Mathematical Economics Exam #2, November 1, 2016

You have until 6:15 to complete this exam. Answer all four questions. Be sure to justify your answers! Each question is worth 25 points, for a total of 100 points. Good luck!

1. Consider the problem of maximizing $u(x, y) = x + \sqrt{y}$ subject to the constraints $x, y \geq 0$ and $px + y \leq 10$ where $p > 0$.
   a) Is constraint qualification satisfied?
   b) Find the solution to the maximization problem.

2. Let $f(x, y, z) = x^2 + 5y + z^3$.
   a) Does this function map $\mathbb{R}^3$ onto $\mathbb{R}$?
   b) Find a point $(x_0, y_0, z_0)$ satisfying $f(x_0, y_0, z_0) = 7$.
   c) Given your choice $(x_0, y_0, z_0)$, is there a differentiable function $g(y, z)$ on some neighborhood of $(y_0, z_0)$ that obeys $x_0 = g(y_0, z_0)$ and $f(g(y, z), y, z) = 7$?
   d) Compute $dg$.

3. Let $f(x, y) = xy^2 + x^3y - xy$. Find all critical points of $f$ and classify them (local max, local min, saddlepoint, other/unknown).

4. Consider the quadratic form $Q(x, y, z) = x^2 + 4xy - 2y^2 + 6yz$ with constraint $x + y + z = 0$.
   a) Find a symmetric matrix that defines this quadratic form.
   b) Use the bordered Hessian to determine whether the quadratic form has a constrained maximum, minimum, or saddlepoint at $(0, 0, 0)$?