Intermediate Microeconomics — Week 12

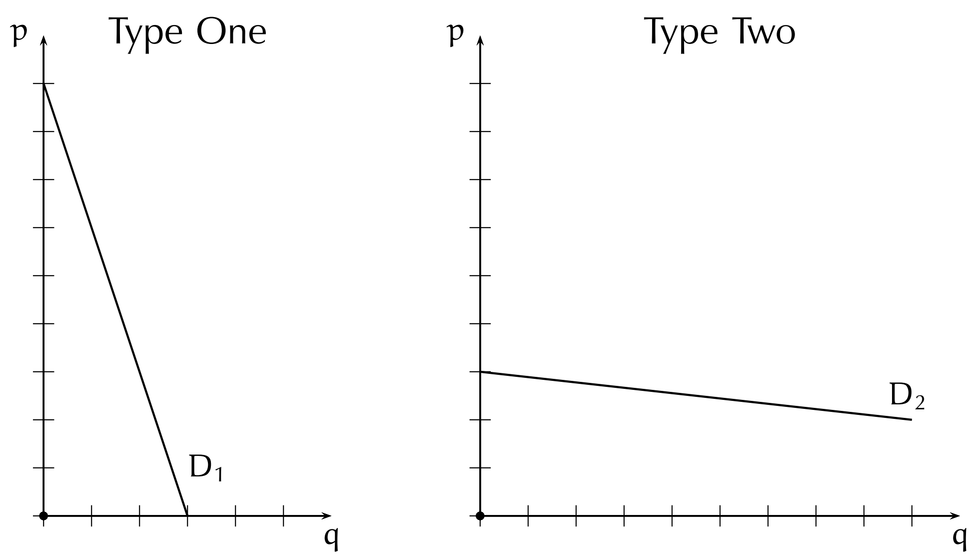
Professor Boyd **November 8 & 10, 2022**

10.4.1 Quantity Discounts

Repeated

One case where quantity discounts are useful is when we have one type of consumer who values small amounts of a product highly, but has little use for large quantities. This is illustrated on the left. A second type of consumer will not pay a high price for the product, but would like to buy a lot of it. This is illustrated on the right.

Quantity Discounts



10.4.2 Incentive Compatibility

Repeated

We want to get the type one consumers (high-value, low-quantity) to pay a high price, and the type two consumers (low-value, high quantity) to pay a low price and buy a lot. To get them to do this voluntarily, we need a price scheme that is **incentive compatible**. That is, it gives type one consumers an incentive to pay a lot, and type two consumers an incentive to buy a lot.

Suppose we set the **regular price** at $p_1 = \$5$, and offer a **discounted price** of $p_2 = \$2$ to anyone who buys at least 6 units.

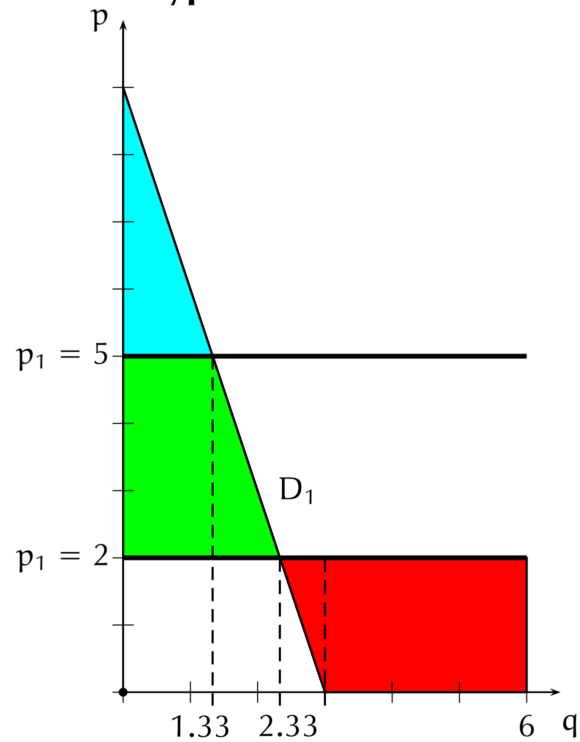
10.4.3 Incentive Compatibility: Type One **Repeated**

Let's consider the type one consumers first. They are willing to pay a higher price than type two, but are not willing to buy much.

At $p_1 = 5$, they buy $q_1 = 1.33$, obtaining consumer's surplus marked in light blue. At $p_2 = 2$, they would like to buy 3 units, but end up having to buy 6. This adds the green area to their surplus, but subtracts the red area.

The green area is $4 + \frac{1}{2}(3)(1) = 5.5$ and the red area is $6 + \frac{1}{2}(2/3)(2) = 6.67$. Type one consumers will lose \$1.17 of surplus if they take the discount. **It is better for them to choose to pay the high price.**

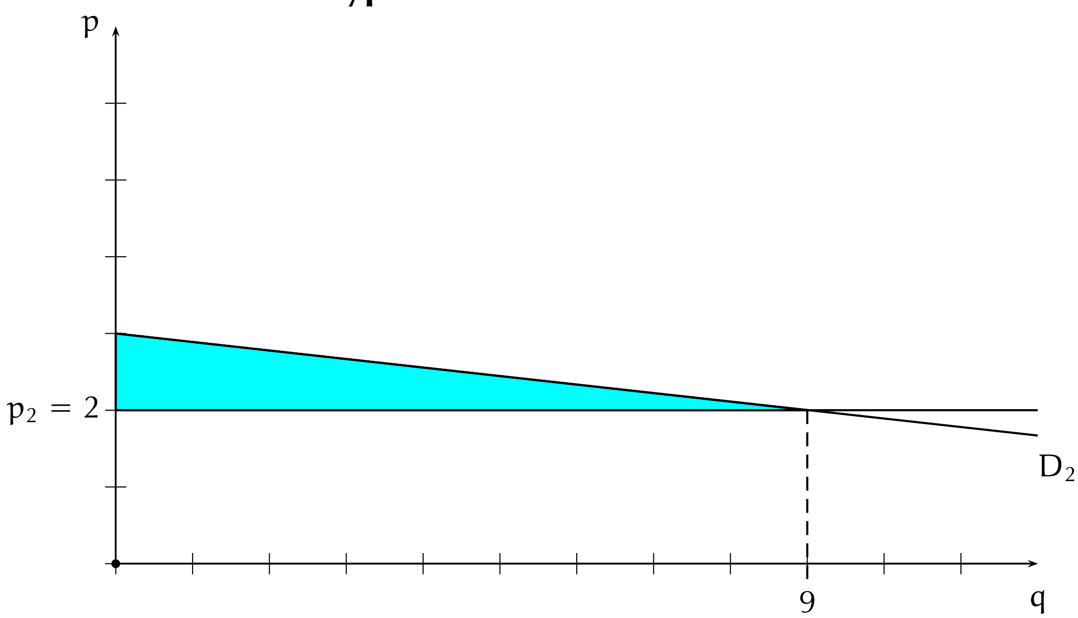
Type One Consumer



10.4.4 Incentive Compatibility: Type Two

We now consider the type two consumers. The higher price, $p_1 = 5$ is above their choke price of \$3. They will only buy this product on discount. At $p_2 = \$2$, type two consumers want to buy 9 units of the product, easily exceeding the minimum purchase requirement of 6 units for the discounted price. Their consumer's surplus of \$4.50 from buying 9 units at \$2 each is shown in light blue. They get consumer's surplus if they buy at the low price, but none at the high price. **It is better for them to choose to pay the low price.**

Type Two Consumer



10.4.5 Discounts Increase Profit

Our pricing scheme is incentive compatible. It succeeds in separating the consumers into two groups: those that choose to pay \$5 (type one), and those that choose to pay \$2 (type two). With a constant $MC = 1$, we have kept the type one consumers at the monopoly price for their demand, while offering a lower price that the type two consumers choose, which brings in additional profit. The firm could do even better by lowering the discounted price a bit and increasing the minimum purchase quantity.

10.4.6 Versioning

A second type of indirect price discrimination is versioning. **Versioning** is offering what is essentially the same product with different options.

One type occurs with airline pricing. Airlines are often not monopolies, but do have market power and can prevent resale. One problem for airlines is separating business customers from leisure customers. One way to this is to charge a lower price for tickets booked well in advance. This exploits the fact that business travel often involves a last-minute decision, while leisure travel is usually planned well ahead of time.

