

Mathematical Economics Exam #1, September 25, 2018

You have until 6:15 to complete this exam. Answer all four questions. **Be sure to justify your answers!** Each question is worth 25 points, for a total of 100 points. Good luck!

1. Consider the following sets in \mathbb{R}^2 .

a) $S_1 = \{\mathbf{x} \in \mathbb{R}^2 : x_1 - x_2 \leq 1, x_1 > 0\}$.

b) $S_2 = B_1(\mathbf{0}) \cup \{\mathbf{x} \in \mathbb{R}^2 : x_2 = 0\}$.

Identify which of the sets are open or closed (if any), and justify your answer. You may find it helpful to sketch the set.

2. Let $\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.

a) Compute the rank of \mathbf{A} .

b) How many vectors \mathbf{x} obey $\mathbf{A}\mathbf{x} = \mathbf{0}$?

c) Does \mathbf{A}^{-1} exist? If so, find it.

3. Define the matrix \mathbf{A} by

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & -1 & 1 \\ 2 & 1 & 0 & 4 \\ 0 & -2 & -4 & -4 \\ 3 & 1 & -1 & 5 \end{pmatrix}$$

and let $V = \{\mathbf{x} : \mathbf{A}\mathbf{x} = \mathbf{0}\}$.

a) Show that V is a vector subspace of \mathbb{R}^4 .

b) Find the dimension of V .

c) Find a basis for V .

4. Consider \mathbb{R}^N with an inner product $\mathbf{x} \cdot \mathbf{y}$. As usual, define the associated norm by $\|\mathbf{x}\| = \sqrt{\mathbf{x} \cdot \mathbf{x}}$. Show that for all $\mathbf{x}, \mathbf{y} \in \mathbb{R}^N$,

$$\mathbf{x} \cdot \mathbf{y} = \frac{1}{4} [\|\mathbf{x} + \mathbf{y}\|^2 - \|\mathbf{x} - \mathbf{y}\|^2].$$