Intermediate Microeconomics — Week 13

Professor Boyd **November 15 & 17, 2022**

Nov. 17, 2022

11.5 Bertrand Oligopoly

Cournot competition applies when producers sell identical goods, or nearly so. Moreover, the model involves all firm selling goods in the same market. There's no product differentiation.

Bertrand competition is primarily a model of differentiated goods, although it also applies in some cases were commodities are not differentiated. Indeed, we will start with that.[1]

Summing up, Bertrand models have the following characteristics:

- Duopoly (two firms), or more
- Firms set their **prices**, not quantities
- Demand determines quantities
- Goods are brought to market simultaneously
- Either Identical or Different Costs

[1] Bertrand competition was first analyzed by Joseph Louis François Bertrand (1822–1900), a French mathematician who also worked on economics and thermodynamics. For the original Bertrand model, see Joseph Bertrand (1883), "Book review of theorie mathematique de la richesse sociale and of recherches sur les principles mathematiques de la theorie des richesses", *Journal de Savants* **67**: 499–508.

11.5.1 Bertrand Duopoly with Identical Goods

We start by examining Bertrand duopoly with identical goods. Since the goods are identical, there's no reason other than price for a consumer to prefer one good to another. To emphasize, when I say they are identical, I'm not just referring to the physical product, but also repair service and other support.

In such a case, consumers have no reason **except price** to prefer one firm over another. In our example, the firms will advertise, and the consumers will flock to the firm with the lowest price.

So consider Target and Walmart selling the Playstation PS5. We will simplify demand drastically to focus on the special aspects of Bertrand competition. The consumers will buy from the store with the lowest price. We take the marginal cost of a single digital edition PS5 as $300. Each consumer can only buy one and there are Q potential consumers. So the quantity demanded is always Q.[2]

[2] Fancier versions of the model, including the original, have a proper demand curve.

11.5.2 Target vs. Walmart: Price War!

The quantity demanded is Q for the store with the lower price, 0 for the store with the higher price. If there's a tie, half the consumers end up buying from Target, the other half from Walmart. Let P_T and P_W be the prices charged by Target and Walmart.

PS5 Demand

Prices	Target	Walmart
$P_T < P_W$	Q	0
$P_T = P_W$	$Q/2$	$Q/2$
$P_T > P_W$	0	Q

Suppose Target sets their price at \$399.99. What is Walmart's best response?

If Walmart charges \$400 or more, no one buys their PS5's, and they earn nothing. If they charge \$399.99, they get half of the market and a producer's surplus of $(399.99 - 300)Q/2 = 49.995Q$. If they charge $P_W < 399.99$, they get producer's surplus of $(p - 300)Q$ which will be better than matching Target as long as $p > 349.995$.

So they undercut Target.

Of course Target responds by undercutting Walmart, and the price war is on!

11.5.3 Where does the Price War End?

It ends in Nash equilibrium, of course!

There are two Nash equilibria, assuming prices in fractional cents are not possible. In one, both firms charge \$300. In the other, both firms charge \$300.01.

Let's look at the latter. It doesn't pay to undercut \$300.01, because you will lose money if $P < \$300$, and you get no surplus if $P = \$300$, while you have a small surplus of $0.005Q$ at \$300.01. If you raise your price, you sell nothing and get no surplus.

An equilibrium at any higher price allows undercutting, so none of those work.

One interesting thing here is that the equilibrium price is almost at marginal cost, only one cent off. This is not unusual when Bertrand equilibria end in a price war. It makes quite a contrast with Cournot duopoly, where the equilibrium price with two firms is quite a bit higher than the competitive price.

11.6 Stackelberg Competition

There is a third model of duopoly behavior, Stackelberg leader-follower competition. It is like Cournot competition in that each firm chooses the quantity to produce, but different in that firm one moves first, and firm two responds to firm one's move.[3]

This means that firm one will take firm two's response into account when choosing their move. The way they do that is to use firm two's reaction function.