A very pure form of versioning has recently been used by Tesla, where they can program their cars to have certain features. They have even been able to alter the number of miles between recharges this way. They sell the exact same vehicle at different prices depending on which features have been enabled.

Such a possibility was foreshadowed by Intel decades ago. They produced two different versions of their new CPU, the DX and SX. The SX was originally less powerful, lacking a special processor. They started out by testing newly produced chips. Those that failed became the SX version, those that passed were DX's. However as yields increased, this production strategy was no longer feasible because there were too many DX chips and not enough SX's. They started disabling the extra features on some DX's to turn them into SX's.

Versioning is also based on setting up an incentive compatible pricing system.

10.4.7 Coupons

Coupons are another form of discounting, but allow some extra flexibility. For one, two different products can be tied together via coupons. We will not go into detail concerning this.

10.5 Pricing Strategy Recap

1. ✓ $P = MC$ (price taking firm, perfect competition) (chapter 8)
2. ✓ Monopoly Pricing: $MR = MC$, Demand determines p (chapter 9)
3. Block Pricing
4. Two-part Tariff
5. ✓ Quantity Discounts
6. ✓ Versioning
7. ✓ Coupons
8. Bundling
9. ✓ Segmenting
10. ✓Perfect Price Discrimination (just done)

10.6 Bundling

Bundling involves selling two or more products as a package (bundle) rather than selling them separately (à la carte). It's frequently used for cable TV. The requirements for bundling are:

1. Market Power
2. No Resale
3. Sale of two or more products with negatively correlated demand.

10.6.1 Positively Correlated Demands

We look at a simple two-good model, where the goods are the cable channels ESPN and A&E. The numbers are the dollar values of each channel to the two consumers (Ada and Bill), and the dollar value of the bundle (both channels together).

	ESPN	A&E	Bundle
Ada	$9	$1	$10
Bill	$10	$1.50	$11.50

Bill has a higher demand price for both ESPN and A&E than Ada does. This is what we mean when we say the demands are **positively correlated**.

10.6.2 Negatively Correlated Demands

We look at a simple two-good model, where the goods are the cable channels ESPN and A&E. The numbers are the dollar values of each channel to the two consumers (Ada and Bill), and the dollar value of the bundle (both channels together).

	ESPN	A&E	Bundle
Ada	\$10	\$1	\$11
Bill	\$9	\$1.50	\$10.50

Here we have switched the demand prices for ESPN so that Ada has a higher demand price than Bill for ESPN, but Bill has a higher demand price for A&E than Ada. This is what we mean when we say the demands are **negatively correlated**.

10.6.3 Á la Carte Pricing with Positively Correlated Demands

Let's return to the case of positively correlated demands and compare á la carte pricing versus bundling.

	ESPN	A&E	Bundle
Ada	\$9	\$1	\$10
Bill	\$10	\$1.50	\$11.50

We'll assume that cable services can be costlessly supplied. This means that revenue and profit are equal.

Each consumer has a choice of buying a single subscription to each channel, or none. If the price is greater than the value, they don't buy. If the price is equal or below the value they do buy. We pick the price that maximizes revenue.

In the case of ESPN, a price above \$10 will mean no sales and no revenue. Above \$9, but \$10 or below will sell to Bill, but not Ada. Revenue is at most \$10. A price of \$9 or below will sell to both. In that case, revenue is maximized by charging \$9 and selling two subscriptions, for a total revenue from ESPN of $2 \times \$9 = \$18$.

Similarly, they can price A&E at \$1, and earn \$2, for a total revenue of \$20 with á la carte pricing.

Even for A&E, it doesn't pay to try to sell only to Bill as the maximum sales price would be \$1.50, which is worse than \$2.

10.6.4 Bundle Pricing with Positively Correlated Demands

The valuations remain the same:

	ESPN	A&E	Bundle
Ada	\$9	\$1	\$10
Bill	\$10	\$1.50	\$11.50

The bundle is worth \$10 to Ada and \$11.50 to Bill. If the cable company only sells ESPN and A&E as part of a bundle, the best they can do is to charge \$10, sell to both, and earn $2 \times \$10 = \$20$ revenue.

The revenue is the same whether the cable company bundles or not. This is because the demands are positively correlated.

10.6.5 Optimal Pricing with Negatively Correlated Demands

We next examine the case where demands are negatively correlated.

	ESPN	A&E	Bundle
Ada	\$10	\$1	\$11
Bill	\$9	\$1.50	\$10.50

Under á la carte pricing, the cable company will again charge \$9 for ESPN and \$1 for A&E. Total sales will again be \$10.

Things are different under bundling. The cable company can now charge up to \$10.50 for the bundle. They sell to both Ada and Bill, for total sales of $2 \times \$10.50 = \$21$. Revenue has increased by 5%.

Bundling can increase revenue when demand is negatively correlated.

There are other possibilities when more people are involved. In the textbook, they look at a case of mixed bundling, where consumer can purchase either á la carte or via a bundle.

10.6.6 Pricing Strategy Update

1. ✓ $P = MC$ (price taking firm, perfect competition) (chapter 8)
2. ✓ Monopoly Pricing: $MR = MC$, Demand determines p (chapter 9)
3. Block Pricing
4. Two-part Tariff
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10.7 Advanced Pricing Strategies

Advanced pricing strategies, block pricing and two-part tariffs, can be used in the following situation:

1. Market Power
2. No Resale
3. Demand curves may be identical or different

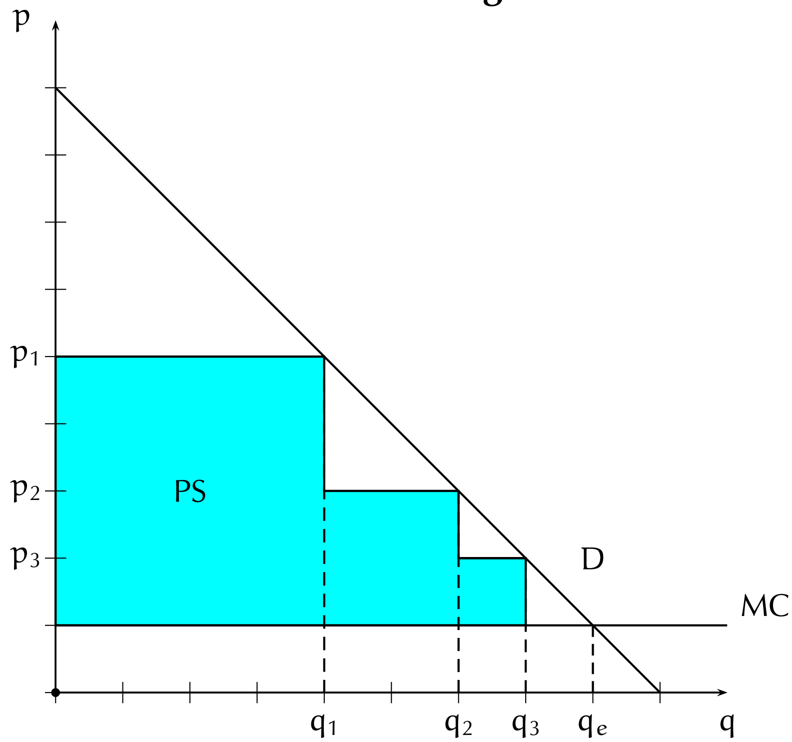
Unlike indirect price discrimination, block pricing applies even when there is no difference in demand between different consumers. Rather, the price setter charges different prices to the same person depending on how much is purchased. Block pricing differs from conventional discounts in an important way. The seller chooses the quantities, not the buyer.

10.7.1 Block Pricing I

This makes the pricing schedule a bit different. The price is $p_1 q_1$ for q_1 units (p_1 each). The second block is $p_2(q_2 - q_1)$ for $(q_2 - q_1)$ units (p_2 each), etc. The point is that the buyer is faced with a take it or leave it decision for each block. They can't take part of a block.

The diagram shows how block pricing works, and how it moves the market toward an efficient quantity q_e.

