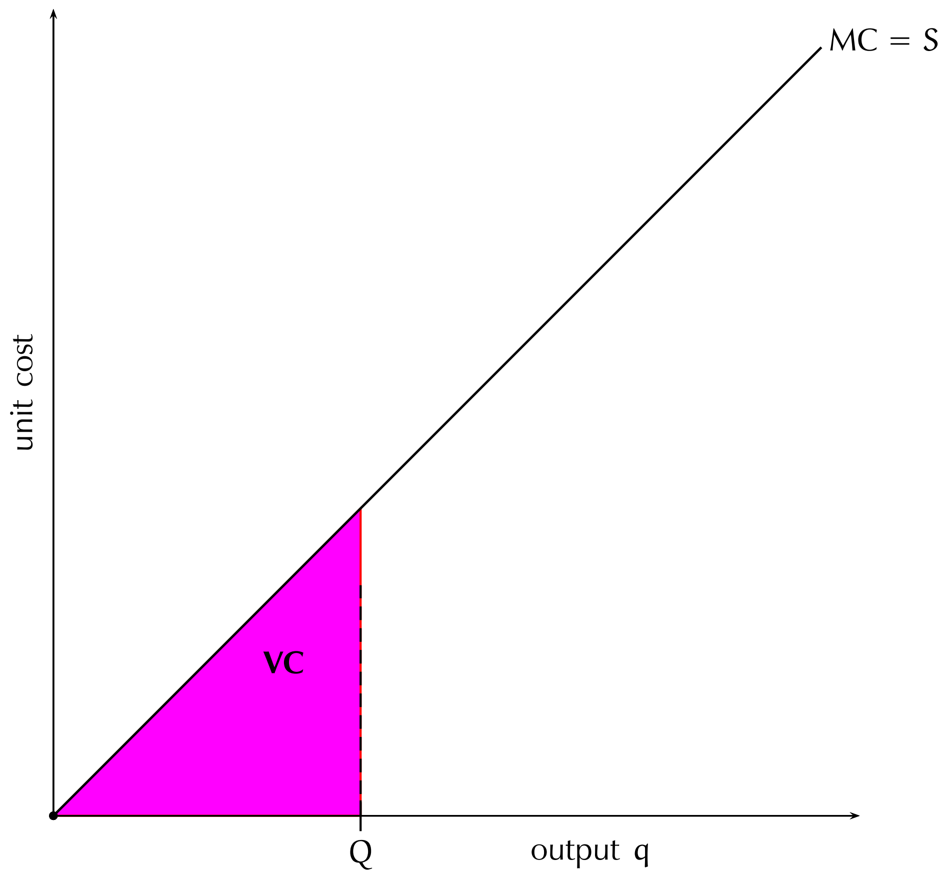
$$\begin{aligned}
\sum_{q=0}^{Q-1} MC(q) = & \big[TC(1) - TC(0)\big] \\
& + \big[TC(2) - TC(1)\big] \\
& + \big[TC(3) - TC(2)\big] \\
& \quad \vdots \\
& + \big[TC(Q-1) - TC(Q-2)\big]. \\
& + \big[TC(Q) - TC(Q-1)\big]. \\
= & \; TC(Q) - TC(0) \\
= & \; TC(Q) - FC \\
= & \; VC(Q).
\end{aligned}$$

The variable cost is the sum of the marginal costs.

8.2.14 Variable Cost and Marginal Cost

Now the sum of the marginal costs is just the area under the marginal cost curve, up to the quantity produced, as shown on the graph.

Marginal and Variable Costs



But the marginal cost curve is also the firms' supply curve. That means that the area under the supply curve is the variable cost.

8.2.15 Variable Cost Using Calculus

We can also use calculus to show that the area under the supply curve is the variable cost. Here $S(q) = MC(q) = d\,TC(q)/dq$, so the area under the supply curve is

$$\begin{aligned}\int_0^Q S(q)\,dq &= \int_0^Q MC(q)\,dq \\ &= \int_0^Q \frac{d\,TC(q)}{dq}\,dq \\ &= TC(Q) - TC(0) \\ &= TC(Q) - FC \\ &= VC(Q).\end{aligned}$$

This also works if there is a choke price. Due to demand constraints, at the choke price, the firm might be called on to supply any quantity up to q^* with $MC(q^*) \leq p^*$. We still integrate the area under marginal cost to the quantity demanded, and the result is still the variable cost for that quantity.

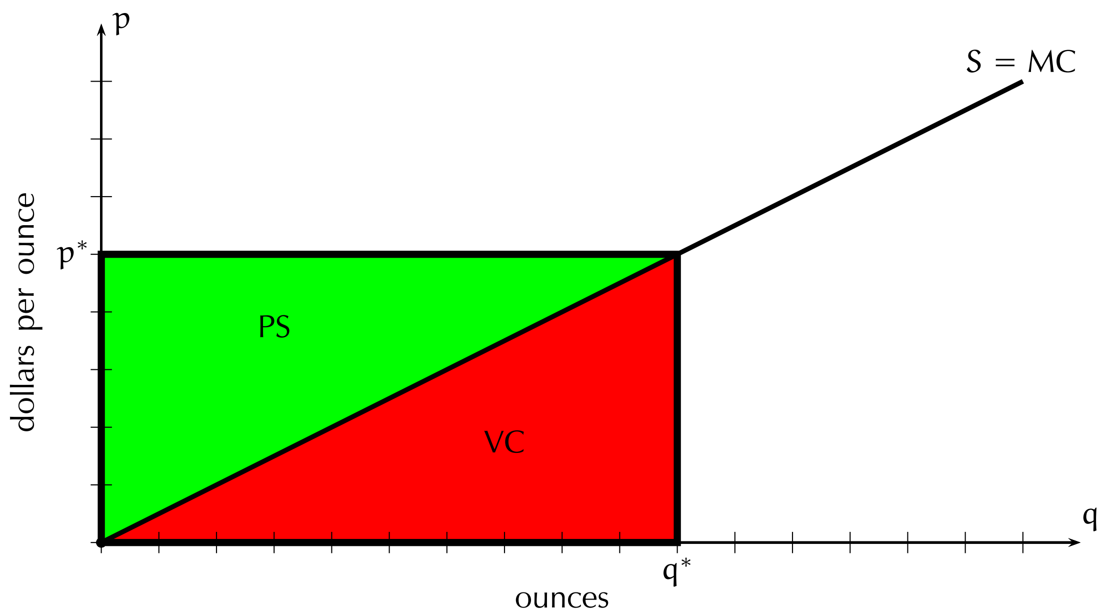
8.2.16 Producer's Surplus

The producer's surplus is the area bounded above by the price and below by the supply (marginal cost) curve. So is the large, heavy-bordered box whose area $p \times q^*$ is the revenue, minus the area under the supply curve, which we now know is the variable cost VC. The producer's surplus is then

$$\begin{aligned} PS &= p^* \times q^* - VC(q^*) \\ &= p^* \times q^* - TC(q^*) + FC \\ &= \text{Profit} + FC, \end{aligned}$$

which is the green area on the diagram.

Producer's Surplus and Variable Cost



8.2.17 Short Run Shutdown Decision

The firm will continue to operate in the short run as long as it continues to earn positive producer's surplus. That is, as long as it covers its variable costs.

If the producer's surplus is negative, it is best for the firm to shutdown immediately. If producer's surplus is positive, the firm continues to operate in the short run. Whether it will be open in the long run depends on econmic profit.

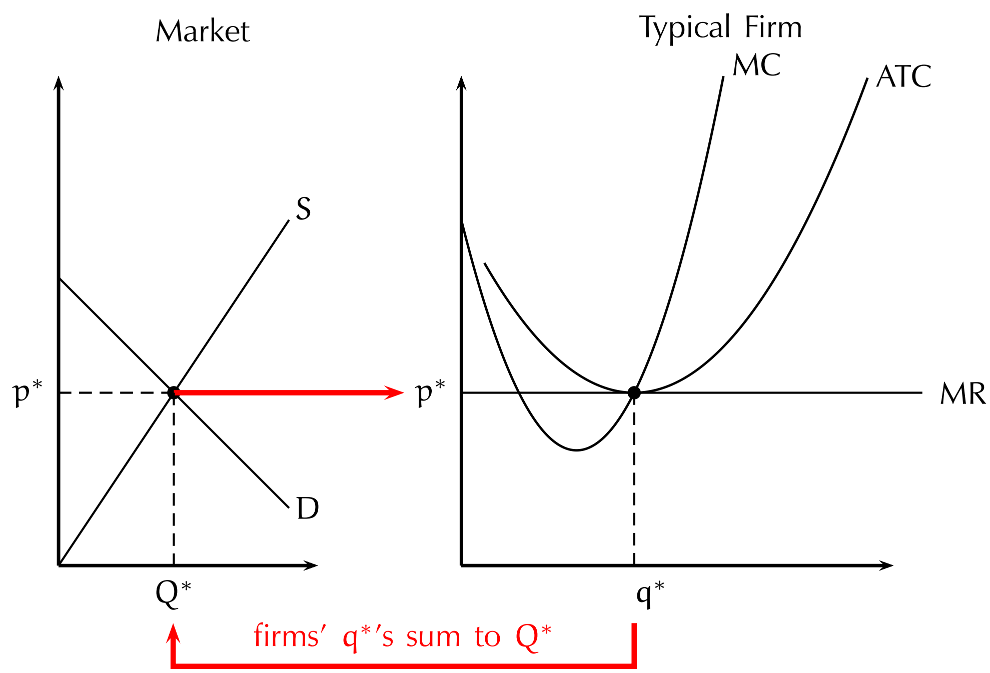
8.3 Long Run Equilibrium

For long run equilibrium in the perfectly competitive model, we must satisfy two conditions:

1. The market clears. Price and market quantity is determined by the supply and demand curves. Firms produce q^*'s so that $p^* = MC(q^*)$.
2. No firm wants to change their current behavior in the long run. This means profit is zero, $p^* = ATC(q^*)$.