Summing up, we have:

- Duopoly (two firms)
- Choose **quantities**, not prices
- Demand determines price
- **Firm one brings goods to market first, followed by firm two.**
- Either Identical or Different Costs

[3] Stackelberg competition was introduced by Heinrich Freiherr von Stackelberg (1905–1946) in his 1934 book, *Marktform und Gleichgewicht*, Springer-Verlag, Wein, New York. It was translated into English as *Market Structure and Equilibrium* (2011), transl. Damien Bazin (Scientific Director), Lynn Urch and Rowland Hill, Springer, Heidelberg, Dordrecht, London, New York. Stackelberg was relatively young when he died from lymphoma.

11.6.1 A Stackelberg Equilibrium I

We'll work out the Stackelberg solution for the case of identical marginal costs, $MC = c = 8$ and a demand curve of $p = 40 - Q/2$. These are the numbers we previously used for the Cournot model of section 11.3.

We previously found the reaction functions were:

$$q_1(q_2) = 32 - \frac{q_2}{2}. \tag{11.3.1}$$

$$q_2(q_1) = 32 - \frac{q_1}{2}. \tag{11.3.2}$$

11.6.2 A Stackelberg Equilibrium II

Rather than treat firm two's choice of quantity as a parameter, we replace q_2 in firm one's demand function by firm two's reaction function. Then

$$\begin{aligned} p &= 40 - \frac{q_1 + q_2(q_1)}{2} \\ &= 40 - \frac{q_1}{2} - \frac{32 - q_1/2}{2} \\ &= 24 - \frac{q_1}{4}. \end{aligned}$$

Marginal revenue for firm one becomes $MR_1 = 24 - q_1/2$. To find firm one's optimal choice, taking firm two's optimal response into account, we simply set this marginal revenue equal to marginal cost ($c = 8$), obtaining

$$24 - q_1/2 = 8 \qquad \text{or} \qquad q_1 = 32.$$

Firm one produces the monopoly output all by itself.

Finally, we plug this into firm two's reaction function to find

$$q_2 = q_2(32) = 32 - 16 = 16.$$

Firm one has a **first-mover advantage**. This translates into twice as much output and twice as much producer's surplus as firm two. Total output in Stackelberg equilibrium is $32 + 16 = 48$, which is more than is produced in Cournot equilibrium ($42\frac{2}{3}$). Consumers benefit from the higher output and resulting lower price (16).

Firm one earns more producer's surplus (256) than in Cournot equilibrium ($227\frac{5}{9}$), while firm two earns less (128).

11.6.3 A More General Stackelberg Equilibrium

Suppose marginal cost is constant at $MC = c$ and demand is $p = a - bQ$, as we used in Cournot competition. We calculated the reaction function for firm two as $q_2(q_1) = q_M - \frac{q_1}{2}$ in equation (11.3.3).

We substitute in demand, obtaining

$$p = a - bq_M + b\frac{q_1}{2} - bq_1 = a - bq_M - \frac{b}{2}q_1.$$

This yields marginal revenue

$$MR_1 = a - bq_M - bq_1.$$

To maximize firm one's profit, we set $MR_1 = MC = c$. Then

$$a - c - bq_M = bq_1, \qquad \text{so} \qquad q_1 = 2q_M - q_M = q_M.$$

The Stackelberg leader always produces the monopoly quantity q_M, which means the follower produces $q_2(q_M) = q_M/2$. Total output is $\frac{3}{2}q_M$, compared to the Cournot output of $\frac{4}{3}q_M$ and the monopoly/cartel output of q_M. Of the three, Stackelberg competition results in the highest output, and so lowest price and highest consumer's surplus.

11.7 Bertrand Equilibrium with Differentiated Products

Another duopoly model is a variation on Bertrand equilibrium where the products are **differentiated**. In other words, they are similar enough to be good substitutes, but not identical. This type of Bertrand model has the following properties.

- Duopoly (two firms)
- Firms set their prices, not quantities
- Demand determines quantities
- Goods are brought to market simultaneously
- Either Identical or Different Costs

11.7.1 A Bertrand Model with Differentiated Products

The demand functions (not inverse demand) are

$$q_1 = 100 - 2p_1 + 2p_2$$
$$q_2 = 150 + 2p_1 - 3p_2$$

The coefficients on p_1 for q_1 and p_2 for q_2 are negative due to the Law of Demand. The positive coefficients on p_2 for q_1 and p_1 for q_2 indicates the two goods are substitutes (positive cross-price elasticity).