Block Pricing



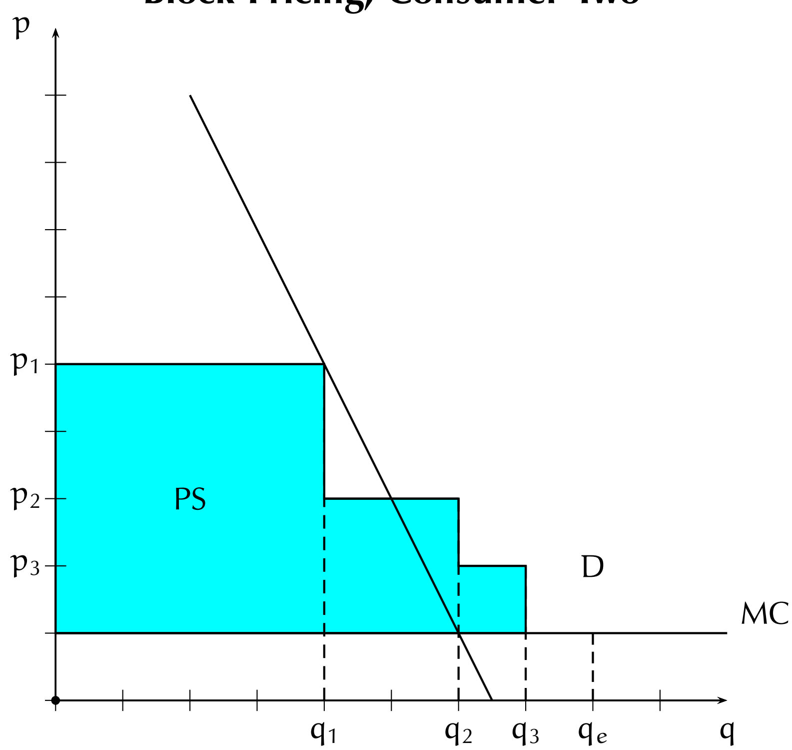
10.7.2 Block Pricing II

The consumer on the previous page bought 3 blocks. Different demand can lead to a different choice. Consumer two, who has a more inelastic demand, will only buy the first block.

The blocks show the same pricing schedule. Consumer two obviously should buy the first block as its on their demand curve. The second one generates zero consumer's surplus, so there's no benefit to buying, and the third block is not worth buying.

If the slope of the demand curve were even slightly flatter, the consumer would buy block two as well as block one.

Block Pricing, Consumer Two



10.7.3 Pricing Strategy Update

1. ✓ $P = MC$ (price taking firm, perfect competition) (chapter 8)
2. ✓ Monopoly Pricing: $MR = MC$, Demand determines p (chapter 9)
3. ✓ Block Pricing
4. Two-part Tariff
5. ✓ Quantity Discounts
6. ✓ Versioning
7. ✓ Coupons
8. ✓ Bundling
9. ✓ Segmenting
10. ✓ Perfect Price Discrimination (just done)

10.7.4 Two Part Pricing

We've now reached the last pricing schedule—two part pricing, also known as a two part tariff. As the name suggests, the price comes in two parts. The first part is the **entry fee**. It's a price you pay in order to be able to buy the product. The second price is a per-unit price, that you pay every time you buy more.

Such fees are commonly seen with utilities. There's a monthly charge to be hooked up to the utility. This fee doesn't depend on how much you use their product. On my Florida City Gas bill its called a "service charge". It costs $12.00 per month. On top of that, I pay for the natural gas I actually use, at a price that's based on their cost of natural gas.

Two part pricing can occur in many ways. We see it in bars (cover charges) and amusement parks such as Disney World (entry fees and prices per ride).

10.7.5 IBM Punch Cards

One interesting example of two part pricing involved IBM. In days past, when computers were room-sized and used punch cards, IBM would rent computers to firms. The rental contract would include services of an IBM technician, necessary, because these large machines often encountered problems.

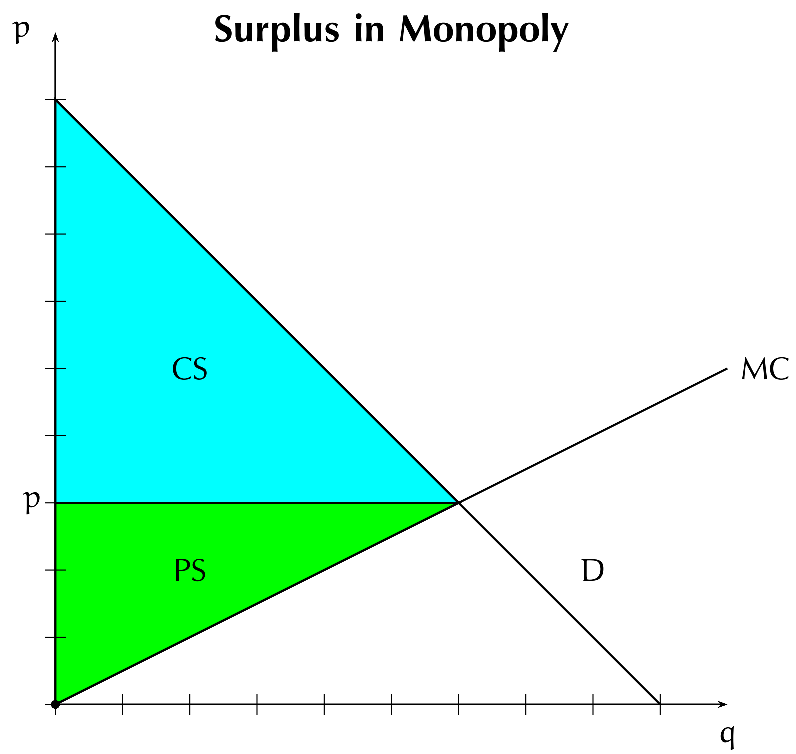
The rental fee was the entry fee part. IBM also charged a premium for the IBM punch cards used to enter programs and for data output. Once punched, a card could not be changed, so the computer needed a steady flow of brand-new punch cards. The cost of the cards was the per unit price. The amount you paid for cards depended on how much you used the computer.

There were cheaper punch cards available. However, if you used a non-IBM punch card, and it was found by the technician when servicing the computer, your service contract would not apply. This gave firms a strong incentive to use IBM punch cards and enforcing the two part pricing.

This was one of the issues in the IBM antitrust case that was resolved in 1956.

10.7.6 Optimal Two Part Pricing: Per Unit Price

Consider a monopoly with linear demand and marginal cost. We set the per unit price p equal to the efficient price. The light blue area is the resulting consumer's surplus.

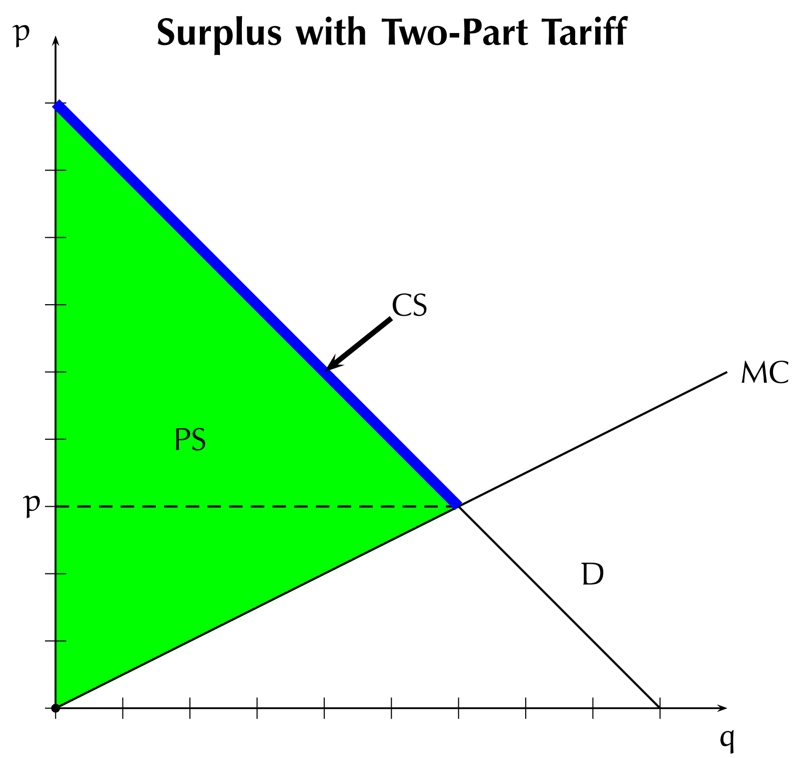


How big can we make the entry fee?