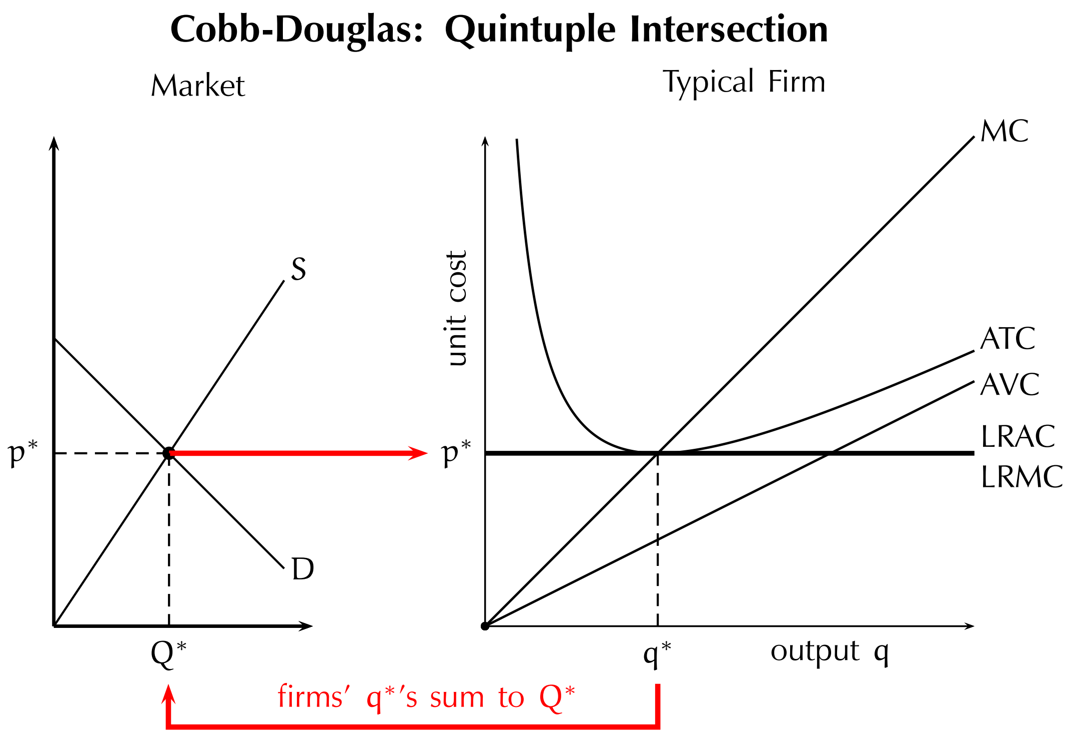
Combining the two conditions, we get a triple intersection of price, marginal cost, and average total cost. It obeys $p^* = MC(q^*) = ATC(q^*)$. See the diagram below.

Long-run Equilibrium: The Triple Intersection



8.3.1 Long Run Equilibrium with Cobb-Douglas Production

The Cobb-Douglas case is pretty similar, again with a triple intersection. Well, it's more than triple due to constant returns to scale, which means the equilibrium price is also the long run average and marginal costs. We have a quintuple intersection!

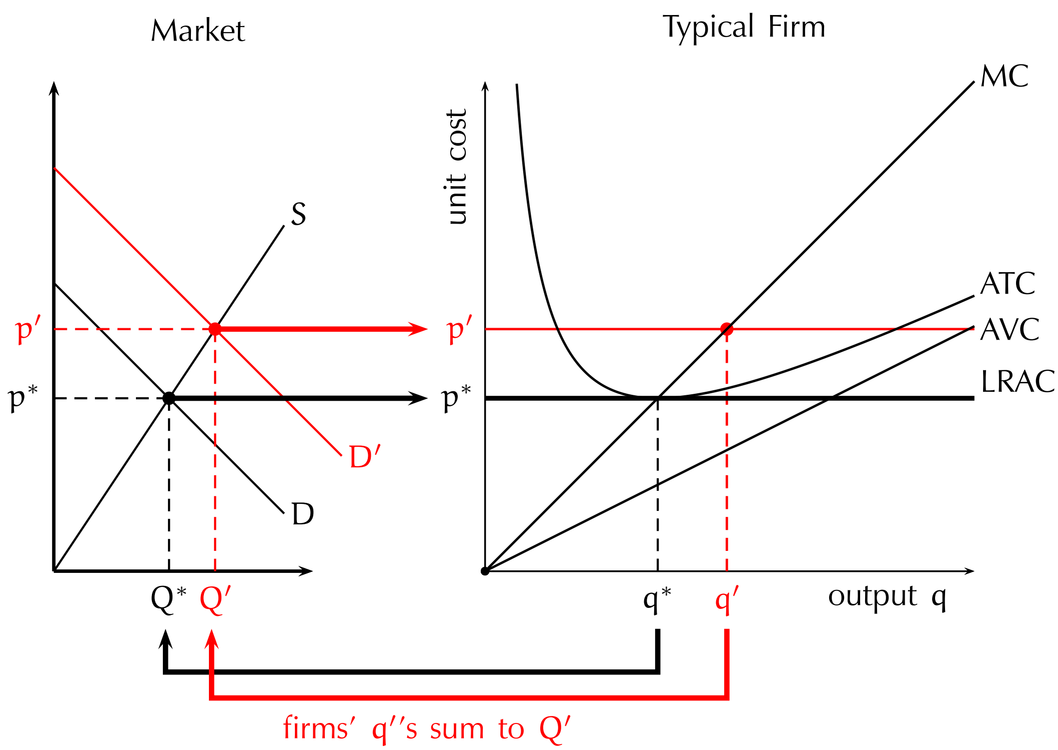


8.3.2 Increase in Demand I: Short Run Effect

Let's use our souped-up supply/demand model to analyze the effect of an increase in demand. We start with a market in long run equilibrium, and increase demand. Demand only appears in the market diagram, so that is where we shift it. Demand increases from D to D′. The market equilibrium price and quantity rise to p′ and Q′.

Each of the firms moves up their short run supply (marginal cost) curve, increasing output to q′. Adding up the increases by each firm, we get an increase in quantity supplied to Q′ from Q, reaching a new short run equilibrium.

Cobb-Douglas: Demand Increases, Short-run Effects

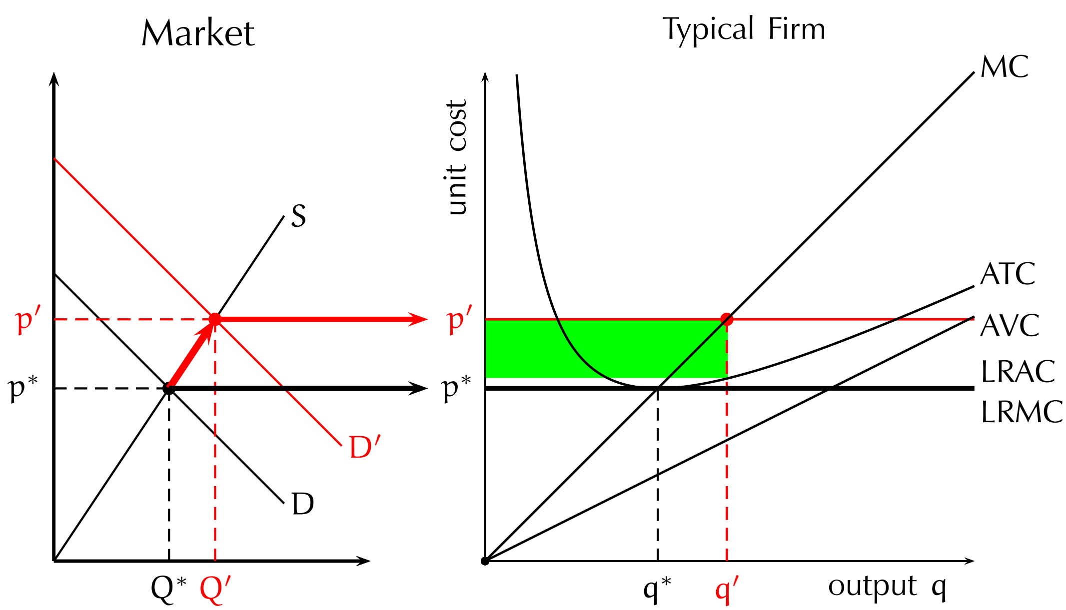


8.3.3 Increase in Demand II: Profit

Are we in a new long run equilibrium? We have to check for profits or losses by the typical firms.

As the green box shows, firms are earning profits. We can expect new firms to enter the industry and expansion by existing firms (along their long run marginal cost curve).

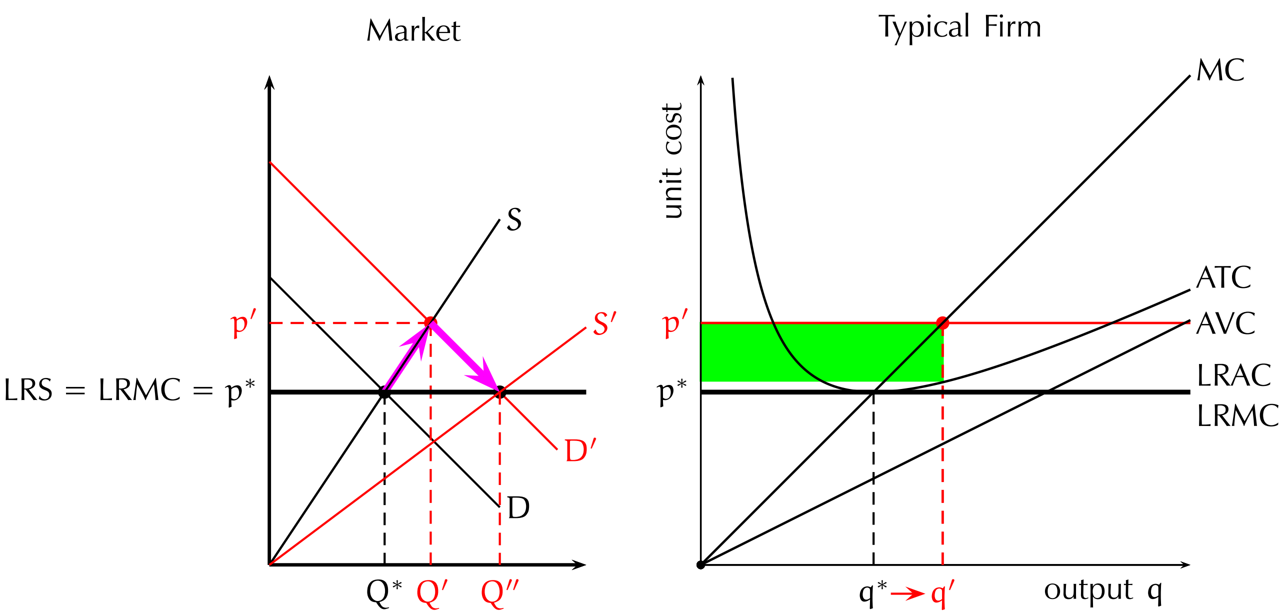
Demand Increase causes Profits



8.3.4 Increase in Demand III: New Long Run Equilibrium

The industry expands in the long run. This shifts the short run supply curve to the right. The new long run equilibrium is attained when profits return to zero. With a horizontal long run marginal cost curve, that happens when the price returns to p^*. Short run supply is permanently increased to S', which crosses the long run marginal cost (supply) curve at the new demand curve. This restores the market to long run equilibrium at (Q'', p^*)

Cobb-Douglas: In Long Run, Entry drives Price Back to p^*



Here we saw that in the long run, supply is more elastic than in the short run. We move from Q to Q' in the short run, from Q to Q'' in the long run. With constant returns to scale, price goes up in the short run but returns to p^* in the long run. Decreasing returns would lead to a long run price increase.

8.3.5 The Model of a Competitive Market

As we saw, the full model includes a long run supply curve in the market diagram. To use the model, we have to decide how a particular change affects the market. Does it affect the market as a whole? Or does it affect firms?

Shifts in demand show up in the market diagram. The effects of the change impact both the market and firm diagrams.

Changes in factor prices or production functions affect the firms first, changing supply. This shifts the supply curves (short and long run) in the market diagram, and may have further long run repercussions for the firms (profits or losses).

8.3.6 Long Run Supply Curve

In many cases, we can use a one-panel version of the perfectly competitive model. This adds a long run supply curve to the supply and demand diagram. We will generally assume that long run supply is positively sloped, but it need not be.