We'll set $MC_1 = MC_2 = c = 10$. For simplicity, we'll assume that fixed costs are zero. Then profit is just producer's surplus, $(p_i - c)q_i$.

11.7.2 Finding the Bertrand Reaction Functions

Our next step is to find the reaction functions which describe the best responses. To that end, we write down the profits (producer's surplus) for each firm in terms of prices.

$$\begin{aligned}
PS_1 &= (p_1 - c)q_1 \\
&= (p_1 - 10)(100 - 2p_1 + 2p_2) \\
&= 100p_1 - 2p_1^2 + 2p_1p_2 - 1000 + 20p_1 - 20p_2 \\
PS_2 &= (p_2 - c)q_2 \\
&= (p_2 - 10)(150 + 2p_1 - 3p_2) \\
&= 150p_2 + 2p_1p_2 - 3p_2^2 - 1500 - 20p_1 + 30p_2
\end{aligned}$$

Then we take the partial derivative of PS_i with respect to p_i, which we set to zero. We obtain

$$\begin{aligned}
0 &= \frac{\partial PS_1}{\partial p_1} = 100 - 4p_1 + 2p_2 + 20 \\
0 &= \frac{\partial PS_2}{\partial p_2} = 150 + 2p_1 - 6p_2 + 30
\end{aligned}$$

11.7.3 The Bertrand Reaction Functions

The Bertrand reaction functions are

$$p_1(p_2) = 30 + \frac{1}{2}p_2$$
$$p_2(p_1) = 30 + \frac{1}{3}p_1.$$

We solve this by substituting $p_2(p_1)$ in the first equation. Then

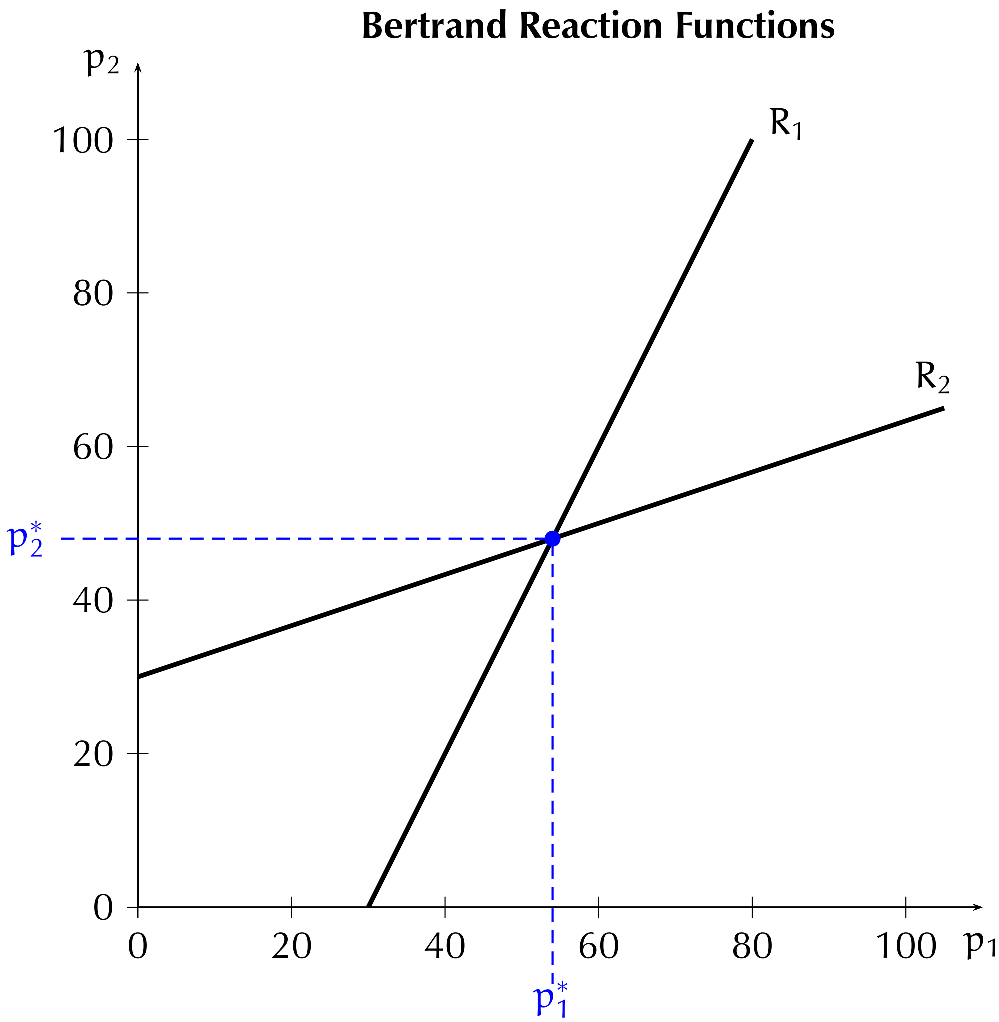
$$p_1 = 30 + \frac{1}{2}\left(30 + \frac{1}{3}p_1\right) = 45 + \frac{1}{6}p_1.$$

