

## Mathematical Economics Exam #2, November 1, 2016

You have until 6:15 to complete this exam. Answer all four questions. **Be sure to justify your answers!** Each question is worth 25 points, for a total of 100 points. Good luck!

1. Consider the problem of maximizing  $u(x, y) = x + \sqrt{y}$  subject to the constraints  $x, y \geq 0$  and  $px + y \leq 10$  where  $p > 0$ .
  - a) Is constraint qualification satisfied?
  - b) Find the solution to the maximization problem.
2. Let  $f(x, y, z) = x^2 + 5y + z^3$ .
  - a) Does this function map  $\mathbb{R}^3$  onto  $\mathbb{R}$ ?
  - b) Find a point  $(x_0, y_0, z_0)$  satisfying  $f(x_0, y_0, z_0) = 7$ .
  - c) Given your choice  $(x_0, y_0, z_0)$ , is there a differentiable function  $g(y, z)$  on some neighborhood of  $(y_0, z_0)$  that obeys  $x_0 = g(y_0, z_0)$  and  $f(g(y, z), y, z) = 7$ ?
  - d) Compute  $dg$ .
3. Let  $f(x, y) = xy^2 + x^3y - xy$ . Find all critical points of  $f$  and classify them (local max, local min, saddlepoint, other/unknown).
4. Consider the quadratic form  $Q(x, y, z) = x^2 + 4xy - 2y^2 + 6yz$  with constraint  $x + y + z = 0$ .
  - a) Find a symmetric matrix that defines this quadratic form.
  - b) Use the bordered Hessian to determine whether the quadratic form has a constrained maximum, minimum, or saddlepoint at  $(0, 0, 0)$ ?