We've seen that long run firm supply curves (long run marginal cost curves) are horizontal when there are constant returns to scale. However, that is often not be the case for the industry. The problem is that when many firms increase their demands for inputs, it is likely that the prices of those inputs rise. Even in the two-factor model, where cost is

$$wL + rK,$$

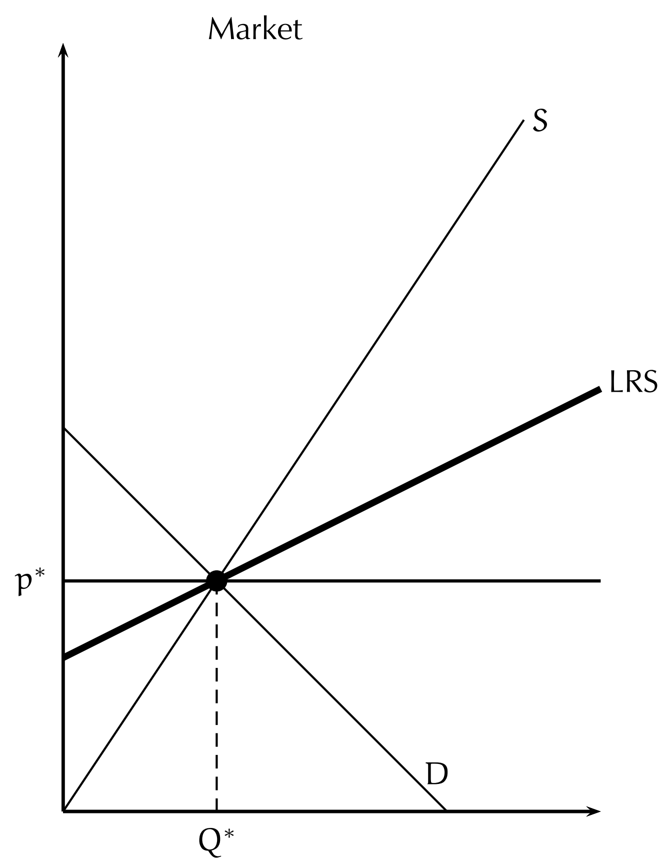
increasing the wage rate w and capital rental rate r will increase the cost of production. For this reason, we usually assume the long run supply curve is positively sloped.

8.3.7 Single Panel: Long Run Equilibrium

The single-panel version of long run equilibrium adds a long run supply curve (LRS) to the market diagram. A steeper short-run supply curve runs through every point of the long run supply curve. The equilibrium again involves a triple intersection, this time of demand, (short run) supply, and long run supply (LRS). This occurs at the point (Q^*, p^*).

The single panel version can be used to study shifts in demand and tax and subsidy problems. It is not enough when dealing with changes in supply.

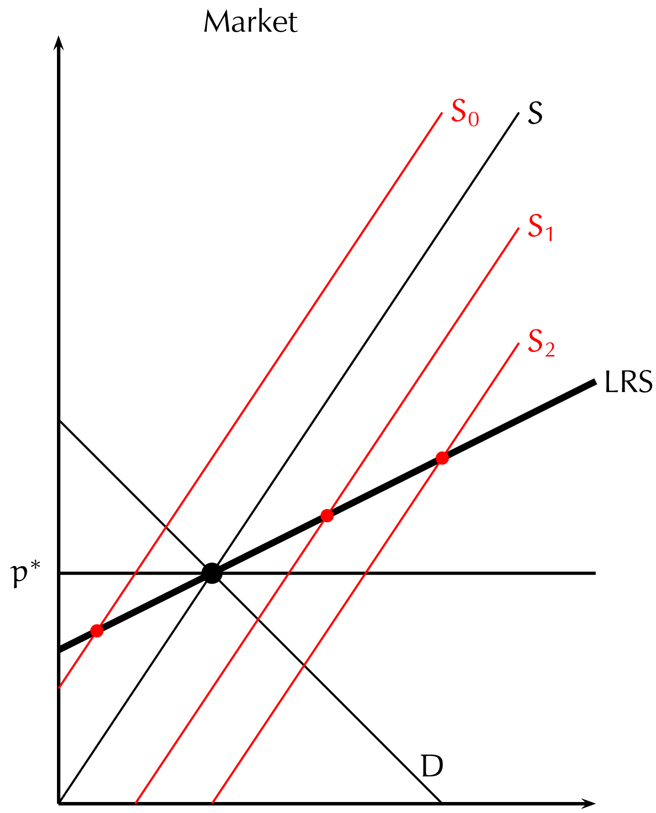
Single-Panel Model with Long-run Supply



8.3.8 Single Panel: Long Run Supply

Here, we've included three of the short run supply curves (red) for illustration (S_0, S_1, S_2) corresponding to fixed capital stocks of $\bar{K}_0$, $\bar{K}_1$, and $\bar{K}_2$.

Relation of Short and Long-run Supply

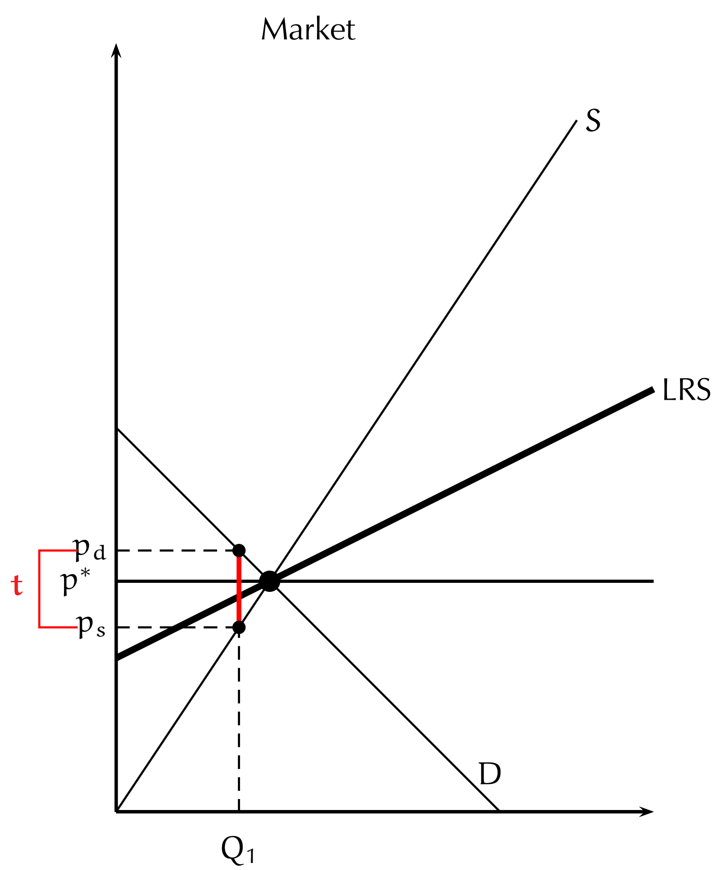


8.3.9 Single Panel: Excise and Sales Taxes in Short Run

Suppose the government imposes an excise or sales tax of amount t. Recall that this drives a wedge between the after-tax price paid by the buyers (p_d) and the after-tax price received by the sellers (p_s). See section 3.6.

I've inserted such a tax wedge (red) in the diagram below. In the short run, the demand price rises to p_d, the supply price falls to p_s, and quantity traded falls to Q_1.

Single-Panel: Short-run Excise and Sales Taxes

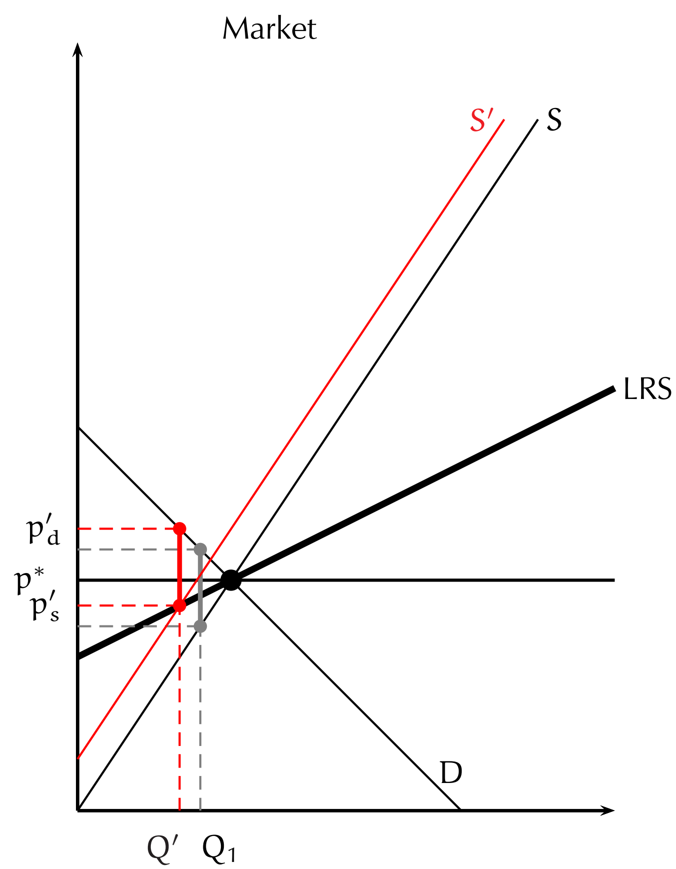


8.3.10 Single Panel: Long Run Tax Wedge

In the long run, the tax wedge ends (red) up sitting between the demand curve and the long run supply curve. This further increases the after-tax buyer's price, while the after-tax sellers price moves up toward p^*, but does not reach it. Compare the position of the short run (gray) and long run (red) tax wedges.

This shows how the fact that long run supply is more elastic that short run supply shifts excise and sales taxes toward consumers in the long run.[1]

Single-Panel: Long-run Excise and Sales Taxes



[1] We saw such a shift (in the opposite direction) with price ceilings on rental housing in week three. I'll just remind you what was found: "... we find rent control limits renters' mobility by 20 percent and lowers displacement from San Francisco. Landlords treated by rent control reduce rental housing supplies by 15 percent by selling to owner-occupants and redeveloping buildings. Thus, while rent control prevents displacement of incumbent renters in the short run, the lost rental housing supply likely drove up market rents in the long run, ultimately undermining the goals of the law."

9.1 Chapter 9: Market Power and Monopoly

Recall our four model markets. We have now finished our exploration of the supply and demand model. Next, we consider monopoly.

Chap.	Type of Market	Number of Firms	Differentiation	Barriers to Entry/Exit
✓8.	Perfect Competition	Many	None	None
9.	Monopoly	One	Unique	A lot
11.	Oligopoly	A Few	Maybe	Some
11.	Monopolistic Competition	Many	Yes	Maybe

We've now finished our study of the perfectly competitive model, the supply and demand model. Some of the tools we developed there will prove useful in studying other types of markets. Next up is monopoly.

9.1.1 What is Market Power?

Market power means control over price. Unlike perfect competition, firms with market power do not rely on the market price. They are **price setters**, able to chose and set their own price.

9.1.2 What is Monopoly?

A **monopolist** is the only seller of a good which has no close substitutes. Monopolists always have some degree of market power.

There is a related concept for buyers. A **monopsonist** is the only buyer of a particular good.[1]

The key fact about monopolists is that they directly face the demand curve. They are not price-takers. Monopolists are price setters and can set any price they want. There only limitation they face is the trade-off between price charged and quantity sold dictated by demand.