Then $(5/6)p_1 = 45$, so $p_1 = 54$. We then compute $p_2(54) = 30 + \frac{1}{3}(54) = 48$. The Bertrand equilibrium is $p_1^* = 54$, $p_2^* = 48$.

Notice that these Bertrand equilibrium prices are no where near the marginal cost of $10. It's very different from the price war case.

11.7.4 Graph of Bertrand Reaction Functions

Unlike the Cournot case, our Bertrand reaction functions have a positive slope.



11.7.5 A More General Bertrand Model

Suppose demand is

$$\begin{aligned} q_1 &= a_1 - b_1 p_1 + d_1 p_2 \\ q_2 &= a_2 + d_2 p_1 - b_2 p_2 \end{aligned}$$

with $a_i, b_i, d_i > 0$. Suppose c_i is the (constant) marginal cost for firm $i = 1, 2$.

Profit for firm i is

$$(p_i - c_i) q_i(p_1, p_2).$$

Taking the p_1 derivative, we obtain the reaction function for firm one

$$\begin{aligned} 0 &= q_i(p_1, p_2) + (p_1 - c_1)(-b_1) \\ &= a_1 - b_1 p_1 + d_1 p_2 - b_1 p_1 + b_1 c_1 \\ 2b_1 p_1 &= a_1 + b_1 c_1 + d_1 p_2 \\ p_1 &= \frac{a_1 + b_1 c_1 + d_1 p_2}{2b_1} > 0. \end{aligned}$$