10.7.7 Optimal Two Part Pricing: Entry Fee

The most we could charge for the entry fee would be virtually the entire consumer's surplus! The consumer will get surplus from purchasing our product, and as long as a little surplus is left, the consumer is willing to buy. As a result, we can charge almost the entire surplus as entry fee.

This transfers nearly all of the surplus to the seller. It's a fully efficient outcome, but not one that benefits the consumer much. They get a small part of the surplus to encourage them to buy, but no more (the blue fringe on the green producer's surplus).



10.7.8 What if the Consumers are Different

With different consumers, each has a different consumer's surplus, yet we must set the same entry fee for all. This complicates the price-setting problem somewhat, but it can still be solved. For a fuller discussion, see Oi (1971).[1]

[1] Walter Oi (1971),"A Disneyland Dilemma: Two-part Tariffs for a Mickey Mouse Monopoly", *Quarterly Journal of Economics*, vol. 85 #1, pp. 77–96.

11.1 Chapter 11: Imperfect Competition

Recall our four model markets. We've now finished our study of monopoly markets.

Chap.	Type of Market	Number of Firms	Differentiation	Barriers to Entry/Exit
✓8.	Perfect Competition	Many	None	None
✓9.	Monopoly	One	Unique	A lot
11.	Oligopoly	A Few	Maybe	Some
11.	Monopolistic Competition	Many	Yes	Maybe

Some of the ideas developed for monopoly will prove useful when examining our next type of market: oligopoly. Both involve price setters, but with the extra complication of strategic interaction for oligopoly.

11.1.1 Oliogopoly

An **oliogopoly** is a market with a few firms. Here we will focus on the strategy used by each firm, and how the other firms react.

Equilibrium occurs in an oliogopoly when each firm uses a strategy that is a **best response** to the strategies used by the other firms. Such an equilibrium, involving mutual best responses, is called a **Nash equilibrium** after John Nash. He not only developed the concept, but showed how it could be used in various circumstances.

We will focus on **duopolies**, where there are two firms. This simplifies the strategic problem a bit as you only have one other firm to worry about.

11.1.2 Equilibrium in an Advertising Game

We consider competition between two airlines, American and United. They will compete by advertising. If an airline advertises, it will increase sales, partly at the expense of its competitors. However, there is also a cost to advertising. The gain in customers is not free. We see this most clearly by comparing the situation where both advertise with that where neither advertise. In the latter, there are no more customers than in the former, but profits are larger due to reduced advertising costs.

The following game matrix shows the payoffs in the advertising game for each pair of strategies. American chooses the column, United chooses the row. The boxes list the payoff for each firm for each strategy pair. Americans' payoffs are in blue, United's in red.

Advertising Game

United \ American	Advertise	Don't Advertise
Advertise	300 \ 200	200 \ 600
Don't Advertise	650 \ 50	600 \ 400

11.1.3 Best Responses: United

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Let's start by finding United's best responses to American's strategies. Suppose American decides to advertise. Then were are in the first column and United has a choice of \$200 million if they advertise and \$50 million if they don't. Obviously, United should respond by advertising.

What if American chooses to not advertise. Then we are in the second column and United has a choice of \$600 million from advertising, and \$400 million from not advertising. Again it is better for United to advertise.

Here advertising is a **dominant strategy** for United, meaning it is **always** the best response.

Advertising Game

United \ American	Advertise	Don't Advertise
Advertise	200 \ 300	600 \ 200
Don't Advertise	50 \ 650	400 \ 600

11.1.4 Best Responses: American

But what about American? What should they do? If United advertises, American can earn $300 million by advertising, but only $200 million if they don't advertise. Advertising is better.

If United does not advertise, American gets $650 million if they advertise and $600 million if they don't. Again, advertising is better. For American, advertising is always a best response. It is a dominant strategy.

Advertising Game

United \ American	Advertise	Don't Advertise
Advertise	200 \ 300	600 \ 200
Don't Advertise	50 \ 650	400 \ 600

11.1.5 Equilibrium in Dominant Strategies

In this case there is only one equilibrium. Both American and United must always advertise, regardless of what the other does. The result is that both advertise, and the payoffs are $300 million for American and $200 million for United.

When a firm has a dominant strategy, they don't need to take into account what the other firm is doing. They just use their dominant strategy.

When both sides use a dominant strategy, they are in Nash equilibrium. Each side makes the best response to the other's strategy. This is true even though the other side's strategy has no effect when you have a dominant strategy.

Dominant strategies for each player will lead to a unique Nash equilibrium. However, there it is often the case that Nash equilibria do not use dominant strategies. We'll see this in Cournot duopoly.

11.2 Cournot Duopoly

We start our study of oligopoly by focusing on the case of two sellers—duopoly. Here there are two firms that share a market without any other competition. One possible strategy is to collude—form a cartel. To collectively produce the monopoly quantity and split the profits.

We will start with a model where the cartel members have the same costs, so we expect the profits to be split equally. But we have to ask whether it is in each side's interest to maintain the cartel, or whether they have a better strategy.

Cournot duopoly consists of two firms that produce the same product. Firms choose how much to produce. Price is determined by the market demand curve based on total quantity produced. Summing up, we have:

- Duopoly (two firms)
- Choose **quantities**, not prices
- Demand determines price
- Goods are brought to market simultaneously
- Either Identical or Different Costs

11.2.1 Cartels in a Cournot Model

The firms can form a cartel to restrict production. Then one or more firms may decide to cheat on the cartel agreement. That means producing more than they agreed.

Let's see how it works.

Our two firms have identical and constant marginal cost $c > 0$. They have no fixed costs (we'll look at that later). The decide to produce amounts q_1 and q_2. Total supply is $Q = q_1 + q_2$. Market demand depends on total supply and inverse demand is assumed linear:

$$p(Q) = a - bQ = a - b(q_1 + q_2).$$

The marginal revenue is $MR(Q) = a - 2bQ$. Setting MR = MC and solving as before, we find the monopoly (cartel) solution

$$Q_M = \frac{a-c}{2b} \qquad \text{and} \qquad p_M = \frac{a+c}{2}.$$

With no fixed cost, profit is equal to producer's surplus, which is $(p_M - c)Q_M$. This is illustrated on the next page.

The producer's surplus can be split equally between the firms by setting $q_1 = q_2 = Q_M/2$. Then

$$PS_1 = (p_M - c)q_1 = (p_M - c)q_2 = PS_2 = \frac{1}{2}(p_M - c)Q_M.$$

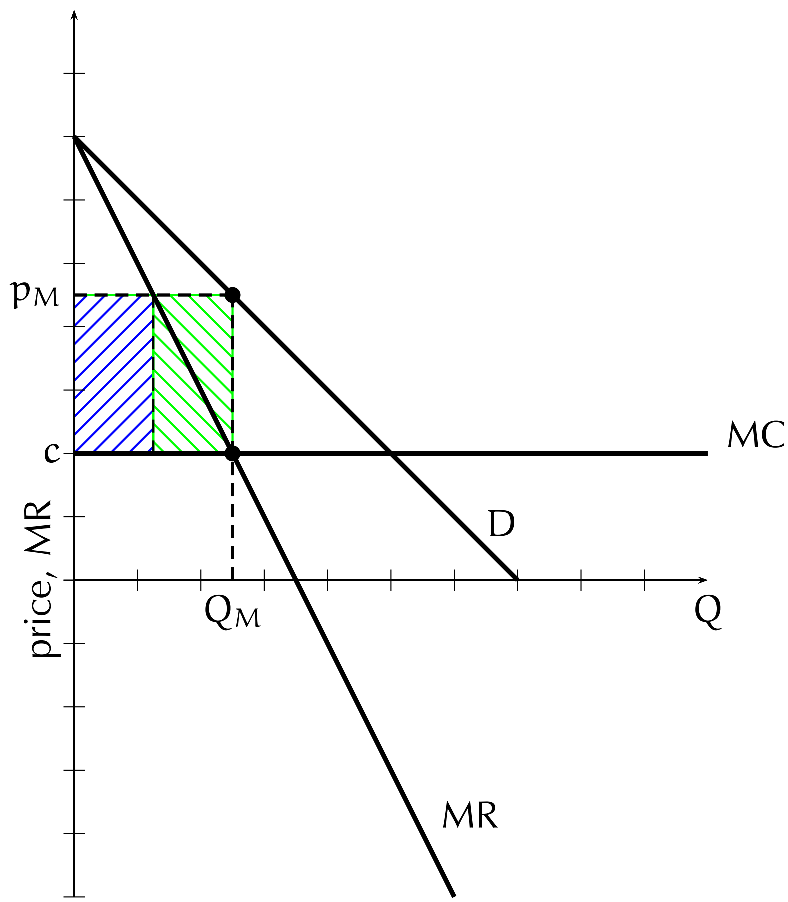
Each firm gets half of the monopoly profit. If the firms were not identical, the split might be different.