[1] The term monopoly, which has been used for centuries, is derived from Greek monopōlion, consisting of "mónos" (singular, alone) and pōlion (to sell). The term monopsony is more recent. It was introduced by Joan Robinson in her 1933 book, "The Economics of Imperfect Competition" where "opsonía" means "purchase".

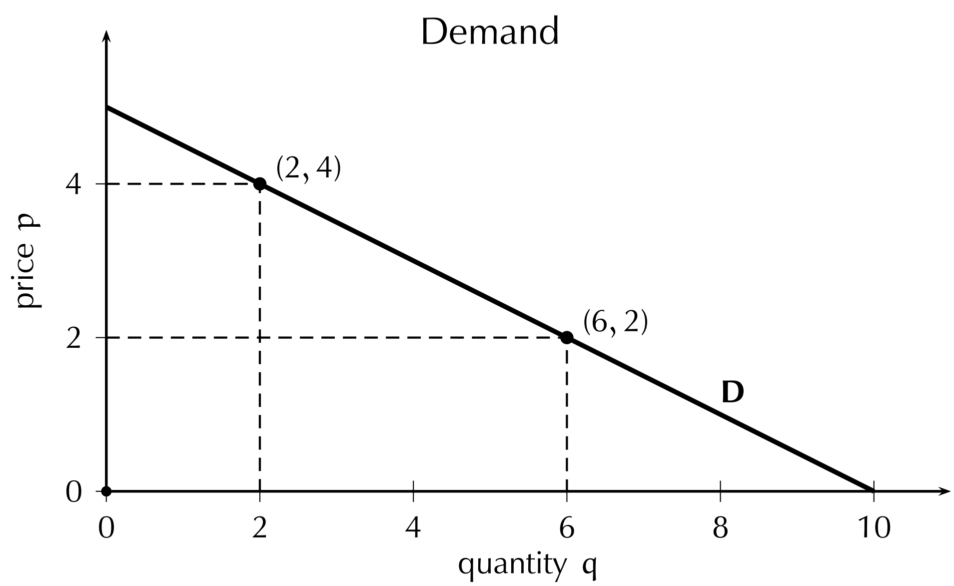
9.1.3 Demand Constrains the Monopolist

A monopolist can raise their price, but volume sold will drop. They can increase volume of sales, but only at the cost of a lower price.

We see this on the demand curve below. If the monopolist charges $2, quantity demanded is 6. The monopolist can raise the price to $4, but quantity demanded falls to 2.

Another way of looking at it is that the monopolist can only sell 2 units at a price of $4. To sell 6 units, the monopolist must lower the price to $2.

Demand: The Price Quantity Trade-off



9.1.4 How Do Monopolies Form?

How does a monopoly form? The key is to prevent the creation of close substitutes—create a barrier to entry. There are several ways this can happen.

1. Grant of monopoly. An example is the Livingston-Fulton steamboat monopoly in NY waters, granted by NY State. Livingston was highly connected and got the monopoly provided he could produce a steamboat with sufficient performance. Fulton made such a steamboat in 1807. The monopoly was ended by the US Supreme Court in 1824 on the grounds that the US Constitution gave rights over such interstate commerce to the US Government, not New York State. Within a year, there were more than seven times as many steamboats operating in NY State as before the decision.
2. Patents and Copyrights. Patents apply to many drugs and other products. Copyright was used to protect the original IBM PC. It was based on a BIOS chip produced by IBM. In 1983, Phoenix Technologies managed to create a functionally equivalent BIOS without copying any IBM code. This lead to the rise of the PC clones, which took over the personal computer market from IBM.

9.1.5 Barriers to Entry

3. Natural monopoly (economy of scale). Economies of scale mean decreasing average total cost. Larger firms can then sell at a profit at lower prices than small firms. This leads to a small number of firms, sometimes one. Microsoft Windows has an economy of scale that makes it the main producer of operating systems for personal computers. Its competitors survive by virtue of special circumstances (reduced costs due to reduced market coverage, subsidies by hardware producers so they can work better with large-scale computer systems).
4. Control of a key input, as the de Beers family gained control of most sources of natural diamonds.
5. Lock-in Effect, Switching Costs. It can be costly to switch from one product to another as many things might have to be redone to use the other product. This can lock you in to a particular technology. It has been claimed (falsely) that this is true for keyboards—that QWERTY keyboard we are all familiar with is technologically inferior to the Dvorak keyboard, but it is too costly to change. However, independent tests of typing speed and accuracy have shown little or no real advantage. Windows vs. Mac is a better example, as is the difficulty of creating alternatives to Facebook, Twitter, etc.
6. Technological advantage, usually transitory.

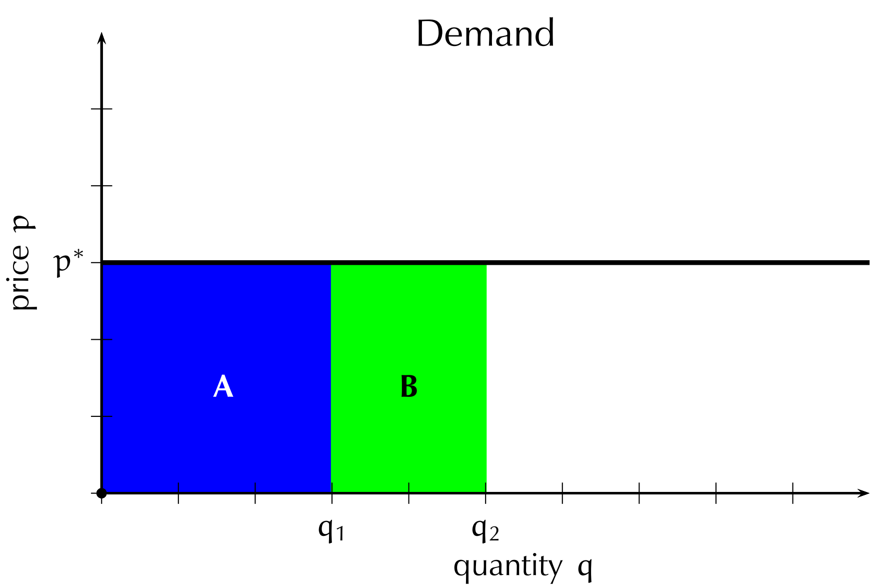
9.1.6 Price Setters and Price Takers

Marginal revenue is quite different for price takers and price setters. We start by considering a price taker facing market price p^*. We consider revenue when they sell q_1 or q_2 units of their product.

If they sell q_1 units, revenue is the area A, which is p^*q_1 dollars. If they sell q_2 units, revenue is the combined area $A + B$, which is p^*q_2 dollars.

The increase in revenue is $(A+B)-A = B$, which is $p^*(q_2-q_1) = p^*\Delta q$ dollars, and marginal revenue is $B/\Delta q = p^*\Delta q/\Delta q = p^*$. For any price taking firm, $MR = p^*$.

Price-taker's Marginal Revenue



9.1.7 Marginal Revenue and Market Power

Things are different when a firm has market power. Such firms face a downward sloping demand curve and are price setters.

If they sell q_1 units, revenue is the area $A + C$, which is $p_1 q_1$ dollars. If they sell q_2 units, revenue is the area $A + B$, which is $p_2 q_2$ dollars.

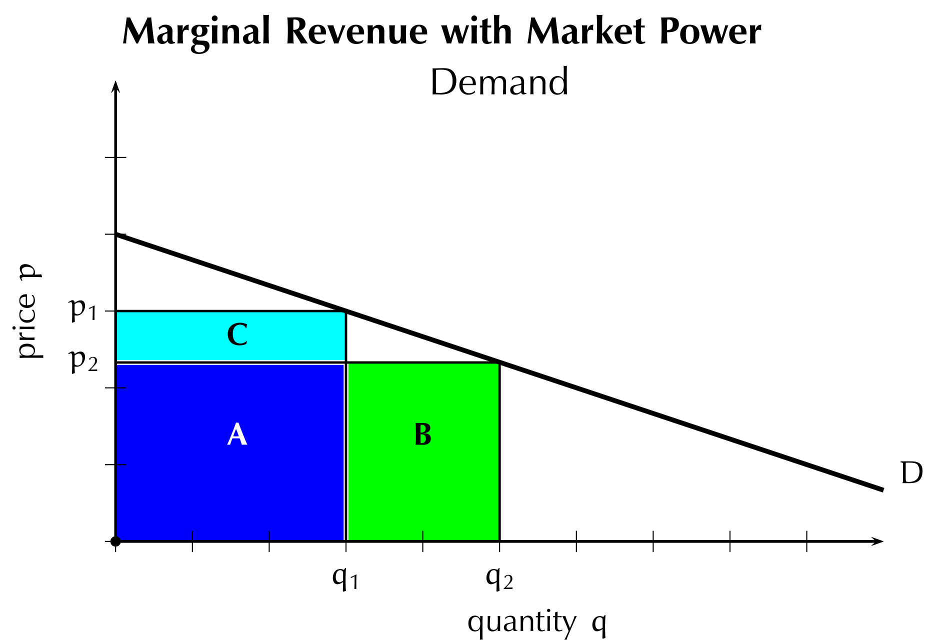
The difference in revenue is $(A + B) - (A + C) = B - C$. This is

$$p_2 \Delta q - (p_1 - p_2) q_2 < p_2 \Delta q.$$

That means that marginal revenue is less than $p_2 < p_1$.

For firms with market power, marginal revenue is not only less than the current price, its less than the new lower price.

The good news about market power is that you get to choose your price. The bad news is that this choice has a cost—marginal revenue is lower when the price is higher.



9.2 Profit Maximization and Market Power

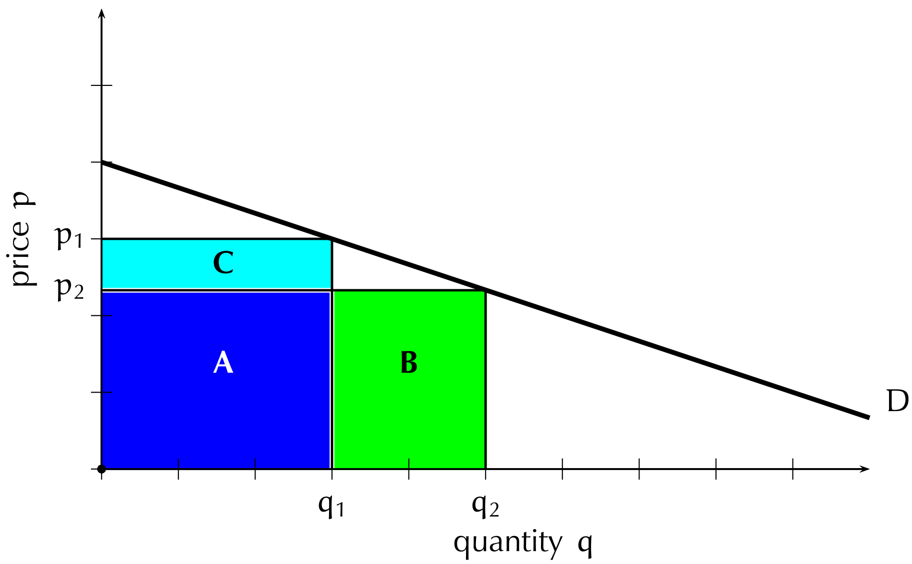
Our first task is to figure out how to maximize profit. Our results will apply to monopolies and some oligopolies. The key is what happens to marginal revenue.

