1. Consider the set
\[ \mathcal{B} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix} \]

a) Is \( \mathcal{B} \) a basis?

b) If \( \mathcal{B} \) is a basis, find the coordinates of the vector \( \mathbf{x} = (1, 2, 3)^T \) in the basis \( \mathcal{B} \). If \( \mathcal{B} \) is not a basis, find a non-zero vector that is not in the span of \( \mathcal{B} \).

2. Let \( \mathbf{A} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \).

a) Compute the rank of \( \mathbf{A} \).

b) What is the dimension of the nullspace (kernel) of \( \mathbf{A} \), \( \{ \mathbf{x} : \mathbf{A}\mathbf{x} = \mathbf{0} \} \)?

c) Find a basis for the nullspace of \( \mathbf{A} \).

3. Consider the following sets:
\[ S = \bigcup_{n=1}^{\infty} (-n, 0), \quad T = \bigcap_{n=1}^{\infty} (-n, +n). \]

a) Is \( S \) an open set? Explain

b) Is \( T \) open? Closed? Explain.

4. Consider the sequence
\[ x_n = \left\{ (-1)^n \frac{n + 1}{n + 3} \right\}_{n=1}^{\infty}. \]

Does this sequence converge? If so, find the limit. If not, does it have any cluster points. Explain.