Similarly, the reaction function for firm two is

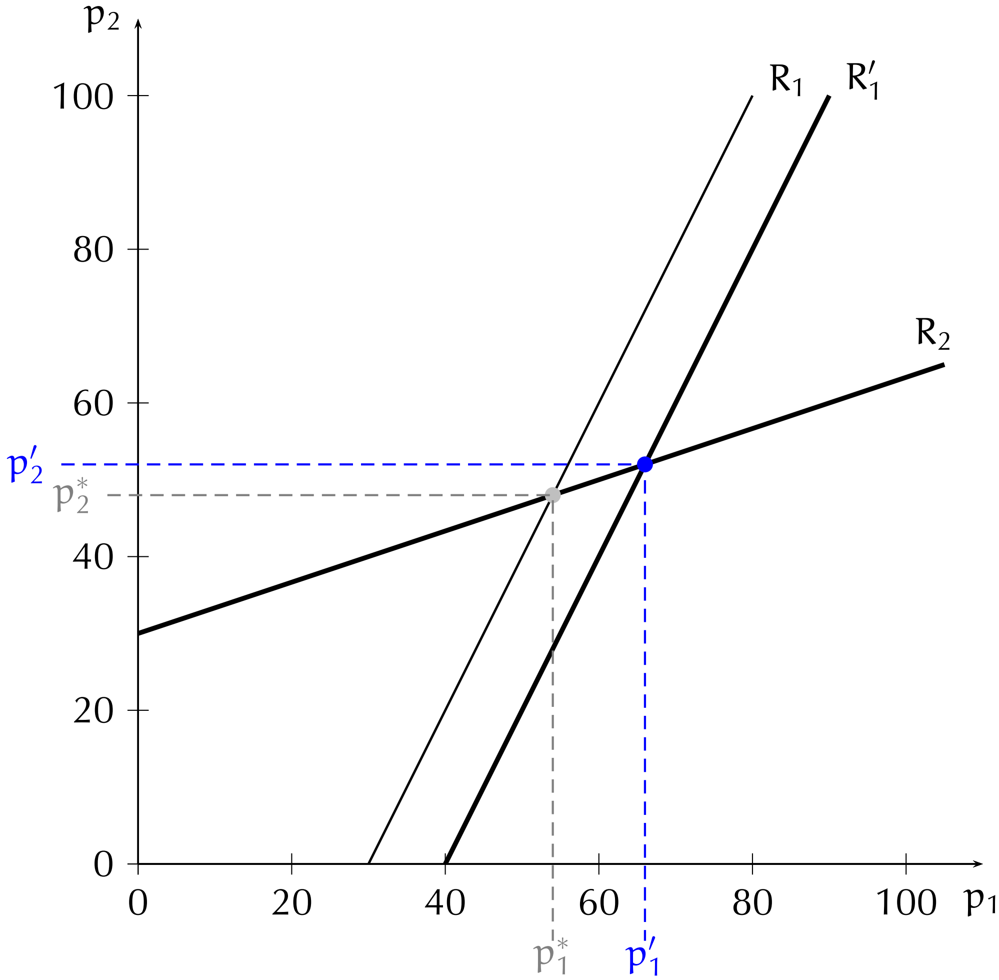
$$p_2 = \frac{a_2 + b_2 c_2 + d_2 p_1}{2b_2}.$$

Increases in marginal cost shift the curve upward.

11.7.6 Increased Cost in the Bertrand Model

Here we graph the effect of a cost increase for firm one. In equilibrium, both firms raise their prices as a result.

Bertrand Equilibrium: Effect of Cost Increase



12.1 Chapter 12: Game Theory

We will look at a number of game theoretic models. This includes revisiting the cartel model, and considering whether a firm can deter entry.

12.1.1 The Coordination Game I

So far, we've examined games with dominant strategies. The coordination game does not have dominant strategies.

Coordination Game: Matrix

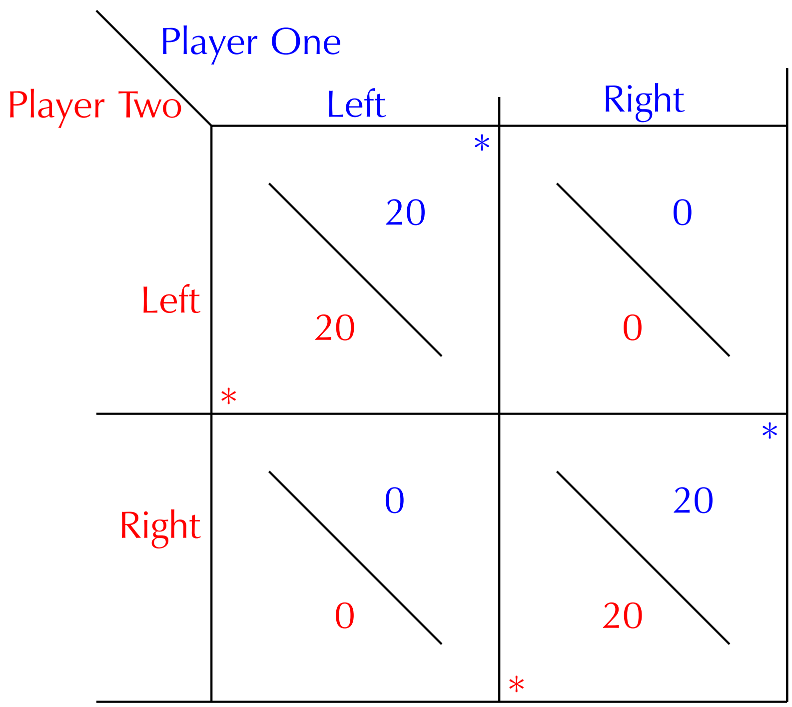
Player Two \ Player One	Left	Right
Left	20 \ 20	0 \ 0
Right	0 \ 0	20 \ 20

We must see what the best responses are.

