Mathematical Economics Midterm #1, October 2, 2000

You have until 4:45 to complete this exam. Answer all five questions. Each question is worth 20 points, for a total of 100 points. Good luck!

1. (Linear Systems) Consider the linear system

$$x + y + z = 13$$
$$x + 5y + 10z = 61.$$

   Show that this system has solutions. How many solutions are there? Can you find a solution where x, y, and z are all positive integers?

2. (Matrices) Let A and B be $n \times n$ matrices. Show that if $(A+B)^2 = A^2 + 2AB + B^2$, then $AB = BA$.

3. (Determinant) Consider the matrix

$$A = \begin{bmatrix} 3 & -4 \\ -1 & 0 \end{bmatrix}$$

   Find all λ so that $\det(A - \lambda I) = 0$.

4. (Linear Systems) Demand for good 1 is $e_1 - ap_1 + bp_2$; demand for good 2 is $e_2 + cp_1 - dp_2$; the supply of good i is s_i. Here a, b, c, d, e_i, and s_i are all positive, and $s_i > e_i$.

   a) What system of equations do you get when you set supply equal to demand in both markets?

   b) What criterion must be met in order to solve for p_1 and p_2?

   c) What additional conditions must be satisfied in order to get positive equilibrium prices p_i?

5. (Bases) Suppose $\mathbf{x} = (1,3,7)^T$. Consider the basis $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ where $\mathbf{b}_1 = (1,1,0)^T$, $\mathbf{b}_2 = (0,2,4)^T$, and $\mathbf{b}_3 = (0,3,9)^T$. Find the coordinates of $\mathbf{x}$ in the basis $\mathcal{B}$ (i.e., find α_1, α_2, and α_3 with $\alpha_1\mathbf{b}_1 + \alpha_2\mathbf{b}_2 + \alpha_3\mathbf{b}_3 = \mathbf{x}$.