9.2.1 Marginal Revenue and Market Power

Firms with market power, price-setters, face a downward sloping demand curve. As a result, marginal revenue is less than price.

We illustrate the situation on the diagram below. If the firm sells q_1 units, revenue is the area $A + C$, which is $p_1 q_1$ dollars. If the firm sells q_2 units, revenue is the area $A + B$, which is $p_2 q_2$ dollars.

Demand and Revenue



The difference in revenue is $(A + B) - (A + C) = B - C$. This is

$$p_2 \Delta q - (p_1 - p_2) q_2 < p_2 \Delta q.$$

That means that marginal revenue is less than $p_2 < p_1$. In contrast with the price-taking firm of perfect competition, marginal revenue with market power is not only less than the current price, but less that the new lower price.

9.2.2 Elasticity and Marginal Revenue

We can quantify the relation between price and marginal revenue. The marginal revenue is

$$\begin{aligned}\frac{B-C}{\Delta q} &= \frac{p_2 \Delta q - (p_1 - p_2) q_1}{\Delta q} \\ &= p_2 - q_1 \frac{p_1 - p_2}{\Delta q} \\ &= p_2 + q_1 \frac{\Delta p}{\Delta q} \\ &\approx p \left(1 + \frac{1}{e_d}\right)\end{aligned}$$

where e_d is the elasticity of demand.

9.2.3 Elasticity and Marginal Revenue: Calculus Version

We can also use calculus to derive the same relation. Revenue is $R(q) = p(q)q$ where $p(q)$ is the inverse demand function. Then

$$\begin{aligned} MR &= \frac{dR}{dq} \\ &= q\frac{dp}{dq} + p \\ &= p\left(\frac{q}{p}\frac{dp}{dq} + 1\right) \\ &= p\left(1 + \frac{1}{e_d}\right). \end{aligned}$$

The formula for marginal revenue that was approximate for marginal revenue over an interval, becomes exact for marginal revenue at a point (the calculus version).

$$MR = p\left(1 + \frac{1}{e_d}\right). \tag{9.2.1}$$

Remember that the two formulas disagree slightly because this one is based on tangents, and the other on finite changes and chords. Finite changes work better when comparing the actual before and after situation. Calculus works better for finding out what's happening at a specific price, not nearby prices.

9.2.4 Conclusions concerning Elasticity and Marginal Revenue

By equation (9.2.1),

$$\begin{aligned} & MR > 0 \\ \Leftrightarrow \quad & 1 + \frac{1}{e_d} > 0, \\ \Leftrightarrow \quad & 1 > -\frac{1}{e_d} \\ \Leftrightarrow \quad & e_d < -1. \end{aligned}$$

It follows that

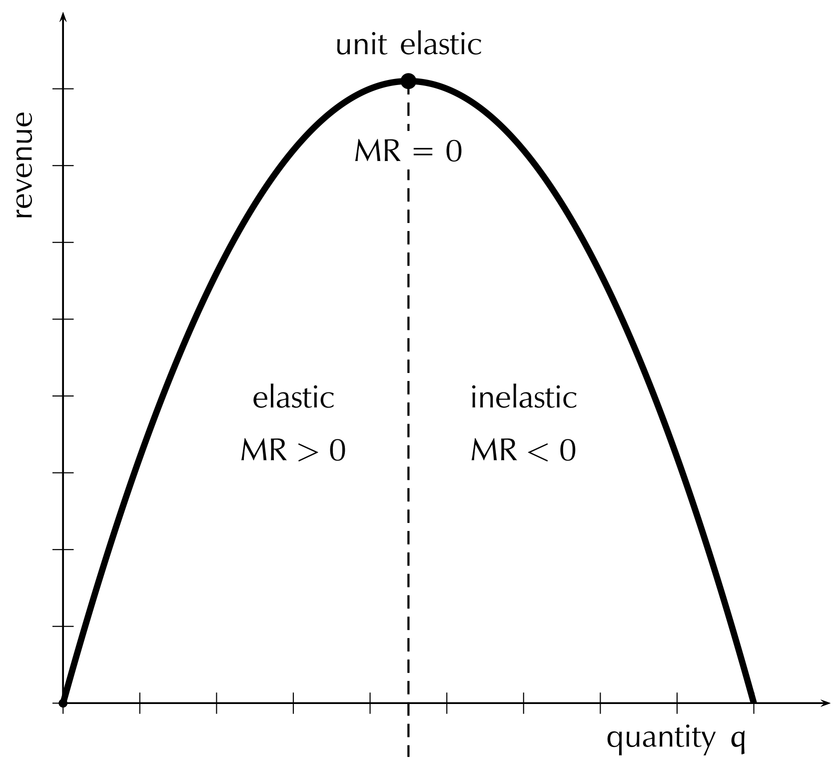
- $MR > 0$ if and only demand is elastic.
- $MR = 0$ if and only if demand is unit elastic.
- $MR < 0$ if and only if demand is inelastic.

9.2.5 A Revenue Hill

The graph shows total revenue $q \times p(q)$. Here the revenue curve is a hill. Revenue is zero when $q = 0$, and at the choke price. In between, it rises to a maximum, then falls back to zero.

The maximum revenue occurs when $MR = 0$. To the left, demand is elastic, with $MR > 0$. To the right, demand is inelastic, with $MR < 0$.

Revenue Hill



9.2.6 Maximize Profit, Not Revenue

10/27/22

We should remember that the firm is trying to maximize profit, revenue minus costs. They do not maximize revenue alone. They are also concerned about costs. As we saw on the first day of class, marginal benefit and marginal cost will be equal at any maximum.

As with price taking firms, the firm's benefit is measured by revenue, so our firm with market power will choose a quantity q^* where marginal revenue is equal to marginal cost.

$$MR(q^*) = MC(q^*)$$

The difference between the two cases is that $p = MR(q^*)$ for price takers (perfect competition) while $p > MR$ for price setters (pretty much every other case).

Of course, marginal cost is always positive, so for firms with market power (price setters),

$$MR(q^*) = MC(q^*) > 0.$$

But then, profit maximization implies that the firm chooses a quantity where demand is **elastic**. That is the only place where $MR > 0$. In other words, they pick a quantity on the left side of the revenue hill, a quantity that is **lower than the revenue maximizing quantity**.

9.2.7 Marginal Revenue and Linear Demand: Equations

In most cases, we can understand principles of monopoly decision-making, and many qualitative aspects of monopoly behavior, by considering linear demand, with inverse demand function

$$p(q) = a - bq$$

where $a, b > 0$. Recall that a is the choke price and b is the absolute slope of the demand curve.

Then total revenue R is

$$R(q) = p(q)q = aq - bq^2.$$

and marginal revenue is

$$MR(q) = \frac{dR}{dq} = a - 2bq.$$

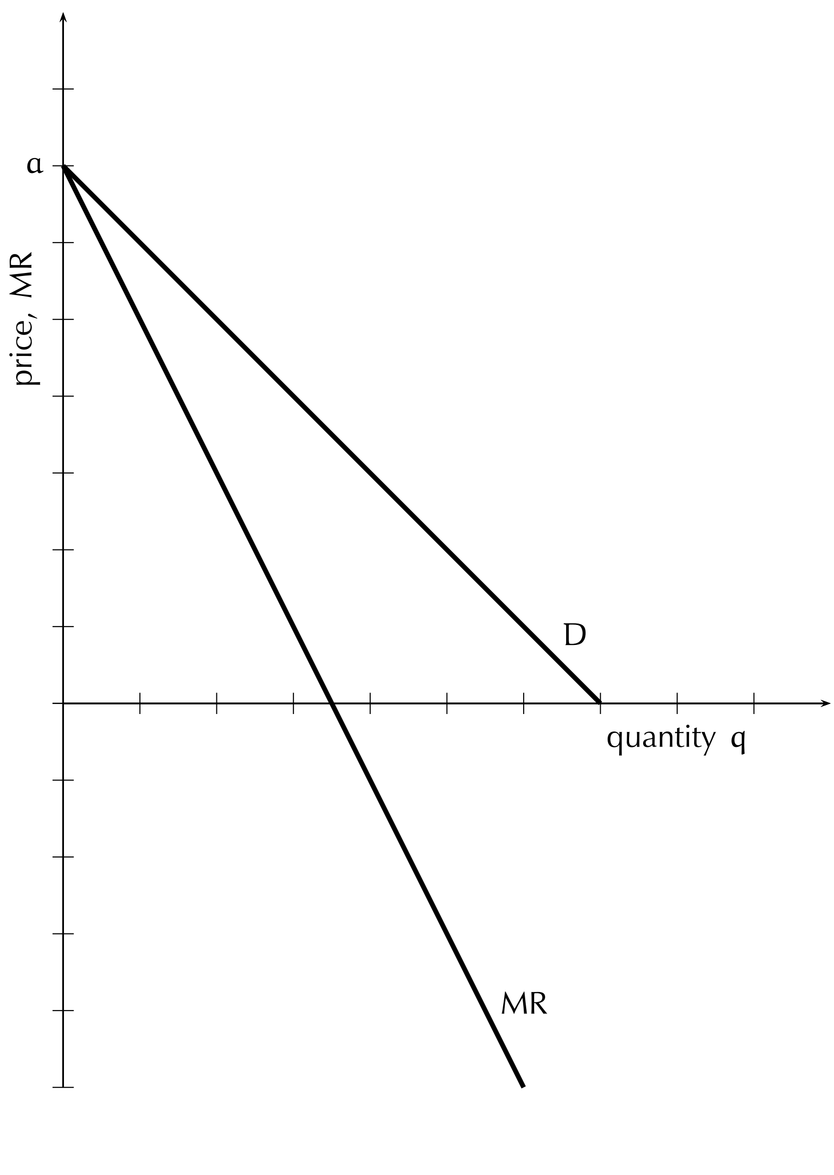
In other words, when demand is linear, the marginal revenue curve is also a straight line with the **same vertical intercept** (a) as the demand curve, and **twice the slope**.

9.2.8 Marginal Revenue and Linear Demand: Graph

Here's a graph of demand and marginal revenue.

The fact that the horizontal intercept of the marginal revenue curve is half that of the demand curve reflects the combination of the same vertical intercept and twice the slope. With twice the slope, marginal revenue gets down to the horizontal axis twice as fast.

