Mathematical Economics Final, December 10, 2020

Answer all five questions. You may use any reasonable shortcuts, and may refer to your notes, my notes, and the textbook. No other reference materials may be used. You many not consult any individuals or search the internet.

You must email me your answers by 7:00pm today (December 10, 2020). To insure maximum credit, **be sure to explain your answers**. Each question is worth 30 points, for a total of 150 points. The questions are not equally hard. Good luck!

1. On $\mathbb{R}^2$, maximize $3x + 5y$ under the constraints $2x + y \le m$, $x \ge 0$ and $y \ge 0$ where $m > 0$. Don't forget to check constraint qualification. Interpret the multiplier for the constraint $2x + y \le m$.
2. Let $u(x_1, x_2) = x_1^2 + x_2^2$. Solve the expenditure minimization problem for $\bar{u} > 0$ and $\mathbf{p} \gg \mathbf{0}$. Compute both the Hicksian demands and the expenditure function e.

$$\begin{aligned} e(\mathbf{p}, \bar{u}) = \min\ & \mathbf{p}\cdot\mathbf{x} \\ \text{s.t. } & u(\mathbf{x}) \ge \bar{u} \\ & x_1 \ge 0, x_2 \ge 0. \end{aligned}$$

3. Consider the differential equation $2\ddot{y} - 3\dot{y} + y = te^t$ with initial conditions $y(0) = 0$ and $\dot{y}(0) = -1$.
   a) Find the general solution of the associated homogeneous equation.
   b) Find a particular solution of the inhomogeneous equation.
   c) Find the solution that obeys the initial conditions.
4. Let $f(x, y) = (xy)^{12}$.
   a) Either show that f is quasiconcave on $\mathbb{R}^2_{++}$ or show that it is not quasiconcave.
   b) Is f homogeneous? If so, what is its the degree of homogeneity. If not, show that it is not homogeneous.
5. Let $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$. Find $\mathbf{A}^{1/2}$