11.2.2 Monopoly or Cartel Solution

With zero fixed cost, the producer's surplus and profit are the same. The green area shows the maximum possible cartel profit. This is the same profit a monopoly would earn in this market. Unlike a monopoly, the cartel profit is then divided among the cartel members (two of them for duopoly).

If each produces half the cartel amount, each gets half the profit, as shown by the dashed line.

Cartel Profits



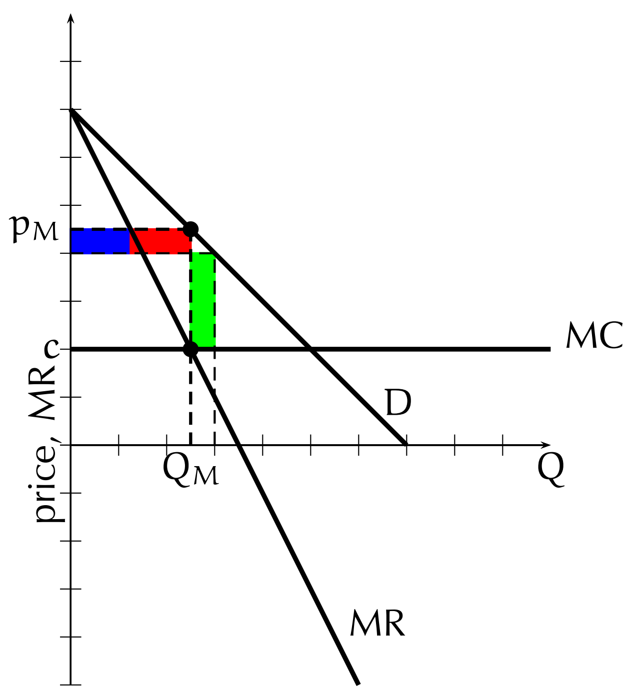
11.2.3 Cartel Incentives

Let's think about the incentives in this two-player cartel. They have maximized monopoly profit by setting MR = MC. We know that the marginal revenue is less then price because the firms receive a lower price when they sell more. Marginal revenue combines the gains from selling more and the losses from selling at a lower price.

But what incentive does each firm face? At the cartel solution, each bears only half of the cost of the lower price. This means that if one firm sells one more unit, they gain the full additional revenue from it (green), and lose only half of loss from a lower price (red). If they were a monopolist, the marginal cost would balance the marginal revenue, but an individual firms marginal revenue is higher than the cartel's because the firm only bears half of the cost from lower prices. The other half (blue) is borne by the other firm.

The fact that the green area is larger than the red means there is an incentive to cheat!

Gains and Losses from Cheating



11.2.4 Putting Numbers in the Cartel Model

Let's put some numbers in to make our model more concrete. Let market inverse demand be $p(q) = 40 - \frac{1}{2}Q$. That is, $a = 40$ and $b = \frac{1}{2}$. Then set marginal cost $c = 8$.

The monopoly solution is

$$p_M = \frac{40+8}{2} = 24 \qquad \text{and} \qquad q_M = \frac{40-8}{2(\frac{1}{2})} = 32.$$

Total profit (producer's surplus) is then

$$(p_M - c)q_M = (24-8)32 = 512.$$

Moroever, splitting output equally,

$$q_1 = q_2 = 16$$

and

$$PS_1 = PS_2 = 256.$$

11.2.5 Firm One Considers Cheating

Suppose firm one considers cheating on the cartel, producing more than agreed. Instead of producing $q_1 = 16$, firm one cheats and produces $q_1 = 20$.

Then total quantity increases to $Q = q_1 + q_2 = 20 + 16 = 36$. Since quantity has increased, the price falls, to $p(36) = 40 - 36/2 = 22$. We now compute the profits.

Firm one earns

$$\pi_1 = (p - c) \times q_1 = (22 - 8) \times 20 = 280$$

while firm two earns

$$\pi_2 = (p - c) \times q_2 = (22 - 8) \times 16 = 224.$$

For firm one, profit has risen from 256 to 280, while for firm two it has fallen from 256 to 224. Our cheating strategy has been successful. Our profits have risen by 24. Part of the cost of the lower price has been borne by the other firm, which lost 32. Note that total profit has fallen.

11.2.6 Maybe Both Firms Cheat

What if they both cheat? Then $q_1 = q_2 = 20$, so $Q = 40$ and $p(40) = 40 - 40/2 = 20$. Since quantity has increased even more, the price falls further.

Profits are again the same for both firms because they produce the same quantity at the same price and same marginal cost. The amount of profit is different. If both cheat, each earns a profit of

$$\pi_1 = \pi_2 = (p - MC) \times q_i = (20 - 8) \times 20 = 240.$$

Profits are lower for both firms than with the cartel solution. However, neither gets the very low profit that would happen if the other firm cheated and they didn't.

11.2.7 A Simple Cartel Game

We now organize our profit information as a game matrix.

Cartel Cheating Game

Firm Two \ Firm One	16	20
16	256 256	280 224
20	224 280	240 240

To find the equilibrium, we have to find each of the best responses.

11.2.8 Dominant Strategies

The best responses are now marked with stars.

Cartel Cheating Game: Best Responses

Firm Two \ Firm One	16	20
16	256 / 256	224 / 280 *
20	* 280 / 224	* 240 / 240 *

Now producing 20 is the dominant strategy for firm one. It is also the dominant strategy for firm two. There is a Nash equilibrium in dominant strategies. Both firms produce 20.

11.2.9 Nash Equilibrium of the Cartel Game

The heavy box marks the Nash equilibrium of the cartel game. Although the firms wanted to form a cartel, and would both benefit from doing so, they have failed.

They failed because each firm has a strong incentive to cheat on the cartel. This incentive was strong enough to break down the cartel.

Later on, we'll ask if there are any ways to neutralize this incentive problem. For now, it is enough to see that forming a cartel is not so simple.

Cartel Cheating Game: Nash Equilibrium

Firm Two \ Firm One	16	20
16	256 \ 256	224 \ 280 *
20	* 280 \ 224	*** 240 \ 240 ***

It's important to remember than when thinking about the stability of the cartel, we're only asking about the welfare of the firms involved. We're not looking at overall welfare, including both consumers and producers. If we did, we find that cheating increases total welfare because we are moving closer to the efficient (competitive) solution.

11.3 Cournot Competition

Our firms in the Cournot setting attempted to form a cartel. It failed. That means they are forced to compete.

How do they compete? We limited their choices to producing 16 or 20. That was enough to see that a cartel would fail. But if we are serious about analyzing the competition they are in, we must consider all other options.

11.3.1 Marginal Revenue in Cournot Competition

Let's find the best responses. Fortunately, we don't have to start from scratch. We can use a short-cut. We can use our knowledge of price setting firms to find the best responses.

Suppose firm two produces q_2. What is firm one's best response.

We know the demand price is

$$p(q_1) = 40 - \frac{q_1}{2} - \frac{q_2}{2} = \left(40 - \frac{q_2}{2}\right) - \frac{q_1}{2}.$$

We have rewritten demand so that changes in q_2 merely shift the vertical intercept.