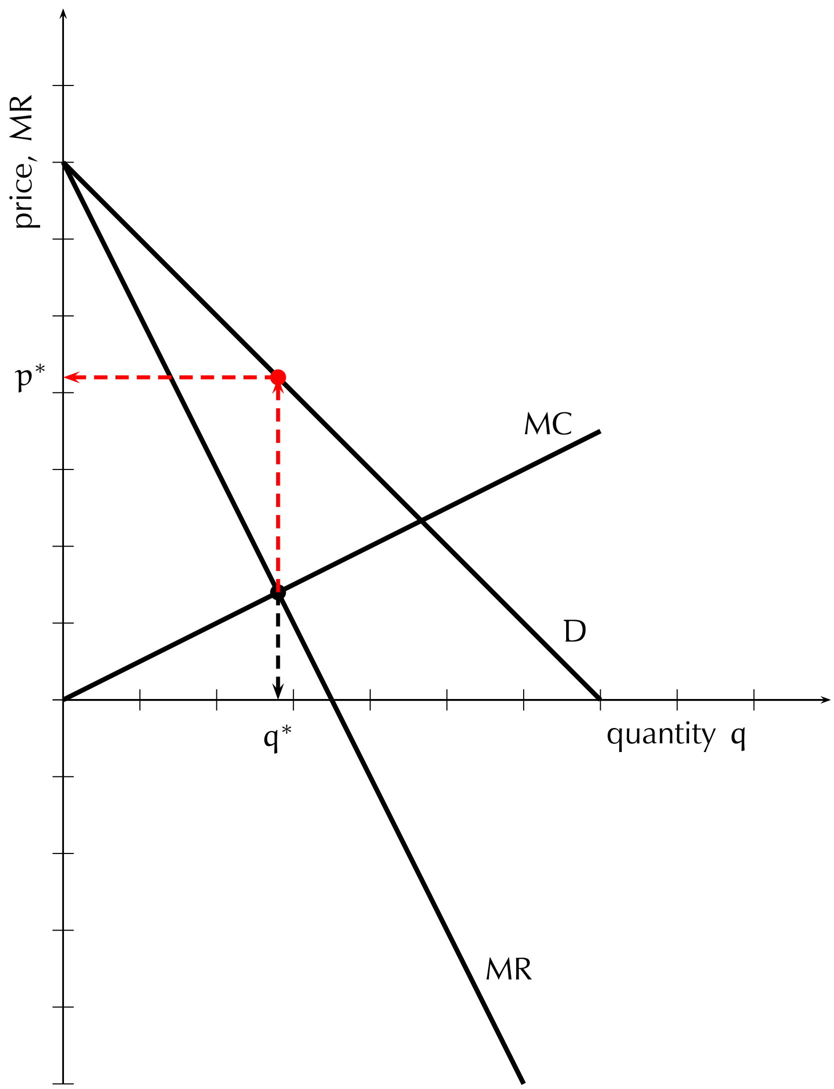
Demand and Marginal Revenue



9.2.9 Profit Maximization with Linear Demand and Marginal Cost

If marginal cost is also linear, we maximize profit by finding the intersection of marginal revenue and marginal cost. This determines the profit-maximizing quantity q^*. Then we read straight up from the intersection to the demand curve and then continue straight over to the vertical axis (red) to find the profit-maximizing price p^* that goes with q^*.

Profit Maximization



9.2.10 Linear Demand and Constant Marginal Cost I

We will often use an even simpler model, where marginal cost is **constant** at $c > 0$. I.e.,

$$\begin{aligned} p(q) &= a - bq \\ MC(q) &= c \end{aligned}$$

To find the monopoly solution, we set marginal cost equal to marginal revenue and solve to find q^*:

$$\begin{aligned} MC(q^*) &= MR(q^*) \\ c &= a - 2bq^* \\ 2bq^* + c &= a \\ 2bq^* &= a - c \\ q^* &= \frac{a-c}{2b} \end{aligned}$$

This only works if the choke price is greater than marginal cost, $a \geq c$. If $c > a$, the marginal revenue and marginal cost curves do not intersect at any $q \geq 0$.

9.2.11 Linear Demand and Constant Marginal Cost II

We then substitute q^* into the inverse demand equation to find the price.

$$\begin{aligned} p(q^*) &= a - bq^* \\ &= a - b\left(\frac{a-c}{2b}\right) \\ &= a - \frac{a-c}{2} \\ &= a - \frac{a}{2} + \frac{c}{2} \\ &= \frac{a}{2} + \frac{c}{2} \\ &= \frac{a+c}{2}. \end{aligned}$$

The profit-maximizing price is the average of the choke price (a) and the marginal cost (c).

Summing up, when demand is linear and marginal cost is constant, the profit-maximizing quantity q^* and price p^* are

$$\boxed{q^* = \frac{a-c}{2b}} \quad \text{and} \quad \boxed{p^* = \frac{a+c}{2}.}$$

9.3 Profit Max is Producer's Surplus Max

Recall that PS = FC + Profit. Since the fixed cost doesn't change in the short run, maximizing profit means maximizing producer's surplus and vice-versa.

9.3.1 Graphing Producer's Surplus under Market Power

Let's think about this graphically. The area between the marginal revenue curve and marginal cost curve is the producer's surplus. We saw half of this earlier, in sections 8.2.12–8.2.16, when we showed that the area under the marginal cost curve is the variable cost.

A similar argument shows that the area under the marginal revenue curve is the total revenue. It's important here that the revenue from zero sales is zero, just as the variable cost from zero production is zero.

9.3.2 Revenue and Marginal Revenue

To see why, we do a calculation. The Fundamental Theorem of Calculus states that the definite integral of a derivative on an interval (here $[0, q]$) is the difference of the original function at the two endpoints. This means

$$\begin{aligned}\int_0^q MR(q)\,dq &= \int_0^q \frac{dR}{dq}\,dq \\ &= R(q) - R(0) \\ &= R(q)\end{aligned}$$

To get the last line, we used the fact that if you don't sell, you don't have any revenue, so $R(0) = 0$, which leaves us with $R(q)$.

9.3.3 Producer's Surplus Area via Calculus

We're now ready to compute producer's surplus, but not in the normal way. Again we use the Fundamental Theorem of Calculus to compute the area between the marginal revenue and marginal cost curves.

$$\begin{aligned}
\int_0^q \big[MR(q) - MC(q)\big]\, dq &= \int_0^q \left[\frac{dR}{dq} - \frac{d\,VC}{dq}\right] dq \\
&= \int_0^q \left[\frac{d(R - VC)}{dq}\right] dq \\
&= R(q) - VC(q) - \big[R(0) - VC(0)\big] \\
&= R(q) - VC(q) \\
&= PS(q).
\end{aligned}$$

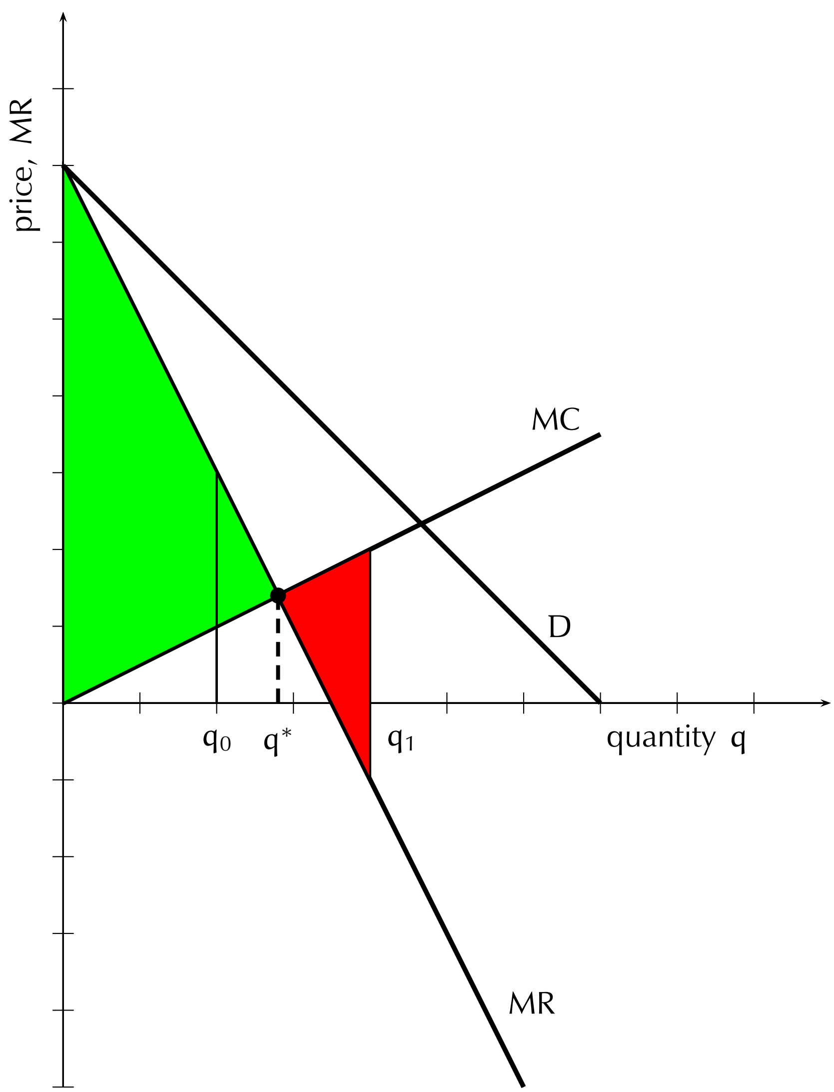
This works for any marginal cost and marginal revenue functions.

9.3.4 A New Graphical View of Producer's Surplus

In other words, the producer's surplus is the area between the marginal revenue and marginal cost curves, up to the quantity produced. Surplus is added if MR > MC, and subtracted if MR < MC.

At q_0, the producer's surplus is the green area left of the vertical line. At q_1, the producer's surplus is the full green are minus the red area. At q^*, we have maximized producer's surplus, which is the full green area.

Producer's Surplus via MR and MC



9.3.5 Comparison with Old Version of Producer's Surplus

The old version of producer's surplus applied to supply/demand models. The consumer's surplus was the area between the market price and the supply curve. We later observed that the supply curve of a competitive firm is the marginal cost curve, which was why the supply curve showed up in producer's surplus.

Now the market price is the marginal revenue curve for a competitive firm.

We can't think in terms of a supply curve for monopoly because there isn't one. But there is a marginal cost curve, and that's what really counts, both in terms of surplus and profit maximization.

9.4 Changes in Demand

To get some intuition for monopoly behavior, we will consider the effects of changes in demand.

9.4.1 Increase in Demand I

Let's see what happens if we increase demand. There are two demand parameters in our model, the choke price (a) and the slope parameter (b). We can increase demand by increasing the choke price, or by reducing the absolute slope, or both.

