

Mathematical Economics Exam #2, Oct. 30, 2018

You have until 6:15 to complete this exam. Answer all four questions. **Be sure to justify your answers!** Each question is worth 25 points, for a total of 100 points. Good luck!

1. Consider the quadratic form $Q(x) = x_1^2 + 2x_1x_2 + x_2^2 + 2x_2x_3 + x_3^2$. Does Q have a maximum, minimum, or neither at $x = 0$ under the constraint $x_2 + x_3 = 0$.
2. Consider the problem of maximizing $x+y$ under the constraints $2x+y \leq 10$, $x+5y \leq 20$, $x \geq 0$, and $y \geq 0$.
 - a) Sketch the feasible set.
 - b) Is constraint qualification satisfied?
 - c) Find all solutions to the maximization problem.
3. Consider the function $f(x, y) = x^{1/3} + y^{1/4}$ defined on the set $F = \{(x, y) \in \mathbb{R}^2 : x^2 + 15y^2 \leq 20\}$. **Note:** I overlooked the fact that negative values are a problem here. I meant $f(x, y) = |x|^{1/3} + |y|^{1/4}$.
 - a) Show that F is a compact set.
 - b) Show that f has both a maximum and minimum value on F .
4. Let $f(x, y, z) = (x^2 + 4y^2)(z + 1)^2$.
 - a) If $(x, y) = (1, 1/2)$ and $f(x, y, z) = 18$, what values may z take.
 - b) When $(x, y) = (1, 1/2)$, can z be written as a C^1 function of (x, y) near $(1, 1/2)$? Explain.
 - c) If you answered (b) affirmatively, call such a function g and find $dg(1, 1/2)$.