Mathematical Economics Final, December 10, 2019

You have until 6:50 to complete this exam. Answer all five questions. **Be sure to justify your answers!** Each question is worth 20 points, for a total of 100 points. Good luck!

1. On $\mathbb{R}^2$, maximize $3x + y$ under the constraint $x^2 + y^2 \leq 10$. Note that there are no non-negativity constraints. Don't forget to check constraint qualification. What do the second order conditions tell you? Is your solution a maximum?

2. Let $u(x_1, x_2) = x_1^{1/2} + x_2^{1/2}$. Solve the expenditure minimization problem for $\bar{u} \geq 0$ and $\mathbf{p} \gg \mathbf{0}$. Identity the Hicksian demands and find the expenditure function e.

$$\begin{aligned} e(\mathbf{p}, \bar{u}) = \min\ & \mathbf{p}\cdot\mathbf{x} \\ \text{s.t. } & u(\mathbf{x}) \geq \bar{u} \\ & x_1 \geq 0, x_2 \geq 0. \end{aligned}$$

3. Consider the differential equation $\ddot{y} - 2\dot{y} - 3y = 3t^2$ with initial conditions $y(0) = -22/9$ and $\dot{y}(0) = 4/3$.

   a) Find the general solution of the associated homogeneous equation.

   b) Find a particular solution of the inhomogeneous equation.

   c) Find the solution that obeys the initial conditions.

4. Let $f(x, y) = (xy)^{1/2}$. Is f concave on $\mathbb{R}^2_{++}$?

5. Let $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} - \mathbf{b}$ where

$$\mathbf{A} = \begin{bmatrix} 3 & 3 & 1 \\ 4 & 4 & 3 \\ 1 & 1 & 2 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}.$$

   Find all vectors $\bar{\mathbf{x}}$ that are steady states of this differential system.