Let's start by reducing the slope while keeping the choke price fixed at a. The equilibrium quantity

$$q^* = \frac{a - c}{2b}$$

is a decreasing function of b, because b is in the denominator. Decreasing b (increasing demand) increases quantity produced (and sold). The equilibrium price

$$p^* = \frac{a + c}{2}$$

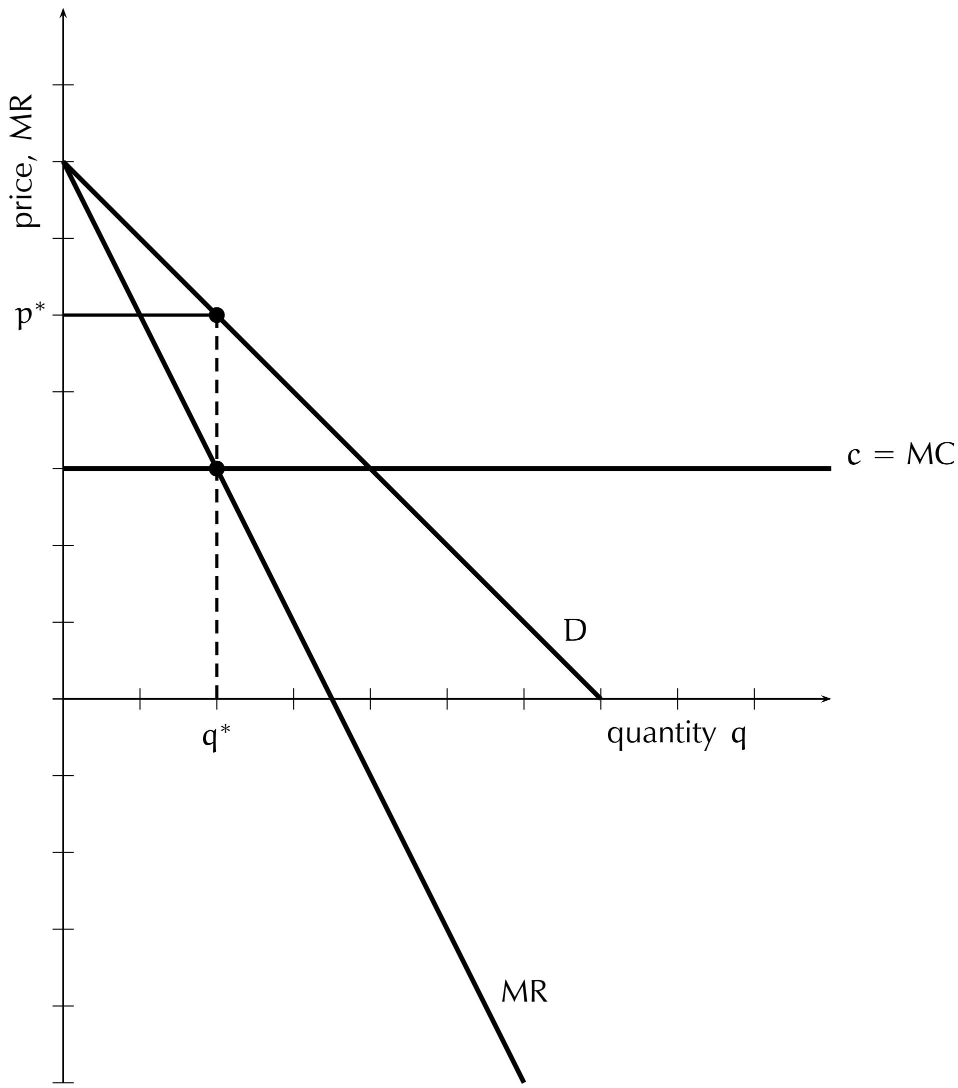
is unaffected by changes in b, so it remains unchanged.

This contrasts with the supply and demand model where an increase in demand increases both price and quantity. It is our first indication that monopoly is different.

9.4.2 Increase in Demand II

We start with a linear demand curve and constant marginal cost. The intersection of MR and MC determines the profit-maximizing quantity q^*, and the demand curve at q^* yields the profit-maximizing price.

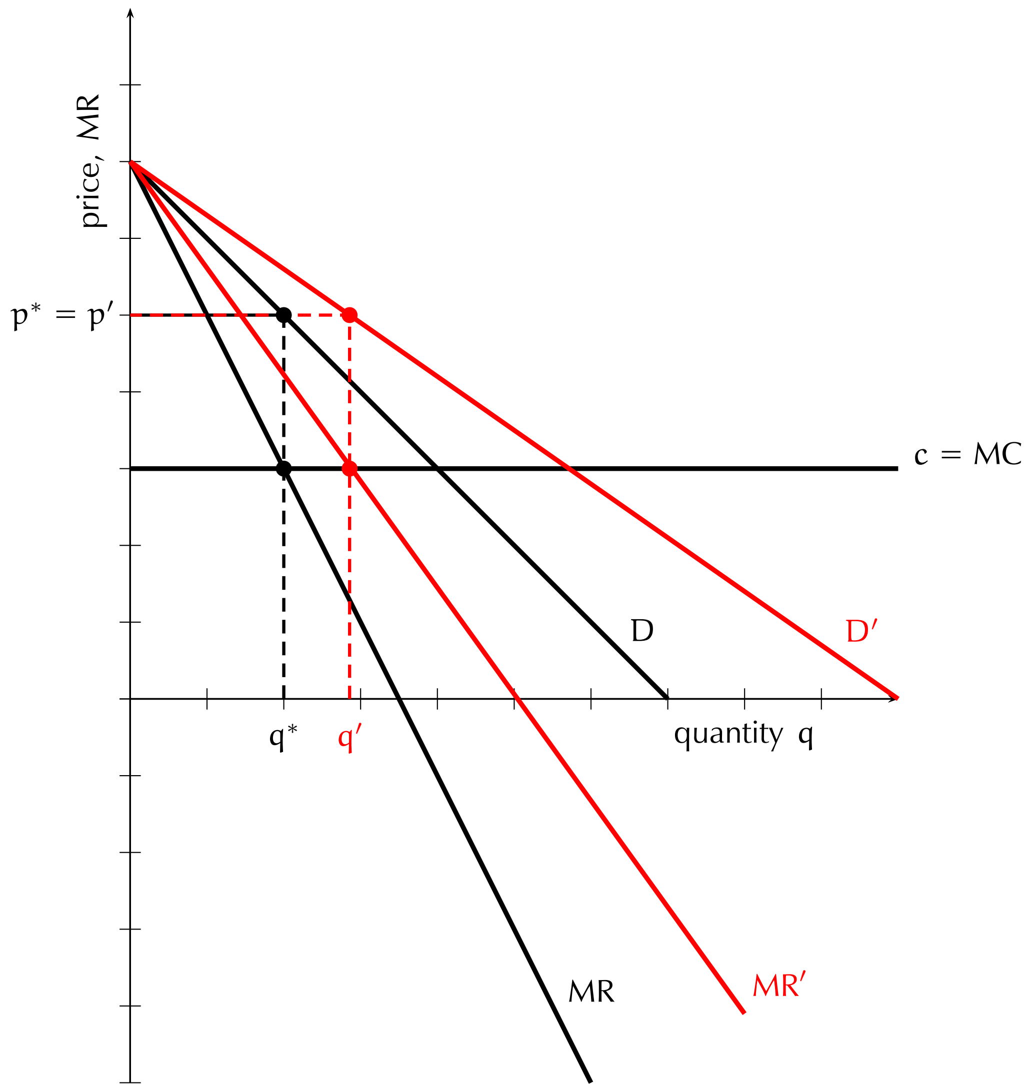
Before Demand Increases



9.4.3 Increase in Demand III

Here's a graph showing how the situation has changed. Both the old (black) and new (red) demand and marginal revenue are illustrated.

Old and New Demand Curves



9.4.4 Increase in Demand IV

We saw earlier that the algebra works out to show the price unchanged as a result of a change in the slope (but not vertical intercept) of linear demand. What about the geometry we saw on the diagram?

Demand and marginal revenue swing out to the right (D' and MR'). The profit-maximizing quantity increases to q'. Interesting, the profit-maximizing price remains unchanged at $p' = p$.

This happens because the profit-maximizing price is halfway between the choke price and marginal cost, and the slope of demand (used to find the price) is also half of the slope of marginal revenue. Since the curves have the same vertical intercept, marginal revenue drops twice as fast and reaches the marginal cost at the same quantity (q') where demand reaches $p = p'$.

9.4.5 Increase in Demand V

Now you may think that with constant marginal cost, the situation is similar to the long run version of constant returns to scale, where supply is perfectly elastic in the long run and the long run price is unaffected by demand.

But in that case, changing the choke price shouldn't matter. But it does! Why?

When we increase the choke price a, the profit-maximizing price also increases

$$p^* = \frac{a + c}{2}$$

as the numerator alone increases.

In this case, the profit-maximizing quantity

$$q^* = \frac{a - c}{2b}$$

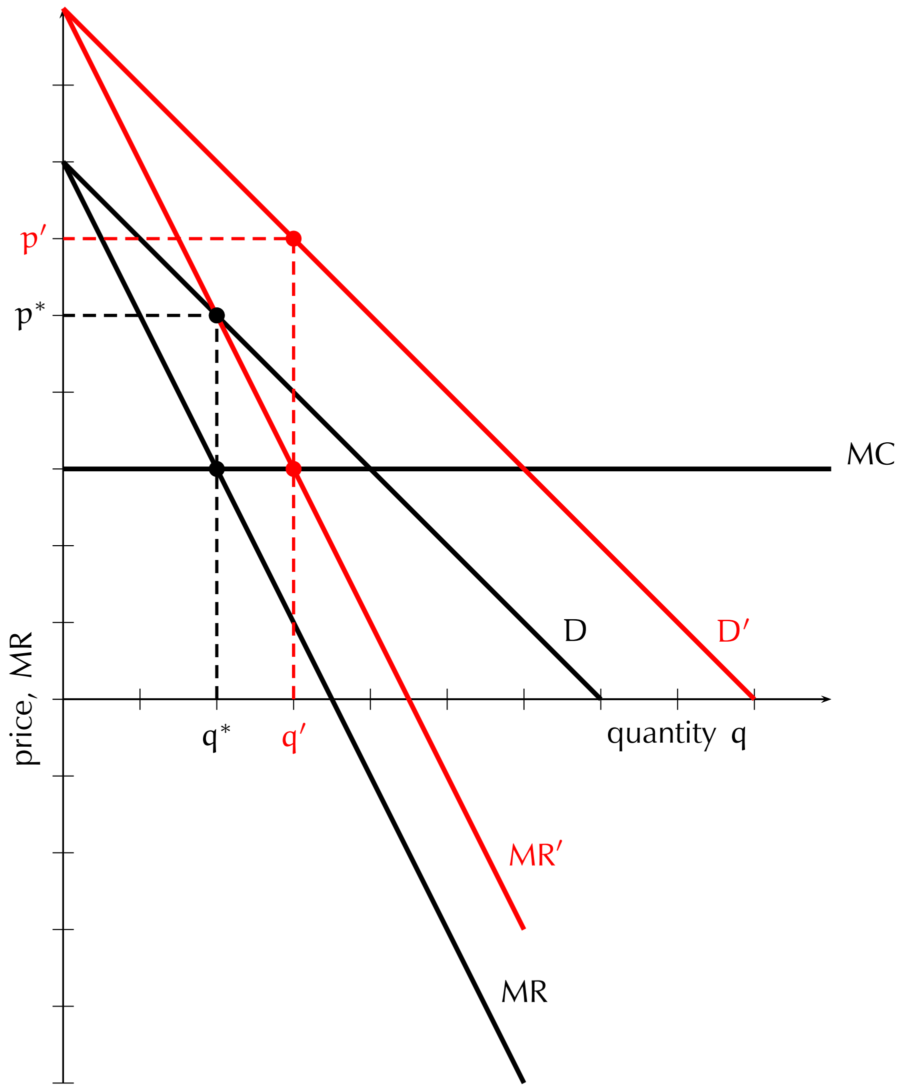
increases. Again, the only change is in the numerator, which increases.

Monopoly really is different from supply and demand.

9.4.6 Increase in Demand VI

Here's a graph of the situation when the choke price alone is increased.