As usual, marginal revenue has the same vertical intercept as demand, and has twice the slope. Recall that firm one cannot affect q_2. It's treated as part of the vertical intercept. Firm one's marginal revenue is

$$MR_1 = (40 - q_2/2) - q_1.$$

11.3.2 Maximum Cournot Profit: Best Response Functions

As usual, we find the maximum profit by setting the marginal revenue equal to the marginal cost of 8.

$$40 - \frac{q_2}{2} - q_1 = 8 \qquad \text{or} \qquad q_1(q_2) = 32 - \frac{q_2}{2}. \tag{11.3.1}$$

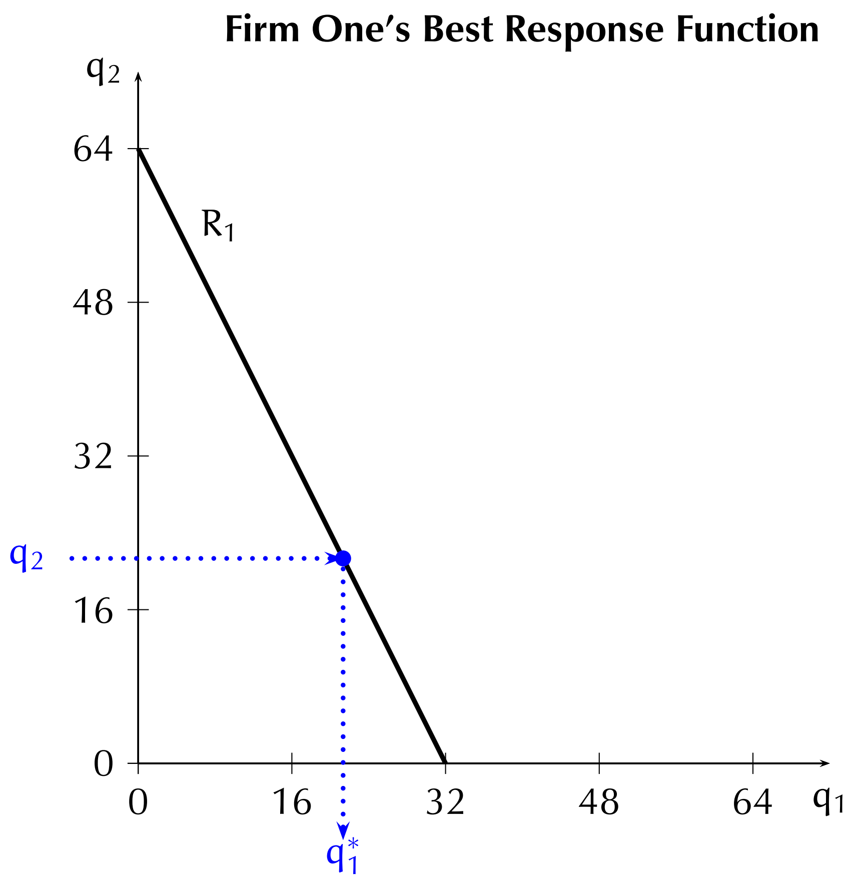
On the right hand side, we've written q_1 as a function of what firm two produces, q_2. This function is called the **reaction function**. For every q_2, it tells us the **best response** by firm one.

There is also a reaction function for firm two. Because of the symmetry of the problem, it takes the same form (the calculations are identical except for switching q_1 and q_2). Firm two's reaction function is

$$q_2(q_1) = 32 - \frac{q_1}{2}. \tag{11.3.2.}$$

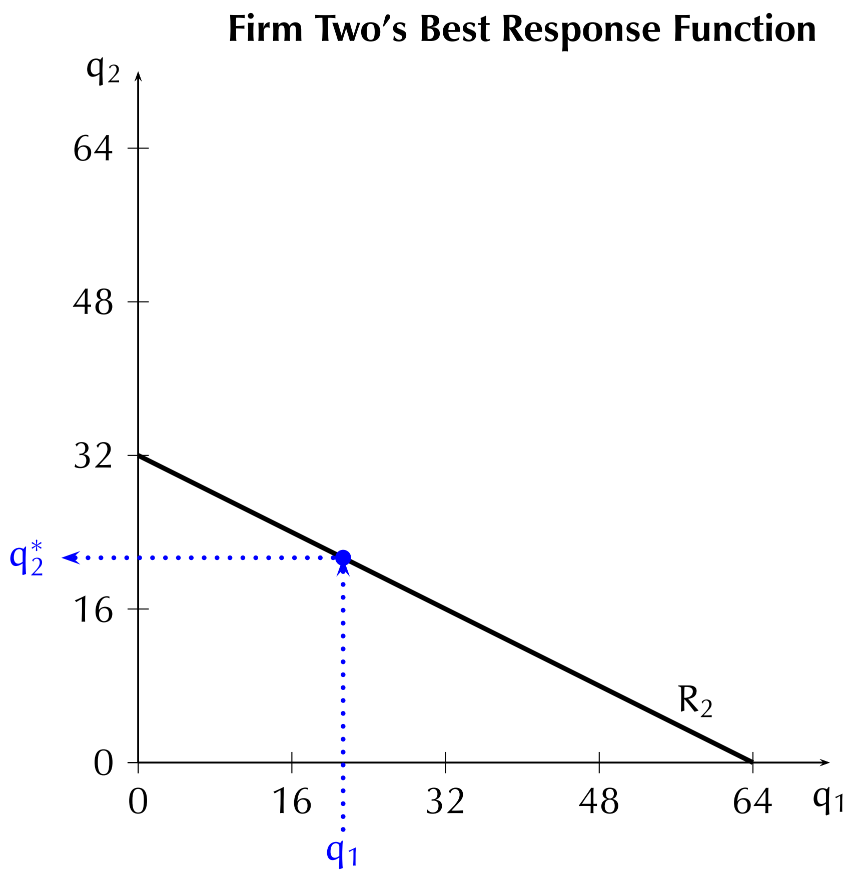
11.3.3 Reading Firm One's Reaction Function

We graph firm two's choice q_2 on the vertical axis. Given a value q_2, we read over to firm one's reaction curve R_1, and then down to find firm one's optimal response q_1^* to firm two's production of q_2.



11.3.4 Reading Firm Two's Reaction Function

We will also graph firm two's reaction function in the same way, keeping q_1 on the horizontal axis and q_2 on the vertical axis. In that case, we start with firm one's production q_1, and read up to firm two's reaction function, and over to find firm two's optimal response, q_2^*.



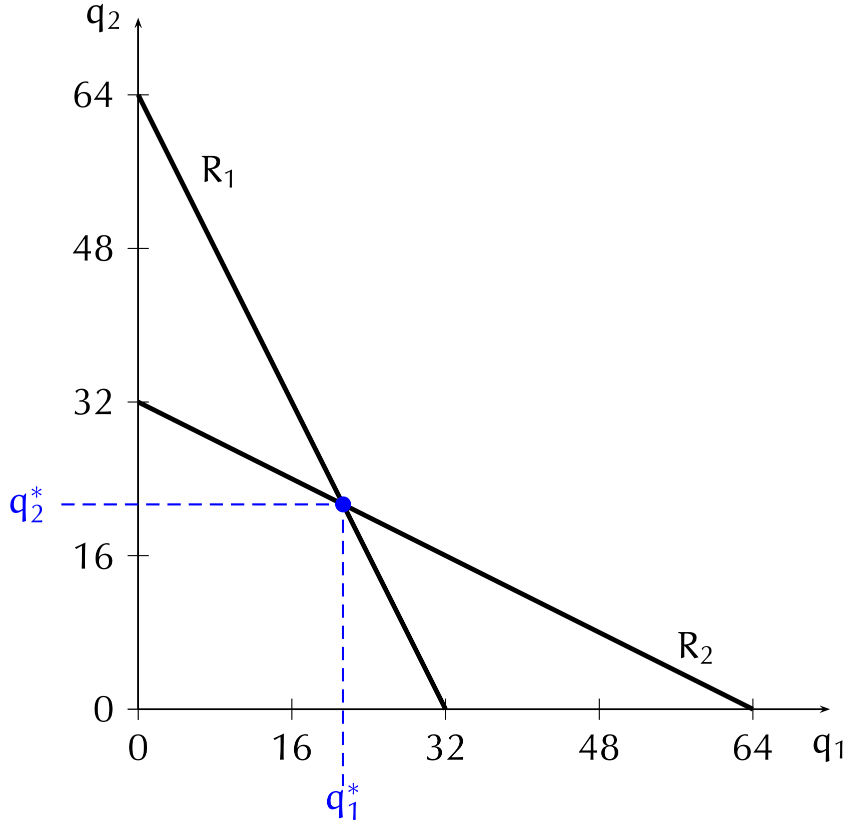
11.3.5 Reaction Functions and Nash Equilibrium

For the Nash-Cournot equilibrium, we must have mutual best responses. We need a combination (q_1^*, q_2^*) where q_1^* is the best response to q_2^* and vice-versa. In other words, we need to find where the two best response curves intersect.

At that intersection, $q_1^* = q_1(q_2^*)$ and $q_2^* = q_2(q_1^*)$. The actual numbers in our model are $q_1^* = q_2^* = 64/3$ and $p = 56/3$.