12.1.2 The Coordination Game I: Best Responses

Player one's best response to **Left** is to choose **Left** (payoff 20) and player one's best response to **Right** is to choose **Right** (payoff 20). The situation is symmetric, and player two's best response are also to copy the other player.

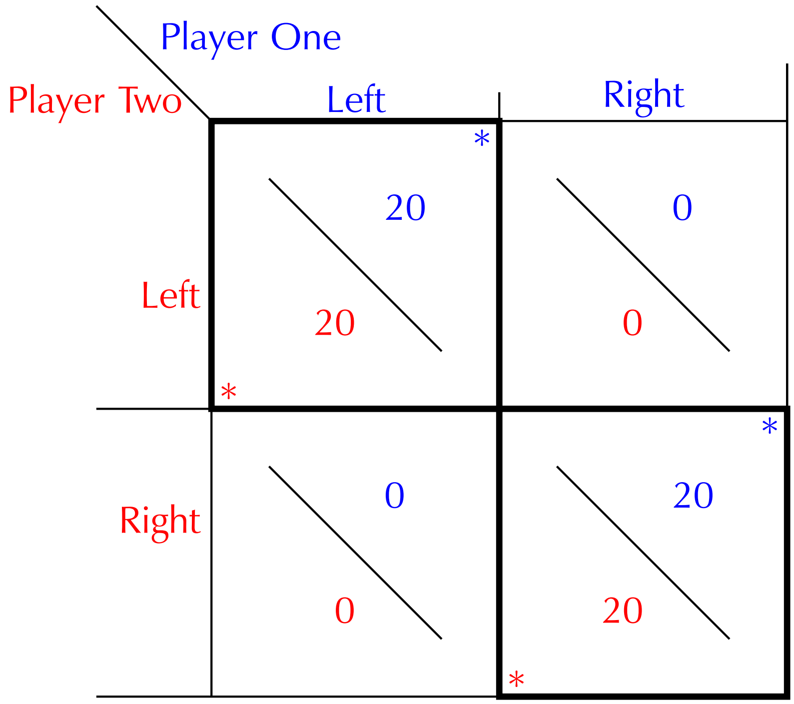
Coordination Game: Best Responses



12.1.3 The Coordination Game I: Nash Equilibria

Because the best response depends on the opponent's move, there are no dominant strategies. There are two Nash equilibria, (L, L) and (R, R).