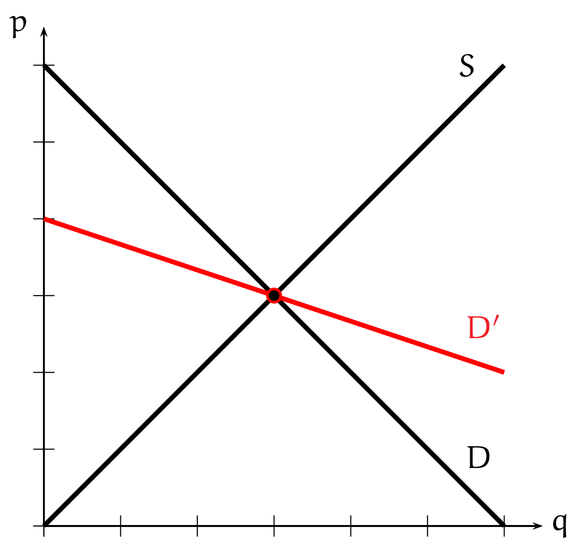
Old and New Demand Curves: Choke Price Increase



9.4.7 Neutral Change in Demand I

There's one more interesting change in demand to look at. We will change both the choke price and slope of demand in a way that keeps the current monopoly price and quantity on the demand curve. If we did that in a supply/demand context, equilibrium price and quantity would be unchanged. Because such a demand change would not affect the equilibrium price, we call it a **neutral change in demand**.

Neutral Demand Change in Supply/Demand Model



When there is market power, neutral changes are not neutral! They affect profit-maximizing prices and quantities. This helps drive home the point that for monopolists (or anyone with market power), supply curves are irrelevant.

9.4.8 Neutral Change in Demand II

So suppose $MC = \$3$ and inverse demand is $p(q) = 7 - q$. Then $MR = 7 - 2q$. We set $MR = MC$, so

$$7 - 2q^* = 3.$$

It follows that $q^* = 2$ and $p^* = 7 - 2 = 5$.

Suppose demand changes to $p'(q) = 9 - 2q$. Note that $(p, q) = (5, 2)$ is still on the demand curve. Now $MR' = 9 - 4q$. We set $MR' = MC = 3$, or

$$3 = 9 - 4q'$$

and solve for $q' = 3/2$. The corresponding price is $p' = 9 - 2q' = 6$. The profit-maximizing price and quantity have changed even though it was still possible to choose the same price and quantity, but doing so no longer maximizes profit. The shift in demand opened up new opportunities

9.4.9 Neutral Change in Demand III

Although it is possible to use the old price and quantity, it's not optimal. We can see that by computing the producer's surplus.

The variable cost is $VC(q) = 3q$. With the new demand curve, producer's surplus at $(2, 5)$ is $10 - VC(2) = 10 - 6 = 4$, while producer's surplus at $(1.5, 6)$ is $9 - VC(1.5) = 9 - 4.5 = 4.5$.

With the old demand curve, $(p', q') = (6, 1.5)$ is not available. The demand price for $q' = 1.5$ is 5.5, not 6, and the quantity demanded for $p' = 6$ is 1, not 1.5.

Even though the original optimal point is on the new demand curve, the change in slope—the change in elasticity—has opened up a new possibility for the monopolist. And the monopolist has taken it!

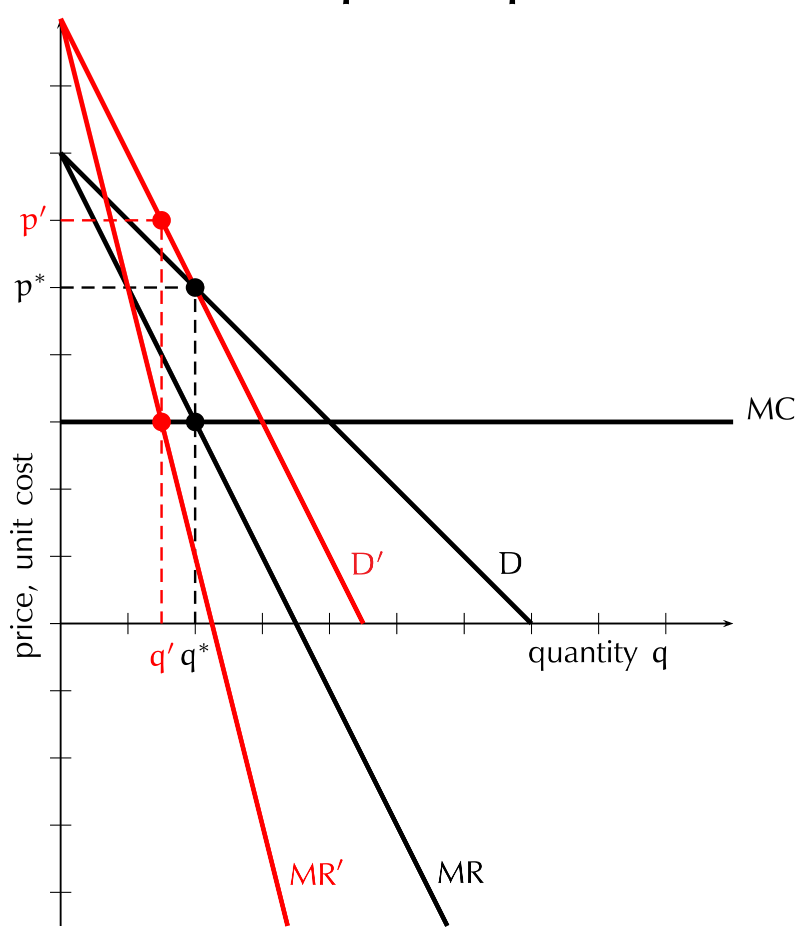
9.4.10 Neutral Change in Demand: Monopoly Solution

The so-called neutral change in demand is not neutral under monopoly. Under monopoly, the optimum depends on the elasticity of demand, and the new demand curve is less elastic where the demand curves interest. That means a new price and quantity are needed.

The new marginal revenue reaches marginal cost at $q' < q^*$, this causes an increase in price.

The reaction to a neutral change in demand means there is **no supply curve for monopoly**.

New Monopolist's Optimum



9.5 Elasticity and Profit Maximization

The neutral change in demand example showed us that the slope of the demand curve plays a role in determining where profit maximization occurs. That suggests that we think in terms of elasticity to sort this out.

Where does elasticity come in? It comes in via the relation between price and marginal revenue. We can use this to sharpen our understanding of monopoly pricing.

9.5.1 Elasticity and Marginal Revenue

Recall the elasticity formula for marginal revenue.

$$\mathrm{MR} = p\left(1 + \frac{1}{e_d}\right).$$

For profit maximization with $\mathrm{MC} = c$, we set

$$\mathrm{MR} = p\left(1 + \frac{1}{e_d}\right) = \mathrm{MC} = c,$$

so

$$p\left(1 + \frac{1}{e_d}\right) = c. \tag{9.5.2}$$

9.5.2 The Lerner Index

Equation (9.5.2) can be rearranged to read

$$\frac{p-c}{p} = -\frac{1}{e_d}.$$

The left side is the markup over marginal cost as a percentage of price, which must be the inverse absolute elasticity of demand. This is known as the **Lerner index**, L.

The Lerner index L measures the degree of market power. It is defined by

$$L = -\frac{1}{e_d} = \frac{1}{|e_d|}.$$

Since we must have elastic demand, $e_d < -1$. It follows that $1 > -1/e_d = L \geq 0$. The last inequality is due to the Law of Demand.

One important fact is that

$$MR = p\left(1 + \frac{1}{e_d}\right) = p(1-L).$$

9.5.3 Constant Elasticity Demand Curves

Some demand curves have a constant elasticity of demand. We saw that with Cobb-Douglas utility, where demand is always unit elastic ($e_d = -1$).

There are other constant elasticity demand curves. Anything of the form

$$p^{-e} = q \qquad \text{or} \qquad p = q^{-1/e}.$$

with $e > 0$ will do.

Here the elasticity of demand is

$$e_d = \frac{p}{q}\frac{dq}{dp} = \frac{p}{p^{-e}}(-ep^{-e-1}) = -e.$$

Since $e > 0$, the elasticity of demand is $-e < 0$, as befits a demand curve. We also need demand to be elastic so that the profit maximization problem has a solution. This requires $-e < -1$, or $e > 1$.

9.5.4 Monopoly Price with Constant Elasticity Demand

In that case, we can solve for the monopoly price in terms of elasticity and marginal cost c. Here $e_d = -e$ with $e > 0$. Then

$$\begin{aligned}
\frac{p-c}{p} &= -\frac{1}{e_d} = \frac{1}{e} \\
p - c &= \frac{p}{e} \\
p\left(1 - \frac{1}{e}\right) &= c \\
p\left(\frac{e-1}{e}\right) &= c \\
p &= c\left(\frac{e}{e-1}\right) \\
p &= c\left(\frac{1}{1-1/e}\right) \\
p &= c\left(\frac{1}{1-L}\right)
\end{aligned}$$

This price is only positive if $e > 1$, reflecting the fact that profit-maximization problem is only possible when demand is elastic.

9.5.5 Monopoly Pricing and the Lerner Index

The relation between price and marginal cost for constant elasticity demand curves can also be written

$$p(1 - L) = MC$$

where $L = -1/e_d$ is the Lerner index.

9.5.6 Sales and Excise Taxes with Monopoly: Marginal Revenue

Recall that a sales or excise tax of size t puts a wedge between demand and supply prices.

$$p_d = p_s + t \qquad \text{or} \qquad p_s = p_d - t$$

After-tax revenue is

$$R_t(q) = \left[p_d(q) - t\right]q.$$

Then marginal revenue with taxation is

$$\frac{dR_t}{dq} = \frac{d\big(p_d(q)q\big)}{dq} - t.$$

The $d(p_d q)/dq$ term is the original marginal revenue MR. That means that a tax of size t reduces marginal revenue by t, to $\text{MR}(q) - t$. It follows that monopoly profit is maximized when

$$\text{MR}(q) - t = \text{MC}(q) \qquad \text{or} \qquad \text{MR}(q) = t + \text{MC}(q).$$

9.5.7 Sales and Excise Taxes with Monopoly: Linear Demand

When demand is linear, $p(q) = a - bq$, we found that marginal revenue is $MR = a - 2bq$. With constant marginal cost $MC = c$ and an excise or sales tax of size t, we must solve the equation

$$MR = t + MC,$$

which we can rewrite as

$$a - 2bq^* = t + c.$$

Then

$$\boxed{q^* = \frac{a - (c + t)}{2b}}$$

and

$$\boxed{p^* = \frac{a + c + t}{2}.}$$

Compared to the no-tax case, the buyer's price has risen by $t/2$, so the seller's price falls by $t/2$. The tax is split evenly. If marginal cost were increasing, we would get a different split, depending on how fast marginal cost increases.

9.5.8 Sales and Excise Taxes with Monopoly: Constant Elasticity I

When consumer have a demand curve with constant elasticity, the situation changes. It is still the case that

$$\mathrm{MR} = c + t,$$

but now

$$\mathrm{MR} = p(1 - L)$$

where $L = -1/e_d$ is the Lerner index.

9.5.9 Sales and Excise Taxes with Monopoly: Constant Elasticity II

Setting marginal revenue equal to tax plus marginal cost, we obtain

$$p(1-L) = t + c,$$

so the monopoly charges

$$\boxed{p = \frac{c}{1-L} + \frac{t}{1-L}.}$$

The increase in the buyer's price due to the tax t is $\Delta p_d = t/(1-L)$. Now $0 \leq L < 1$ at the monopoly quantity. It follows that $t/(1-L) \geq t$. For example, $L = 1/2$ implies $\Delta p = 2t$, while $L = .8$ yields $\Delta p = 5p$.

More than 100% of the tax is paid by the buyers, and the more elastic demand is, the bigger the tax share will be. This is something that would never occur in a supply and demand model.

October 27, 2022