In this context, Cournot was the first to find the equilibrium, way back in 1838.[1]

Nash Equilibrium from Reaction Functions



[1] I believe this is the first appearance of this equilibrium concept in a formal model. Antoine Augustin Cournot (1801–1877) examined what we now call Cournot competition in his 1838 book, "Recherches sur les Principes Mathématiques de la Théorie des Richesses."

11.3.6 A General Cournot Duopoly with Identical Firms

Consider a Cournot duopoly where marginal cost is constant at $MC = c$ and inverse demand obeys $p = a - bQ = a - bq_1 - bq_2$. Marginal revenue for firm one is

$$MR_1 = (a - bq_2) - 2bq_1.$$

Setting $MR_1 = MC = c$, we can solve for firm one's reaction function.

$$q_1(q_2) = \frac{a-c}{2b} - \frac{q_2}{2} = q_M - \frac{q_2}{2}$$

where $q_M = (a - c)/2b$ is the monopoly quantity.

Similarly, the reaction function for firm two is

$$q_2(q_1) = q_M - \frac{q_1}{2}. \tag{11.3.3}$$

Notice that the intercepts of the reaction functions are q_M and $2q_M$.

11.3.7 Equilibrium in Cournot Duopoly with Identical Firms

By symmetry, the Nash-Cournot equilibrium (q_1^*, q_2^*) obeys $q_1^* = q_2^* = q_c$. Then

$$q_1(q_c) = q_c, \qquad \text{so} \qquad q_c = q_M - \frac{q_c}{2}.$$

It follows that each firm produces

$$q_c = \frac{2}{3} q_M$$

and that total output is

$$Q = 2q_c = \frac{4}{3} q_M.$$

Production is higher than with a monopoly or cartel, and the price is corresponding lower.

11.3.8 Changing Marginal Cost

Let's complicate the model a bit. Suppose the firms have different marginal costs, c_1 and c_2. The marginal revenue curves are unchanged, so we must solve

$$\begin{aligned} MR_1 &= \left(32 - \frac{q_2}{2}\right) - q_1 = c_1 \\ MR_2 &= \left(32 - \frac{q_1}{2}\right) - q_2 = c_2 \end{aligned}$$

which yields the reaction functions:

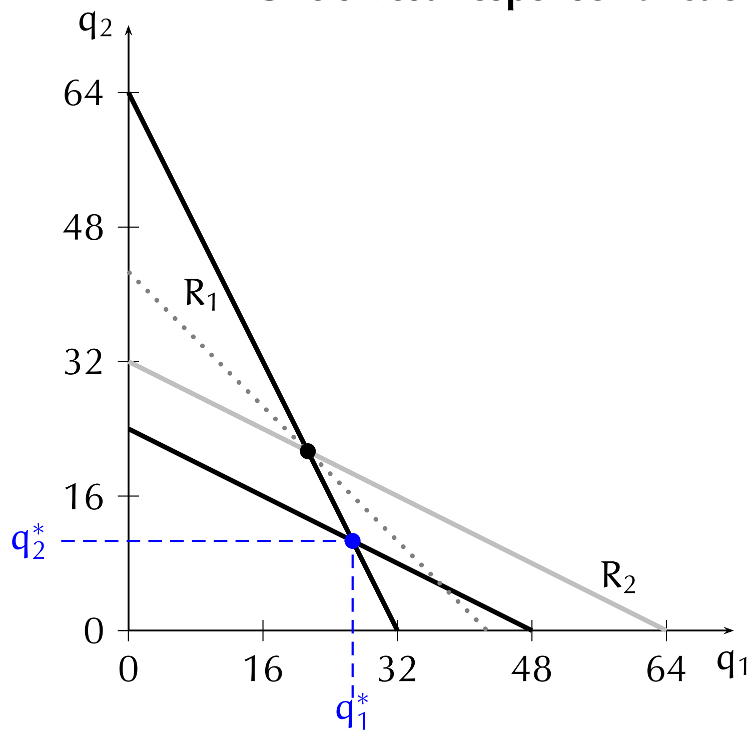
$$\begin{aligned} q_1(q_2) &= (40 - c_1) - \frac{q_2}{2} \\ q_2(q_1) &= (40 - c_2) - \frac{q_1}{2} \end{aligned}$$

11.3.9 The Effect of a Change in Marginal Cost

Set $c_1 = 8$ and $c_2 = 12$. We have increased the marginal cost for firm two. This moves firm two's reaction function inward, leading to increased production by firm one, and decreased production by firm two.

In fact, total output is down, which you can tell by the position of the new equilibrium relative to the dotted 45° line though the old equilibrium. This dotted line shows all the (q_1, q_2) that keep total output constant at the old level. The new equilibrium is lower, meaning lower total output and a higher price.

Firm One's Best Response Function



Generally speaking, increased marginal cost shifts the reaction functions inward, decreasing marginal cost shifts them outward.

11.3.10 Calculating the Nash-Cournot Equilibrium I

With $c_1 = 8$ and $c_2 = 12$, the reaction functions are:

$$q_1(q_2) = 32 - \frac{q_2}{2}$$
$$q_2(q_1) = 24 - \frac{q_1}{2}$$

In equilibrium, (q_1, q_2) must solve both equations simultaneously. We substitute the second equation in the first (line 3):

$$\begin{aligned}
q_1 &= q_1(q_2) \\
&= 32 - \frac{q_2}{2} \\
&= 32 - \frac{q_2(q_1)}{2} \\
&= 32 - \frac{24 - q_1/2}{2} \\
&= 20 + \frac{q_1}{4} \\
\frac{3}{4}q_1 &= 20 \\
q_1 &= \frac{80}{3}.
\end{aligned}$$

11.3.11 Calculating the Nash-Cournot Equillibrium II

To find q_2, we substitute in the other reaction function. This yields

$$q_2 = 24 - \frac{q_1}{2} = 24 - \frac{80}{6} = \frac{32}{3}.$$

Finally, total output is

$$Q = \frac{80}{3} + \frac{32}{3} = \frac{112}{3}.$$

and the resulting price is

$$p = 40 - \frac{Q}{2} = \frac{64}{3} \approx 21.3.$$

11.3.12 Nash-Cournot Equilibrium with Identical Firms

Returning to the original problem, with $c_1 = c_2 = 8$, we have two options. We can solve it the same way as $c_1 \neq c_2$, or we can take a shortcut.

A shortcut is available with firms are identical. Because each firm solves the same problem, we should look for a solution where each firm produces the same equilibrium output. I.e., we should set $q_1 = q_2 = q$.

In that case,

$$\begin{aligned} MC &= MR \\ 8 &= 40 - \frac{q}{2} - q \\ \frac{3q}{2} &= 32 \\ q &= \frac{64}{3}. \end{aligned}$$

Now $q_1 = q_2 = 64/3$, so total quantity is $Q = 128/3$. It follows that price is $p = 56/3$ and producer's surplus is $PS_1 = PS_2 = (56/3 - 8)(64/3) = 2048/9 = 227\frac{5}{9}$.

11.3.13 What About Fixed Costs?

We assumed that fixed cost is zero. What happens if there are fixed costs? The cartel solution tells us monopoly producer's surplus is 512. in the Nash-Cournot solution, each firm has a surplus of $227\frac{5}{9} \approx 227.6$.

This means that a monopolist could still make a profit at any fixed cost below 512, while the duopolists make losses even if the fixed cost is only 228. If there is a fixed cost, it can serve as a barrier to entry.

To make this a little clearer, if the fixed cost was 280, a monopolist would make a profit of 132. If would not benefit a second firm to enter this industry as duopoly profit would be negative.

Fixed costs can be barriers to entry.

11.4 Cournot Competition with Many Firms

So let's run with this idea and ask, for a given demand, marginal cost, and fixed costs, how many firms will enter the industry.

11.4.1 Cournot Competition with Many Firms: The Model

So let's take a model with N identical firms. We label the firms $i = 1, \dots, N$, and the quantities produced $q_1, \dots, q_N$. Each firm has the same constant marginal cost, c. Inverse demand is $p = a - bQ$ where

$$Q = q_1 + q_2 + \cdots + q_N$$

is the total quantity produced by the firms.