Coordination Game: Nash Equilibria

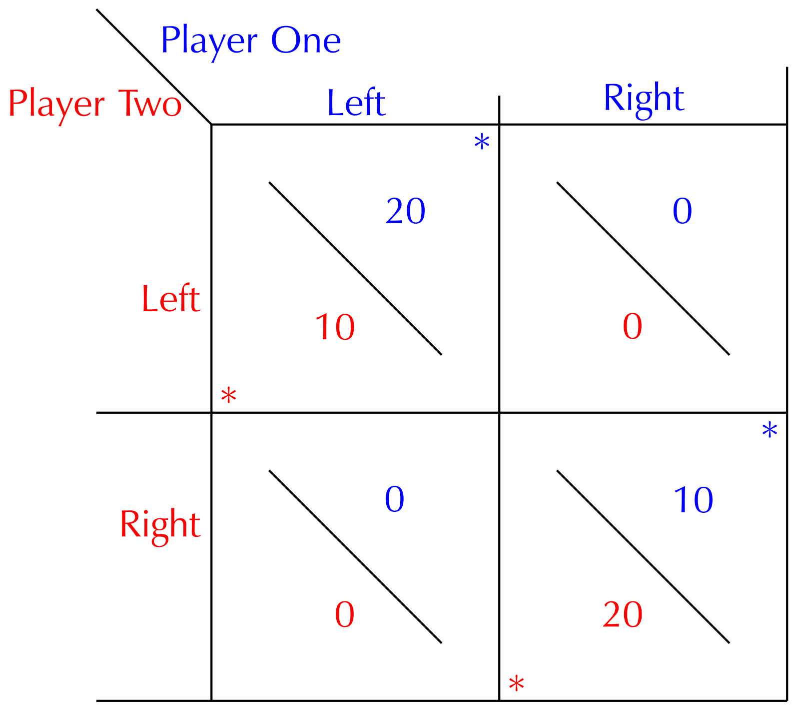


One way to handle the fact that there are two Nash equilibria is to allow the players to communicate prior to play. They can then agree on whether to pick **Left** or **Right**, avoiding any bad outcomes.

12.1.4 The Coordination Game II

Let's change the payoffs in the coordination game a bit. I've added the best responses. They haven't changed, although the payoffs have changed.

Coordination Game II, with Best Responses



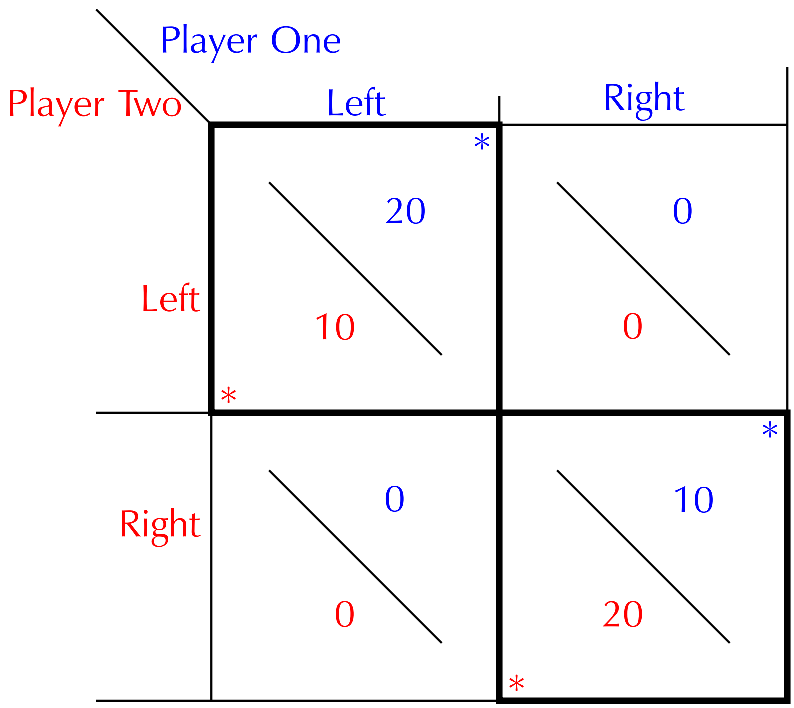
12.1.5 The Coordination Game II: Nash Equilibria

The resulting Nash equilibria are the same, but now the players care which equilibrium they get. In the original version, both players had a payoff of 20 in each equilibrium. Here player one gets 20 in the (L, L) equilibria, but only 10 in the (R, R) equilibria.

The situation is reversed for player two. This creates a conflict if they attempt to decide on an equilibrium ahead of time. Player one prefers (L, L), player two prefers (R, R).

How can they resolve this conflict?

Coordination Game II: Nash Equilibrium



12.1.6 The Coordination Game II: Randomization

One way people deal with such conflicts is to randomize in a fair way. For example, player one flips a fair coin (i.e., that flips to "heads" half the time, and "tails" half the time). Player two calls it. If player two's call is correct, she chooses the equilibrium they coordinate on. If it is tails, player one chooses.

In this augmented version of the game, there is a 50% chance they players will choose (L, L), and a 50% chance they pick (R, R).

Then player one has a 50% chance of getting 20 and a 50% chance of getting payoff 10. Player two is in the exact same situation, with a 50% chance of getting 20 and a 50% chance of getting 10. The situations are exactly the same. That makes it fair.

November 18, 2022