Mathematical Economics Exam #1, September 27, 2016

You have until 6:15 to complete this exam. Answer all four questions. **Be sure to justify your answers!** Each question is worth 25 points, for a total of 100 points. Good luck!

1. Consider the vector $\mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$. The collection $\mathscr{B}$ is defined by

$$\mathscr{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\} = \left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix} \right\}.$$

   a) Show that $\mathscr{B}$ is a basis.

   b) Find the coordinates of $\mathbf{x}$ in the basis $\mathscr{B}$.

2. Consider the sequence defined by $x_n = (-1)^n + 1/n$.

   a) Does this sequence converge? Explain why or why not?

   b) Find a convergent subsequence of $\{x_n\}_{n=0}^{\infty}$

3. Consider the linear system

$$\begin{aligned} w + x + y + z &= 1 \\ w + 2x + 3y + 4z &= 1 \\ w + 4x + 9y + 16z &= 1. \end{aligned}$$

   a) How many solutions does the system have?

   b) Find all solutions to the system.

4. Consider $S = \{(x_1, x_2, x_3) : -1 < x_i < +1 \text{ for } i = 1, 2, 3\}$.

   a) Sketch the set S.

   b) Using the Euclidean norm on $\mathbb{R}^3$, determine whether S is a closed set, open set, both, or neither. Justify your answer.