11.4.2 Cournot Competition with Many Firms: Best Responses

By symmetry, we expect every firm to produce the same amount in equilibrium. This will simplify our problem. Consider the problem for firm i. Given a total output level Q_{-i} produced by the other firms, the demand price is $p = (a - bQ_{-i}) - bq_i$. The resulting marginal revenue for firm i is

$$MR = \left(a - bQ_{-i}\right) - 2bq_i.$$

We set this equal to marginal cost to find firm i's reaction function

$$\begin{aligned} c &= \left(a - bQ_{-i}\right) - 2bq_i \\ 2bq_i &= a - c - bQ_{-i} \\ q_i &= \frac{a - c - bQ_{-i}}{2b}. \end{aligned}$$

Firm i's best response only depends on the total quantity produced by the other firms. It does not depend on how that produced is distributed among the other firms.

11.4.3 Cournot Competition with Many Firms: Quantity

Now we use our shortcut and set $q_i = q^*$ for each i. This means $Q_{-i} = (N-1)q^*$. Then

$$\begin{aligned} q^* &= \frac{a - c - b(N-1)q^*}{2b} \\ &= \frac{a-c}{2b} - \frac{N-1}{2}q^* \\ \frac{N+1}{2}q^* &= \frac{a-c}{2b} = q_M \end{aligned}$$

where q_M is the monopoly quantity. It follows that in equilibrium every firm produces

$$q^* = \frac{2}{N+1}q_M.$$

The total quantity produced is

$$\begin{aligned} Q = Nq^* &= \frac{2N}{N+1}q_M \\ &= \left[\frac{2N+2}{N+1} - \frac{2}{N+1}\right] q_M \\ &= \left[2 - \frac{2}{N+1}\right] q_M. \end{aligned}$$

Monopoly is the $N = 1$ case, where $Q = q_M$. Duopoly has $N = 2$, with $Q = \frac{4}{3}q_M$, as before. With three firms ($N = 3$), total production is $Q = \frac{3}{2}q_M$. Output continually increases with the number of firms. The corresponding price falls. This combination means consumer's surplus increases.

11.4.4 Cournot Competition with Many Firms: Price

The Nash-Cournot price with N firms is

$$\begin{aligned} p &= a - bQ \\ &= a - b\frac{2N}{N+1}q_M \\ &= a - b\frac{2N}{N+1}\frac{a-c}{2b} \\ &= a - \frac{N}{N+1}a - c \\ &= \frac{N+1-N}{N+1}a + \frac{N}{N+1}c \\ &= \frac{1}{N+1}a + \frac{N}{N+1}c. \end{aligned}$$

In other words, equilibrium price is a **weighted average** of the choke price a and common marginal cost c.

11.4.5 Cournot Competition with Many Firms: Varying the Number of Firms

Now

$$\begin{aligned} p &= \frac{1}{N+1}a + \frac{N}{N+1}c \\ Q &= \frac{2N}{N+1}q_M = \frac{N}{N+1} \cdot \frac{a-c}{b}. \end{aligned}$$

As mentioned earlier, for $N = 1$, this is the monopoly solution. For $N = 2$, it is the Cournot duopoly solution.

For N very large, $N/(N+1)$ approaches 1, so $Q \to (a-c)/b$, the efficient quantity. As for the price, it approaches marginal cost: $p \to c$. For large numbers of firms, we find the outcome of Cournot competition is close to perfect competition.

When N is in-between, we can regard this as a model of monopolistic competition—in spite of having identical products.

11.4.6 What about Fixed Cost?

As we saw earlier, fixed cost can limit entry. As N increases, the price falls, as do the quantities produced by individual firms. This means the producer's surplus falls with N also. Since $p \to c$, it goes to zero.

This allows the fixed cost to determine the number of firms. The actual number of firms that can operate without losing money would be the largest number N where each firm's profit is non-negative. Since each firm's profit decreases with N to zero, there has to be such a number.

11.4.7 Cournot Competition with Many Firms: Number of Firms

A firm will only stay be in the industry if they make zero or more profit. That is, if the producer's surplus exceeds the fixed cost. It follows that if N is the last firm that enters it must make zero or more profit, and if there are $(N + 1)$, there must be a loss.

We look for a the largest N with

$$
\begin{aligned}
(p - c)q^* - FC &\geq 0 \\
\left(\frac{a + Nc}{N + 1} - c\right) \frac{1}{N + 1} \frac{a - c}{b} &\geq FC \\
\left(\frac{a - c}{N + 1}\right) \frac{1}{N + 1} \frac{a - c}{b} &\geq FC \\
\frac{1}{(N + 1)^2} \left(\frac{(a - c)^2}{b}\right) &\geq FC \\
\frac{1}{FC} \left(\frac{(a - c)^2}{b}\right) &\geq (N + 1)^2 \\
\frac{a - c}{\sqrt{b\,FC}} - 1 &\geq N
\end{aligned}
$$

That is, we want the largest integer N that obeys:

$$
\frac{a - c}{\sqrt{b\,FC}} - 1 \geq N
$$

11.4.8 Cournot Oligopoly with Many Firms and Differing Costs I

If the costs are different, but marginal cost is constant for every firm, the Nash-Cournot equilibrium is a only little different.

Suppose firm i has marginal cost c_i. I will repeat the calculation, which very similar to the identical cost case. The general rule is that Nc is replaced by

$$\sum_{i=1}^{N} c_i = c_1 + \cdots + c_N$$

while a bare c is replaced by the average value of the c_i, which we call $\bar{c}$ and define by

$$\bar{c} = \frac{1}{N}\sum_{i=1}^{n} c_i = \frac{1}{N}(c_1 + \cdots + c_N).$$

This allows us to write the sum of c_i as $N\bar{c}$.

Thus

$$\begin{aligned} p &= \frac{a + N\bar{c}}{N+1} \\ &= \frac{a + c_1 + c_2 + \cdots + c_N}{N+1} \\ &= \frac{a + N\bar{c}}{N+1} \\ Q &= \frac{N}{N+1}\frac{a - \bar{c}}{b} \end{aligned}$$

For N very large, $N/(N+1)$ approaches 1, so $Q \to (a - \bar{c})/b$ and $p \to \bar{c}$. For large numbers of firms, Cournot competition approaches perfect competition.

11.4.9 Cournot Oligopoly with Many Firms and Differing Costs II

The situation with fixed costs is more complex. Suppose both the marginal and fixed costs vary between firms. Call them c_i and FC_i. Then a firm will only stay be in the industry if they make zero or more profit. That is, if the producer's surplus exceeds the fixed cost:

$$(p - c_i)q_i^* \geq FC_i\,.$$

For firm i, this now means

$$\begin{aligned} (p - c_i)q^* &\geq FC_i \\ \left(\frac{a + N\bar{c}}{N+1} - c_i\right)\frac{N}{N+1}\frac{a - c_i}{b} &\geq FC_i \\ \left(\frac{a + N(\bar{c} - c_i) - c_i}{N+1}\right)\frac{N}{N+1}\frac{a - c_i}{b} &\geq FC_i \\ N\left(\frac{a + N(\bar{c} - c_i) - c_i}{(N+1)^2}\right)\frac{a - c_i}{b} &\geq FC_i\,. \end{aligned} \tag{11.4.4}$$

That's not a transparent equation! The problem is there is a trade-off between fixed and marginal costs. If we eliminate the trade-off by making one of them the same across firms, the meaning becomes clear.

What equation (11.4.4) tells us is that firms will enter if they make a profit. If they all had the same fixed cost, it would mean that the firms with the lowest marginal cost will be in the industry.

If they all have the same marginal cost, the firms with the lowest fixed costs will be in the industry.

11.4.10 Cournot Competition: Summing Up

The Cournot model covers quite a lot of ground. It not only gives a model of duopoly and oligopoly ($N = 2$ or a small number), it is a model of monopoly ($N = 1$), monopolistic competition (N larger, but not too large), and even approaches perfect competition (N very large).

When we add in fixed costs, the resulting profit condition, equation (11.4.4), even determines N, and so the type of entry.

The profit condition also allows for richer firm structures than may be apparent. It allows for industries that allow firms with low marginal cost and high fixed cost (big firms) and firms with high marginal cost and low fixed cost (small firms) to coexist in the same market.

It still remains a relatively simple model (e.g., constant marginal cost), and yet is new enough that economists presenting models with these features at economics conferences felt it necessary to explain the details when I was a graduate student. This is true of most of the models we cover from chapters 11 and 12.

November 10, 2022