

## Mathematical Economics Exam #2, November 7, 2019

You have until 6:15 to complete this exam. Answer all four questions. **Be sure to justify your answers!** Each question is worth 25 points, for a total of 100 points. Good luck!

1. Consider the problem of maximizing  $u(x, y) = x - e^{-y}$  subject to the constraints  $x, y \geq 0$  and  $px + y \leq 10$  where  $p > 0$ .
  - a) Is constraint qualification satisfied?
  - b) Set up the Lagrangian for this problem and find the first-order conditions.
  - c) Now find the solution to the maximization problem.
2. Let  $A = \{(x, y) : x^2 + y^2 \geq 1, x^2 + y^2 \leq 5\}$ .
  - a) Is  $A$  closed? Explain.
  - b) Is  $A$  bounded? Explain.
  - c) Is  $A$  compact? Explain.
  - d) Is  $A$  connected? Explain.
3. Let  $f(x, y) = xy^2 + x^3y - 2xy$ . Find all critical points of  $f$  and classify them (local max, local min, saddlepoint, other/unknown).
4. Consider the quadratic form  $Q(x, y, z) = -x^2 + xy + y^2 + yz + z^2/2$  with constraint  $x + y + z = 0$ .
  - a) Find a symmetric matrix that defines this quadratic form.
  - b) Use the bordered Hessian to determine whether the quadratic form has a constrained maximum, minimum, or saddlepoint at  $(0, 0, 0)$ ?